International Portfolios with Nominal Rigidities and Capital Accumulation

Ebrahim Rahbari *

London Business School

May 20, 2009

Abstract

We analyse the portfolio implications of open economy models, paying particular attention to the role of sticky prices and capital accumulation. Equities and bonds are used to hedge fluctuations in the real exchange rate and human capital returns. Robust and home biased portfolios only obtain if bonds are used to hedge real exchange rate risk, while equities hedge human capital risk. With endogenous capital accumulation, domestic equities are a good hedge for real exchange risk, while nominal bond returns are closely related to the nominal, but not necessarily the real exchange rates. As a result, bond and equity portfolios are excessive and unstable in a wide class of models. Introducing capital adjustment costs improves the performance of the model, sometimes substantially, but still falls short of accounting for observed positions. The empirical analysis finds some support for a hedging explanation of equity home bias and suggests that the failure of the model may stem from its inability to produce realistic behaviour of equity returns and real exchange rates.

*Mailing address: London Business School, Department of Economics, Regent's Park, London, NW1 4SA, United Kingdom. Email: erahbari.phd2004[at]london.edu. The author is grateful to Francisco Gomes, Francesca Monti, Anna Pavlova, Fabrizio Perri, Andrew Scott, Cedric Tille and especially Nicolas Coeurdacier and Helene Rey for helpful comments.
Keywords: International Bond and Equity Portfolios, Capital Accumulation, Nominal Rigidities
JEL Codes: F21, F30, G11, F41

1 Introduction

Recent decades have seen substantial increases in international gross asset positions and capital flows (see e.g. Lane and Milesi Feretti (2001, 2007)). Given the large size of the outstanding positions, changes in valuations of these assets can potentially have a large effect on macroeconomic outcomes. However, until recently, economists were not able to shed much light on the determinants of these asset positions, as standard open economy models contained very simple portfolio decisions, if any at all. What is more, stylised facts on the nature of international asset positions were thin on the ground. It is therefore not surprising that the literature has mostly focussed on trying to explain "equity home bias", the well documented fact that a large fraction of domestic equity is held by domestic owners (see French and Poterba (1991) and Table 1). This fact has been called a bias, as the basic CAPM type optimising model of international portfolio choice would imply full diversification (Lucas (1982)), suggesting that all agents hold the same risky portfolio with the share of domestic equity held equal to the share of the country in the world market portfolio. Many explanations have since been put forward to account for the observed bias, including frictions in the trading of goods or assets or informational frictions (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpee (2007) for surveys of the literature). Another strand of the literature, however, has argued that once we go beyond the basic models that have been used to study international asset positions and introduce features which have been shown to be important in macroeconomic analyses of closed economies, equity home bias can be explained in the context of standard macroeconomic models, without introducing additional frictions that pertain only to an international dimension. In most of these models, asset positions can be understood as positions derived from an explicit diversification motive, and positions driven by motives to hedge against consumption expenditure risk and risk arising from nontradable income, most notably human capital[1]. Thus, Heathcote and Perri (2008) show that simply introducing capital accumulation is enough to generate home bias

[1] In many settings, consumption expenditure and the real exchange are perfectly correlated and in those cases we will use the latter expression as a synonym for the former.

Table 1

Country	Equity Market Cap as % of World Market Cap	% domestic in total equity	Home Bias
Canada	3.5	76.6	73.1
France	4.2	68.8	64.6
Germany	2.9	57.5	54.6
Italy	1.9	57.1	55.2
Japan	13.2	91.9	78.7
UK	7.3	65.0	67.7
US	40.5	82.2	41.7

Source: CPIS, December 2005

in equity positions, as the response of investment implies that domestic equity holdings are good hedges against consumption expenditure and human capital risk. However, in their model, portfolios are also very sensitive to changes in parameters and portfolios generally exhibit excessive home bias for realistic parameter values.

In all of the work discussed above, the portfolio decision of agents essentially only consists of choosing the share of domestic equity held. A major benefit of more recent work has been to allow for a more extensive menu of assets[2]. Thus, Coeurdacier et al. (2008) and Engel and Matsumoto (2008) develop models which are able to generate relatively realistic degrees of equity home bias and robust equity and bond portfolios. In the former paper, the key innovation is to allow for trade in real bonds in addition to capital accumulation, while the latter paper introduces sticky prices and trade in foreign exchange forwards, while abstracting from capital accumulation. In both papers, the basic mechanism that generates equity home bias and robust portfolios is similar and consists of two distinct features. Firstly, equities are good at hedging human capital risk, due to the possibility of capital accumulation in Coeuracier et al (2009) and due to sticky prices in Engel and Matsumoto (2009). Secondly, bonds are used to hedge real exchange rate risk. This is because the real bonds traded in Coeurdacier et al (2009) are perfectly correlated with real exchange rate risk, while in Engel and Matsumoto (2008), equities are a poor hedge for real exchange rate

[2]Devereux and Sutherland (2007a, 2008a) and Tille and van Wincoop (2008) present an essentially identical solution method that allows a general class of open economy model with multiple assets to be solved using standard algorithms, while Devereux and Saito (2005), Evans and Hnatkovska (2006) and Judd et al. (2002) describe alternative solution approaches.

risk.

We argue that a realistic macroeconomic model of international portfolio choice should feature both endogenous investment, trade in nominal bonds and endogenous monetary policy. Models which allow for capital accumulation are, of course, standard in closed economy business cycle analysis, going back at least to the seminal contributions of Kydland and Prescott (1982) and Long and Plosser (1983), owing to the fact that the volatility of investment accounts for a significant fraction of the volatility of output. For open economies, there are at least two additional reasons that call for the inclusion of investment. Firstly, one of the key functions of the international capital market is to finance international investment. Secondly, net exports are to found to be strongly countercyclical and investment is key in generating this correlation, both theoretically and in the data (Backus et al (1992, 1994)). Nominal rigidities and Taylor type monetary policy rules are by now the standard way to introduce real effects of monetary policy into macroeconomic models (see Gali (2008) and Woodford (2003) for introductions). What is more, Devereux and Sutherland (2007) have shown that monetary policy affects asset returns and positions in many settings. Finally, the bulk of bond issuance is in nominal bonds.

We find that once we allow for both capital accumulation and some nominal risk in bond trading, the portfolio implications of the models are often overturned. In particular, we find that equity home bias is often excessively large or negative and that bond positions can vary from large positive to large negative positions in domestic bonds. More generally, portfolios are unstable and sensitive to changes in parameters. The reason is twofold: Firstly, with endogenous investment, domestic equities are often a good hedge for real exchange rate risk: As in Heathcote and Perri (2008) and Coeurdacier et al (2008), equity returns tend to be high when the real exchange rate appreciates. Secondly, nominal bonds are usually not much better at hedging real exchange rate risk, and often worse, due to nominal risk. Equity returns and bond returns are frequently highly correlated, implying that both equity and bond positions are heavily affected by the desires to hedge real exchange rate risk and generating extreme positions. Our benchmark model features productivity and interest rate shocks, local currency pricing, trade in equities and one period nominal bonds and a central bank that is moderately aggressive in targeting CPI inflation. However, as we noted above, the instability of portfolios is primarily due to the response of investment and the result is thus remarkably robust, spanning most of the plausible parameter space, as well as allowing for trade in long term bonds, investment shocks, producer currency pricing or different monetary policy reaction functions.

In Engel and Matsumoto (2008a), sticky prices and the absence of capital accumulation imply that domestic equities are a good hedge for human capital risk, but not real exchange rate risk. In response to productivity shocks, nominal rigidities imply that firms cannot lower prices to stimulate demand to the same extent as with flexible prices. As a consequence, labour demand falls and, with a sufficient degree of price rigidity, so do human capital returns. Firm relative profits rise due to higher productivity and lower wages and so do relative equity returns. Both real and nominal exchange rates initially depreciate, implying that relative bond returns are negative. In response to a positive shocks to the Home nominal interest rate, relative equity returns respond negatively, while human capital returns rise. The real and nominal exchange rate appreciate, so that relative bond returns rise. Thus, long positions in domestic bonds can hedge real exchange rate risk in response to both sources of uncertainty, while long positions in domestic equity hedge human capital risk.

In models with a strong investment response, such as Heathcote and Perri (2008), Coeurdacier et al (2008), or our benchmark case, both long positions in domestic bonds and domestic equities can be a good hedge for both human capital and real exchange rate risk. The reason is that, now, in response to a Home productivity shock, relative dividends and relative equity returns fall, as higher productivity induces high investment which is paid out of dividends. Note that, while *relative* equity returns fall, domestic equity returns still rise in *absolute* terms, but foreign equity returns rise even more, as higher domestic output in the future reduces the relative price of the Home good. Home bias in investment implies that the rise in investment increases the demand for Home labour, and ultimately, relative labour income, while the behaviour of real and nominal exchange rates is qualitatively unchanged. Thus, domestic equity pays high returns, when the real exchange rate appreciates, and so do domestic bonds. However, there is a second reason that portfolios are unstable. In our setup, the bonds traded are one period nominal bonds. Thus, unlike in Coeurdacier et al. (2008), relative bond returns are not perfectly correlated with real exchange rate risk for two reasons. Firstly, relative bond returns are related to the *nominal* exchange rate, not the real exchange rate. Secondly, bonds only have a one payoff, while real exchange rate risk depends on the infinite future series of real exchange rate movements. Since bonds are not a perfect hedge for real exchange rate risk, equity positions will in general also be affected by the motive to hedge fluctuations in real exchange rate, and we show that this effect is large in virtually all settings. However, one may speculate that increasing the maturity of the bonds would remove at least one source of the difference between nominal bond returns and the real exchange rate. However, we show that this is not the case, and that long term bonds are often less closely correlated with real exchange rate risk than short term bonds. This is because, as we look further into the future, prices become more and more flexible and the relation between real and nominal exchange rates becomes weaker. Increases in the degree of price rigidity and monetary policy effectiveness do increase the association between nominal and real exchange rates and thus the hedging properties of bonds, but only unrealistically high values of these parameters increase this relation enough to generate robustly home biased portfolios.

Above we described that the strong response of investment complicates the portfolio decisions of agents. Reducing the size of the response of investment could thus present one way to improve the portfolio implications of these models. We show that this is indeed the case. Adding capital adjustment costs to the model reduces the response of investment and improves the portfolio implications across all specifications, often substantially. In response to productivity shocks, the reduced response of investment implies that relative equity returns now rise. Crucially, the reduced investment response continues to imply that relative labour income moves in the opposite direction, and home equity thus remains a good hedge for human capital risk. However, as in the model with sticky prices, equities are no longer a good hedge for human capital, while nominal bonds are, inducing optimal portfolios to be robust and exhibiting home bias in equity positions.

In our empirical analysis, we use data on G7 countries to estimate the relevant covariances of equity and bond returns with human capital and real exchange rate risk. We find that these hedging demands can in fact explain equity home bias in the US, Japan, and to some extent the UK, but not in other countries. Equity home bias is in all countries driven by the motive to hedge human capital risk and not real exchange rate risk, confiming some predictions of the model. This empirical exercise also highlights two areas, where the model and data are quite far apart: Firstly, equity return innovations are not volatile enough and too correlated with the sources of risk in our model. Secondly, real exchange rate risk in the model is too large.

2 Model

There are two symmetric countries, Home (H) and Foreign (F), indexed by i. Each country is specialised in the production of a composite good using a continuum of country specific intermediate goods. Intermediate goods are produced using labour and capital. The factors of production and the intermediate goods are immobile between countries, but composite goods are traded. In addition to trade in composite goods, countries trade nominal bonds and equities.

2.1 Households

Country i is inhabited by a representative consumer with a utility function that is separable in consumption and labour:

$$U_i = \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{i,t+j}^{1-\sigma}}{1-\sigma} - \iota \frac{L_{i,t+j}^{1+\omega}}{1+\omega} \right), \tag{1}$$

where $C_{i,t}$ is the consumption aggregator of country i, and $P_{C,t}^i$ is the consumption price index. The discount rate β, the intertemporal elasticity of substitution σ, the Frisch labour supply elasticity ω and the parameter ι governing labour supply in the steady state are common across countries. The consumption aggregator for country i is defined as:

$$C_{i,t} = \left[a_C^{1/\phi} \left(C_{i,t}^i \right)^{\frac{\phi-1}{\phi}} + (1-a_C)^{1/\phi} \left(C_{j,t}^i \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{2}$$

where $a_C > 1/2$ is the consumption home bias parameter and ϕ is the elasticity of substitution between Home and Foreign goods. These preferences imply the following consumption price indices:

$$P_{C,t}^i = \left[a_C \left(P_{i,t}^i \right)^{1-\phi} + (1-a_C) \left(P_{j,t}^i \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{3}$$

where $P_{j,t}^i$ is the price in country i of the composite good produced in country j. All prices are quoted in terms of the local currency. The optimal allocation across consumption goods is then given by:

$$C_{i,t}^i = a_C \left(\frac{P_{i,t}^i}{P_{C,t}^i} \right)^{-\phi} C_{i,t} \qquad C_{j,t}^i = (1-a_C) \left(\frac{P_{j,t}^i}{P_{C,t}^i} \right)^{-\phi} C_{i,t} \tag{4}$$

where $C_{j,t}^i$ denotes consumption of good j by agent i. The first order conditions for labour supply are then:

$$\iota l_{i,t}^{\omega} = \left(\frac{W_{i,t}}{P_{C,t}^i} \right) C_{i,t}^{-\sigma}, \tag{5}$$

which describe the standard condition that the marginal disutility of labour today has to equal the marginal utility of consumption times the real wage.

Consumption of the Home and Foreign aggregate consumption goods are given by:

$$C^i_{j,t} = \left(\int_0^1 \left(C^i_{j,t}\left(k\right) \right)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $C^i_{j,t}\left(k\right)$ is consumption of the kth intermediate good in country j by the agent in country i and ε is the elasticity of substitution between varieties in consumption. Optimal consumption of Home and Foreign intermediate goods then implies:

$$C^i_{j,t}\left(k\right) = \left(\frac{P^i_{j,t}}{P^i_{j,t}\left(k\right)} \right)^{\varepsilon} C^i_{j,t}, \tag{7}$$

where $P^i_{j,t}$ is the price in country i of the good produced in country j. The price indices for Home and Foreign composite goods are:

$$P^i_{j,t} = \left(\int_0^1 \left(P^i_{j,t}\left(k\right) \right)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}}, \tag{8}$$

and the real exchange rate Q is defined as:

$$Q_t = \frac{Z_t P^F_{C,t}}{P^H_{C,t}} \tag{9}$$

2.2 Capital Accumulation

At time t, each country possesses a capital stock $K_{i,t}$. Country specific capital stocks depreciate at rate δ and are augmented by country specific investment $I_{i,t}$:

$$K_{i,t+1} = \left(1-\delta\right) K_{i,t+1} + I_{i,t}, \tag{10}$$

where investment in country $i's$ capital stock is a CES aggregate of the Home and Foreign composite good:

$$I_{i,t} = \left[a_I^{1/\phi}\left(I_{i,t}^i\right)^{\frac{\phi-1}{\phi}} + \left(1-a_I\right)^{1/\phi}\left(I_{j,t}^i\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{11}$$

where $a_I > 1/2$ signifies local bias in investment. To save on notation, we assume that the elasticity of substitution between Home and Foreign goods in investment is the same as for consumption. The Home and Foreign composite investment goods are produced using by a CES composite of Home and Foreign varieties:

$$I_{j,t}^i = \left(\int_0^1 \left(I_{j,t}^i\left(k\right)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{12}$$

where, again, we assume that substitution elasticities for investment are equal to those for investment. We do not distinguish between the elasticities of substitution in consumption and investment to save on notation. Demand for investment is then given by:

$$I_{j,t}^i\left(k\right) = \left(\frac{P_{j,t}^i}{P_{j,t}^i\left(k\right)}\right)^{\varepsilon} I_{j,t}^i, \tag{13}$$

where the price indices for Home and Foreign goods are as described above.

At time t, the stock of capital in each country is rented out in a spot market at a rental rate $R_{i,t}^K$. Investment is then chosen to equate the expected discounted payoff from investment to its marginal cost at time t:

$$1 = \beta E_t\left[\varpi_{t,t+1}^i \frac{1}{P_{I,t}^i}\left(R_{i,t}^K + \left(1-\delta\right)P_{I,t+1}^i\right)\right], \tag{14}$$

where $\varpi_{t,t+1}^i$ is the stochastic discount factors applied at time t to discount date $t+1$ profits.

2.3 Firms

Each country contains a continuum of firms, each producing a differentiated intermediate good, indexed by k, using capital and labour. The production function is given by:

$$Y_{i,t}(k) = A_{i,t}(L_{i,t}(k))^{\alpha}(K_{i,t}(k))^{1-\alpha}, \tag{15}$$

where $A_{i,t}$ is the exogenous level of productivity in country i and α is the elasticity of output with respect to labour. Firms maximise the present discounted value of profits and choose labour and capital in aggregate country specific spot markets to minimise the cost of production. Consequently, the labour to capital ratio is the same for all firms in a given country:

$$\frac{K_{i,t}(k)}{L_{i,t}(k)} = \frac{1-\alpha}{\alpha}\frac{W_{i,t}}{R_{i,t}^{K}}. \tag{16}$$

Total demand for each intermediate good is composed of domestic and foreign demand for consumption and investment. Firms take this demand as given and choose prices in local currency in Home and Foreign to maximise profits. However, they can only reset prices with a probability of $1-\theta$ every period. A firm reoptimising in period t will then choose a price $\widetilde{P}_{i,t}^{j}$ that maximises the current market value of profits generated while the price remains in effective, taking all other prices as given:

$$\begin{aligned}
\widetilde{P}_{H,t}^{H}(k) &= \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{H}\varrho_{H,t}\left(P_{H,t+l}^{H}\right)^{\varepsilon}Y_{H,t+l}^{H}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{H}\left(P_{H,t+l}^{H}\right)^{\varepsilon}Y_{H,t+l}^{H}} && (17)\\
\widetilde{P}_{H,t}^{F}(k) &= \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{H}\left(P_{H,t+l}^{F}\right)^{\varepsilon}\varrho_{H,t}Y_{H,t+l}^{F}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{H}S_{t+l}\left(P_{H,t+l}^{F}\right)^{\varepsilon}Y_{H,t+l}^{F}} && (18)\\
\widetilde{P}_{F,t}^{H}(k) &= \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{F}\left(P_{F,t+l}^{H}\right)^{\varepsilon}\varrho_{F,t}Y_{F,t+l}^{H}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{F}\frac{1}{S_{t+l}}\left(P_{F,t+l}^{H}\right)^{\varepsilon}Y_{F,t+l}^{H}} && (19)\\
\widetilde{P}_{F,t}^{F}(k) &= \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{F}\left(P_{F,t+l}^{F}\right)^{\varepsilon}\varrho_{F,t}Y_{F,t+l}^{F}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi_{t,t+l}^{F}\left(P_{F,t+l}^{F}\right)^{\varepsilon}Y_{F,t+l}^{F}}, && (20)
\end{aligned}$$

where $Y_{i,t}^{i}(k) = C_{i,t}^{i}(k)+I_{i,t}^{i}(k)$, $Y_{i,t}^{j}(k) = C_{i,t}^{j}(k)+I_{i,t}^{j}(k)$ and $\Theta_i(\cdot)$ is the total cost function of a firm in country i. As usual, optimal prices thus depend on discounted marginal costs over the expected lifetime of the price set. The nature of price rigidities implies that Home and Foreign goods prices evolve according to:

$$P_{j,t}^{i} = \left(\theta \left(P_{j,t-1}^{i} \right)^{1-\varepsilon} + (1-\theta) \left(\widetilde{P}_{j,t}^{i} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{21}$$

2.4 Financial Markets

There is trade in Home and Foreign one period nominal bonds and Home and Foreign equity. Nominal bonds pay one unit of the domestic currency in each period and are in net zero supply. Owners of equity of country i receive a claim to country i dividends $D_{i,t}$ which are composed of profits earned by the firm plus the rent earned for using capital, minus investment spending:

$$D_{i,t} = \Pi_{i,t} + R_{i,t}^{K} K_{i,t} - P_{I,t}^{i} I_{i,t}$$

The gross nominal returns in domestic currency for bonds and equity are:

$$R_{i,t+1}^{S} = \frac{D_{i,t} + P_{i,t+1}^{S}}{P_{i,t}^{S}} \qquad R_{i,t+1}^{B} = \frac{1}{P_{i,t}^{B}} \tag{22}$$

where $R_{i,t}^{S}$ $\left(R_{i,t}^{B}\right)$ is the return on holdings of country i equity (bonds) and $P_{i,t}^{S}$ $\left(P_{i,t}^{B}\right)$ are the prices of country i equity (bonds). The total supply of equity in each country is normalised to unity. It is assumed that in period 0 each household owns the stock of domestic equity and that bond positions are zero. The budget constraint for Home in period t is then given by:

$$\begin{aligned} & S_{H,t}^{H} P_{H,t}^{S} + S_{F,t}^{H} P_{F,t}^{S} S_t + B_{H,t}^{H} P_{H,t}^{B} + B_{F,t}^{H} P_{H,t}^{B} S_t \\ = \; & W_{H,t} L_{H,t} + S_{H,t-1}^{H} \left(D_{H,t} + P_{H,t}^{S} \right) + S_{F,t-1}^{H} \left(D_{F,t} + P_{F,t}^{S} \right) S_t \\ & + B_{H,t-1}^{H} + B_{F,t-1}^{H} S_t - P_{C,t}^{H} C_{H,t}, \end{aligned} \tag{23}$$

where $S_{j,t}^{i}$ $\left(B_{j,t}^{i}\right)$ are holdings of country j equity (bonds) by country i in period t. The budget constraint of the Foreign agent can be written analogously.

2.5 Market Clearing

The market clearing condition for goods are given by:

$$
\begin{aligned}
C_{H,t}^{H} + C_{H,t}^{F} + I_{H,t}^{H} + I_{H,t}^{F} &= Y_{H,t} && (24) \\
C_{F,t}^{H} + C_{F,t}^{F} + I_{F,t}^{H} + I_{F,t}^{F} &= Y_{F,t} && (25)
\end{aligned}
$$

For assets, we have:

$$
\begin{aligned}
S_{H,t}^{H} + S_{H,t}^{F} &= 1 \qquad S_{F,t}^{H} + S_{F,t}^{F} = 1 && (26) \\
B_{H,t}^{H} + B_{H,t}^{F} &= 0 \qquad B_{F,t}^{H} + B_{F,t}^{F} = 0 && (27)
\end{aligned}
$$

2.6 Monetary Authorities

We assume that the monetary authority sets the nominal interest rate according to a simple rule that is subject to shocks. In Devereux and Sutherland (2008), these shocks are interpreted as financial market shocks, but as in their paper, the role of these shocks more generally here is to introduce shocks to domestic inflation rates which are not related to productivity. Devereux and Sutherland (2007) also provide a more extensive discussion of the role of the monetary policy rule. In our benchmark case, monetary policy is set according to:

$$
R_{i,t}^{N} = \frac{1}{\beta} \left(\frac{P_{C,t}^{i}}{P_{C,t-1}^{i}} \right)^{\gamma} \exp\left(m_{i,t}\right), \tag{28}
$$

where γ determines the monetary policy responsiveness to inflation and m_t is a mean zero shock to interest rates. It is worth pointing out that, while in Devereux and Sutherland (2008), firms set prices according to producer currency pricing and the central bank stabilises producer price inflation rates, we assume that prices are set according to local currency pricing and the central bank stabilises consumer price inflation.

2.7 Exogenous Processes

There are two sources of uncertainty per country. Productivity shocks in each country evolve according to:

$$\log(A_{i,t+1}) = \rho_A \log(A_{i,t}) + \varepsilon_{A,i,t+1}, \tag{29}$$

where $\varepsilon_{A,i,t+1}$ is a mean zero shock to productivity, and ρ_A governs the persistence of productivity. The interest rate shock is represented by:

$$m_{i,t+1} = \rho_M m_{i,t} + \varepsilon_{M,i,t+1} \tag{30}$$

where $\varepsilon_{M,i,t+1}$ is a mean zero shock to interest rates, and ρ_M governs the persistence of interest rates.

2.8 Solution Method

We solve for a linear approximation to the equilibrium of this model around the nonstochastic steady state of this model, applying the methods developed by Devereux and Sutherland (2008). This method is based on the insight that the first order dynamics of the system only depends on the steady state portfolio. In turn it can be shown that the steady state portfolio depends on the first order dynamics of the non portfolio equations of the model combined with a second order approximation of the portfolio equations.

3 Relative Asset Returns and Hedging Demands

In this section, we show that it is possible to relate optimal asset holdings in reduced form to parts driven by motives to hedge real exchange rate and human capital risk. Firstly, note that all assets are equivalent to a first order:

$$E_t\left[\widehat{R}^H_{S,t+j}\right] = E_t\left[\widehat{R}^H_{B,t+j}\right] = E_t\left[\widehat{R}^F_{S,t+j}\right] = E_t\left[\widehat{R}^F_{B,t+j}\right] = \frac{1}{\beta}, j \geq 1,$$

implying zero linearised return differentials in expectation. As mentioned above, our solution approach uses second order approximations to the portfolio Euler equations and can thus arrive at optimal portfolios, despite the first order equivalence. Secondly, *realised* relative returns in terms of Home currency for Home one period nominal bonds are, in linearised form:

$$\widehat{R}_t^B = \widehat{R}_{H,t}^B - \widehat{R}_{F,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1} = \widetilde{E}_t \left[-\widehat{Z}_t \right], \tag{31}$$

where $\widetilde{E}_t = E_t - E_{t-1}$ [3]. Thus, Home bonds will have a positive relative payoff when the Home currency unexpectedly appreciates, while they will offer a negative relative payoff if the Home currency unexpectedly depreciates. Relative linearised equity returns are determined by the sum of the discounted expected future relative dividends:

$$\widehat{R}_t^S = \widehat{R}_{F,t}^S - \widehat{R}_{H,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\, \widetilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{Z}_{t+j} \right) \right], \tag{32}$$

The linearised stochastic discount factors for Home and Foreign households are $-\sigma \left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t} \right) - \left(\widehat{P}_{C,t+1}^H - \widehat{P}_{C,t}^H \right)$ and $-\sigma \left(\widehat{C}_{F,t+1} - \widehat{C}_{F,t} \right) - \left(\widehat{P}_{C,t+1}^F - \widehat{P}_{C,t}^F \right)$. When markets are complete, the stochastic discount factors are equalised between Home and Foreign agents in all states of nature. With incomplete markets, however, the stochastic discount factors will in general differ between different investors. However, as is shown in the appendix, the stochastic discount factor of the Home and the Foreign agent are identical at this level of approximation, once we express them in the same units:

$$\begin{aligned} & E_t \left[-\sigma \left(\widehat{C}_{H,t+1} - \widehat{C}_{H,t} \right) - \left(\widehat{P}_{C,t+1}^H - \widehat{P}_{C,t}^H \right) \right] && (33) \\ = \; & E_t \left[-\sigma \left(\widehat{C}_{F,t+1} - \widehat{C}_{F,t} \right) - \left(\widehat{P}_{C,t+1}^F - \widehat{P}_{C,t}^F \right) \right] + \widehat{Z}_t - \widehat{Z}_{t+1}. && (34) \end{aligned}$$

In our benchmark model, there are two sources of uncertainty and two assets per country. With zero initial net foreign asset positions, it can then be shown that markets are complete

[3] Since assets are equivalent to a first order, relative returns and relative return innovations are identical and we will use both terms interchangeably.

to a first order approximation and we have perfect risk sharing. This means that, to a first order approximation, the decentralised equilibrium could be found as a solution to a planner's problem with the appropriate welfare weights. It also implies that the Backus-Smith-Kollmann (BSK) condition holds in linearised form:

$$-\sigma\left(\widehat{C}_{H,t}-\widehat{C}_{F,t}\right)=\widehat{P}^{H}_{C,t}-\widehat{S}_{t}-\widehat{P}^{F}_{C,t}=-\widehat{Q}_{t}. \tag{35}$$

Thus, relative consumption is perfectly correlated with the real exchange rate. Note that unlike in closed economies, complete markets do not imply that the marginal utility of consumption of different agents is perfectly correlated, as it is now efficient for agents to consume less when prices are high. Secondly, the portfolios derived are independent of the covariance matrix of innovations. The appendix then shows that the relative linearised budget constraint in terms of Home currency can be written as:

$$\widehat{NFA}_{H,t}=\widehat{NFA}_{H,t-1}\frac{1}{\beta}+\widehat{\xi}_{H,t}+\widehat{NX}_{H,t}, \tag{36}$$

where the net foreign asset position of the Home agent, $NFA_{H,t}$, is given by Foreign assets held by the Home agent minus Home assets held by the Foreign agent:

$$NFA_{H,t}=S^{H}_{F,t}P^{S}_{F,t}Z_{t}+B^{H}_{F,t}Z_{t}-S^{F}_{H,t}P^{S}_{H,t}-B^{F}_{H,t}P^{B}_{H,t}. \tag{37}$$

$\xi_{H,t}$ is the excess return on the Home portfolio defined as the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in Home equity:

$$\begin{aligned}\xi_{H,t} \quad = \quad & \left(S^{H}_{H,t-1}-1\right)\left(P^{S}_{H,t}+D_{H,t}\right)+S^{H}_{F,t-1}\left(P^{S}_{F,t}+D_{F,t}\right)S_{t}\\ & +B^{H}_{H,t-1}+B^{H}_{F,t-1}S_{t}-NFA_{H,t-1}R^{S}_{H,t}.\end{aligned} \tag{38}$$

$NX_{H,t}$ are Home net exports defined as:

$$NX_{H,t} = Y_{H,t}^{H} P_{H,t}^{H} + Y_{H,t}^{F} P_{H,t}^{F} S_t - I_{H,t} P_{I,t}^{H} - P_{C,t}^{H} C_{H,t}. \tag{39}$$

Using (35) and (36), we can then derive a partial equilibrium expression for equity holdings as a function of hedging motives (for details of the derivation, see the appendix):

$$S = \frac{1}{2} - \frac{1}{2}\frac{(1-\beta)\,C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^Q, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} - \frac{1}{2}\frac{W\,L}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)}, \tag{40}$$

where real exchange rate risk, $\widehat{R}_t^Q$, is defined as $\widehat{R}_t^Q = \left(1 - \frac{1}{\sigma}\right) \widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \widehat{Q}_{t+j}$, and the relative return on human capital, $\widehat{R}_t^W$, is equal to $\widehat{R}_t^W = \widehat{R}_{H,t}^W - \widehat{R}_{F,t}^W - \widehat{Z}_t + \widehat{Z}_{t-1}$ $= (1-\beta)\,\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right)$. $cov_{R_t^B}(x, y)$ is the covariance between x and y, conditional on relative bond returns and, D, W, L, S are steady state dividends, wages, labour input, and equity holdings, respectively.

While condition (40) is not structural, it provides useful intuition about the drivers of equity positions in this model. Firstly, note that this condition is more general than the specific model discussed here. In fact, it is implied by a fairly general class of models, i.e. all dynamic models with first order complete markets, trade in equities and one period nominal bonds and labour income as the only nontradable source of income.

According to (40), the optimal equity position can be broken into three terms. The first term reflects the pure diversification motive of a logarithmic investor with no labour income ($\sigma = 1$ and $WL = 0$) and would imply here that half of each equity stock is held by each investor $\left(S = \frac{1}{2}\right)$, due to symmetry. The second term arise from a motive to hedge movements in personal consumption expenditures which are here perfectly correlated with the real exchange rate[4]. The BSK condition, (35), indicates that optimal risk sharing implies that consumption falls when the real exchange rate appreciates and the fall in consumption is larger the larger is the intertemporal elasticity of substitution. For high values of

[4]Note that a rise in Q here denotes a real exchange rate *depreciation*

the intertemporal elasticity of substitution $(\sigma < 1)$, relative consumption falls so much in response to a rise in the real exchange rate that relative consumption expenditure also falls. Since optimal consumption spending would be lower in the case when the real exchange rate appreciates, agents would, ceteris paribus, prefer to hold an asset that has low payoffs when the real exchange rate appreciates. For the more realistic case of low intertemporal elasticity of substitution ($\sigma > 1$), agents would prefer to hold an asset that pays more when the real exchange rate appreciates. The last term arises from a motive to hedge movements in the return to human capital and it implies that agents would prefer to hold an asset that has high payoffs when the returns to human capital are low, and vice versa. Finally, it is important to note that equities are not the only assets that can be used to hedge these sources of risk. In this model, nominal bonds are also traded and these can also be used to hedge some or all of these sources of risk. Consequently, it not the unconditional comovement of equity returns with the sources of risk that determines asset positions, but their comovement conditional on the relative returns of nominal bond.

It is instructive at this point to compare this expression with the equivalent ones implied by other previous work. In Heathcote and Perri (2008), there are no nominal rigidities and equities are the only asset traded. Since their benchmark model also only features one shock per country, markets are still effectively complete. This implies that the reduced form expression analogous to (40) would be very similar, with the main difference being that the equity position only depends on the the unconditional comovement between relative equity returns and the sources of risk. In Coeurdacier et al. (2008), prices are flexible and infinitely lived real bonds are traded instead of nominal bonds. This implies that while the relevant covariances are conditional on bond returns, it is the real bond returns that we have to condition on. Since the payoff of the real bonds is perfectly correlated with current and future real exchange rates, movements in the real exchange rate are completely hedged using real bonds and the equity positions are unaffected by the conditional covariance between relative equity returns and real exchange rate risk. Coeurdacier and Gourinchas (2008) provide an analysis of a very general class of models in a static setting. Their expressions would thus often look similar to (40), with the key difference being that the terms would only involve second moments of contemporaneous innovations. Finally, Engel and Matsumoto (2008) analyse a model with nominal rigidities and complete markets in a dynamic setting which would imply the same partial equilibrium expression that we present here.

A similar expression can be derived for bond holdings:

$$B = -\frac{1}{2}\frac{C}{D}\frac{cov_{R_t^S}\left(\widehat{R}_t^Q, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)} - \frac{1}{2}\frac{WL}{(1-\beta)\,D}\frac{cov_{R_t^S}\left(\widehat{R}_t^W, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)}, \tag{41}$$

where B is the steady state position in Home bonds and $cov_{R_t^S}(x, y)$ is the covariance between x and y, conditioning on relative equity returns. Positions in Home bonds are thus similarly affected by the motives to hedge real exchange rate and human capital risk[5]. Investors will take a larger position in Home bonds, ceteris paribus, if they offer a relatively high payoff, when the real exchange rate appreciates or returns to human capital are low. Note that, once again, it is the residual risk that matters, once we have hedged some risk with equity positions.

Finally, note that the expressions for bond and equity positions feature covariance variance ratios and thus look very much like regression coefficients, a property we will exploit in the empirical section below.

4 Portfolios

In this section, we compute optimal portfolios in a variety of settings, focussing in particular on the effects of sticky prices and capital accumulation. We show that models that can generate robustly home biased equity portfolios have two properties: i) equity positions are relatively little affected by real exchange rate risk, ii) relative equity return innovations are conditionally negatively correlated with return innovations of human capital. We show that

[5]Since bonds are in net zero supply, perfectly diversified positions imply zero gross positions in the Home and Foreign bond, which explains the absence of a constant term in this expression.

models with sticky prices and trade in one period nominal bonds in the spirit of Engel and Matsumoto (2008a,b) can potentially fulfil both criteria. Similarly, models with endogenous capital accumulation and trade in long term real bonds satisfy both properties. Virtually all variations of the more general model which allows for capital accumulation and trade in nominal bonds also imply that domestic equities are a good hedge for human capital risk. However, we find that this class of models has great difficulty to fulfil the first criterion, for two reasons. Firstly, the response of investment to shocks implies that domestic equity provides a good hedge for real exchange rate risk. Secondly, due to nominal risk, nominal bonds are no longer a perfect hedge for real exchange rate risk. Thus, equity positions are used to hedge this source of risk and the resulting portfolios become very unstable or excessive. Our benchmark model features trade in one period nominal bonds and equities, capital accumulation, sticky prices, local currency pricing and a monetary policy rule that responds to contemporaneous CPI inflation. In this model, both equities and bonds are highly correlated and both assets are good hedges for both sources of risk. As a result, portfolios are very sensitive to parameter values and usually excessive. However, we can relax many assumptions of the benchmark model and still retain the conclusion that the resulting portfolios are not realistic, chiefly because equity positions are affected by real exchange rate risk. Thus, we experiment with many different parameter configurations, different pricing regimes, monetary policy rules, trade in long term bonds or different types of shocks. However, these results also suggest that one potentially promising avenue to generate more robust portfolio implications could be to reduce the response of investment. And indeed, once we introduce adjustment costs to investment, the portfolio implications of all models improve, sometimes substantially.

4.1 Impulse Responses

To build some further intuition for the portfolio implications of different types of models, it is useful to consider separately the impulse responses implied by models with capital accumulation ("benchmark model") and without ("sticky price model"). The model without capital accumulation is identical to the benchmark model, with the exception that we set both the rate of investment and the rate of depreciation to zero, so that the capital stock is fixed in both countries. Due to the symmetry of the model, it is without loss of generality to focus on impulse responses to relative Home productivity and monetary shocks, where a relative Home productivity shock is defined as a positive one standard deviation shock to Home productivity, with a negative shock to Foreign productivity of equal size.

Figure 1 displays the responses of relative labour income, relative dividends and nominal exchange rates to a relative Home shock to productivity and the interest rate in the sticky price model. We see that, in response to a productivity shock, relative labour income falls, if prices are sufficiently rigid. This is because firms' marginal costs fall and, consequently, they would like to cut prices in order to expand demand. Since not all of them can do so, firms' output cannot rise as quickly and labour demand and relative wages fall. Relative Dividends equal relative profits and increase, both because productivity has increased and labour income has fallen[6]. The nominal exchange rate initially depreciates.

With a positive interest rate shock, we have the opposite implications. Relative labour income rises, while dividends fall and the nominal exchange rate appreciates. From above, we know that human capital risk depends on the response of the infinite series of relative labour income, while relative equity returns depend on the response of relative dividends and relative bond returns depend on the nominal exchange rate. It can then be shown that relative returns to human capital and one period nominal bonds are negative in response to a relative productivity shock, while relative returns to equity are positive. In response to an interest rate shock, relative returns to human capital and bonds are positive, while relative equity returns are negative. Thus, a long position in domestic equity could potentially provide a good hedge for human capital risk. Bonds, however, may potentially not be a very effective asset to hedge this source of risk, as the response to productivity shocks would call for a long position in foreign bonds (and a corresponding short position in domestic

[6] Since the nominal exchange rate initially depreciates, there is a counteracting effect from exchange rate changes, but it is quantitatively dominated by the increase in production and the reduction in labour costs.

bonds), while monetary shocks would be better hedged with a long position in Home bonds.

In figure 2, we investigate the analogous responses in the benchmark model, with capital accumulation[7]. Relative labour income now initially responds positively to a Home relative productivity shock. This is because now, in addition to consumption demand, there is demand for investment. An increase in Home productivity induces a rise in Home investment which increases Home labour demand, due to investment home bias. The rise in relative investment dominates the increase in profits, so that relative dividends fall. The response of the nominal exchange is qualitatively similar to the sticky price model, implying that the nominal exchange rate initially depreciates, but appreciates in the long run. In response to a relative monetary shock, we again have largely opposite responses, with relative labour income falling and dividends increasing, while the nominal exchange rate appreciates. As a consequence, both relative bond and relative equity returns are positive in response to productivity shocks, while both are negative in response to interest rate shocks. Thus, long positions in both domestic bonds and domestic equity could, at least qualitatively, hedge human capital risk. At this stage, it is worth emphasising that a negative response of relative equity returns to productivity shocks does not in general apply that Home equity returns respond negatively. Rather, both Home and Foreign equity returns are positive in response to productivity shocks, but the increase in Foreign equity returns is quantitatively larger. This is because the increase in the production of the Home good leads to a persistent fall in relative prices.

In figures 3 and 4, we evaluate the ability of equities and bonds to hedge real exchange rate risk in the two models. In the sticky price model, the real exchange rate depreciates in response to a productivity shock, while it appreciates in response to a monetary shock. The response of relative dividends and the nominal exchange then imply that a long position in domestic bonds would be a good hedge for real exchange rate risk. Equities have limited ability to hedge real exchange rate risk here, as productivity shocks would require a home

[7]The impulse responses shown are for the benchmark model which features both sticky prices and capital accumulation, but the responses are qualitatively similar in a model without nominal ridigities.

biased equity portfolio, while monetary shocks would imply an equity portfolio with negative home bias. In the benchmark model, the response of the real exchange rate is qualitatively similar. As the response of investment implies that dividends fall, while labour income increases in response to a productivity shock, both long positions in Home bonds or equities could potentially hedge real exchange risk.

Thus, we find that in models with sticky prices, equities and bonds have quite different hedging properties, with long positions in equities being good hedges for human capital risk, while long positions in domestic bonds being potentially better hedges for real exchange rate risk. In models with capital accumulation, the response of investment implies that both assets have similar properties and both long positions in domestic bonds and equities could be used to hedge human capital and real exchange rate risk. In the rest of this section, we investigate whether this qualitative pattern is confirmed by a more rigorous quantitative evaluation.

4.2 Calibration

We adopt a benchmark calibration that closely follows the literature on international business cycles and portfolios (e.g. Backus et al. (1994), Chari et al. (2002), Coeurdacier et al. (2007, 2008), Heathcote and Perri (2008)). The model is assumed to run at quarterly frequency and we thus set the discount factor β to 0.99 which implies an annual steady state real interest rate of 4.1%. The rate of depreciation is set to $\delta = 0.025$ implying an annual rate of depreciation of around 10%. The degrees of consumption and investment home bias are set to $a_C = a_I = 0.85$, implying a steady state import/GDP ratio of 15%. The elasticity of output with respect to labour, α, is set to 0.66. The elasticity of substitution between individual varieties, ε, is set to 10, implying a steady state markup of 11%. The elasticity of substitution between Home and Foreign goods, ϕ, is set to 1.5 and the risk aversion coefficient is set to be $\sigma = 2$. The parameter governing the disutility of labour is set to $\iota = 9.7$. We set $\theta = 0.75$, implying that one quarter of firms can change prices during every quarter and an average life of prices of one year. The elasticity of labour supply, ω, is set to one. The

Table 1

Calibration

Parameter	Interpretation	Value
β	Discount Rate	0.99
σ	Risk Aversion	2
ϕ	Substitution Elasticity Home/Foreign	1.5
a_C	Consumption Home Bias	0.85
a_I	Investment Home Bias	0.85
δ	Depreciation Rate	0.025
α	Capital Share in Production	0.66
i	Disutility of Labour	9.7
θ	1/Average Duration of Prices	0.75
ε	Substitution Elasticity Domestic Varieties	10
ω	Labour Elasticity	1
ρ_A	Persistence of Productivity Shock	0.91
ρ_M	Persistence of Monetary Shock	0

persistence of the productivity and interest rate shocks are set to $\rho_A = 0.91$ and $\rho_M = 0$, respectively. As mentioned above, markets are complete to a first order which implies that portfolios are not sensitive to changes in the covariance matrix. However, the covariance matrix does matter for the simulated volatility and the comovement of the variables in the system. In our benchmark, the covariance matrix is given by:

$$\Sigma = \begin{bmatrix} 0.012 & 0.0054 & 0 & 0 \\ 0.0054 & 0.012 & 0 & 0 \\ 0 & 0 & 0.012 & 0 \\ 0 & 0 & 0 & 0.012 \end{bmatrix}.$$

4.3 Benchmark Portfolio and Previous Literature

In the previous section, we saw that models with sticky prices imply that domestic bonds and equities have different hedging properties with respect to real exchange rate and human capital risk, while their response is qualitatively similar in models with capital accumulation.

We can thus speculate that models without capital accumulation can more easily produce robust portfolios. Here, we evaluate the quantitative portfolio implications of these models. Table 2 shows the portfolios implied by several models of interest. We also relate asset positions to parts driven by each of the two sources of risk identified above, using the reduced form expression derived in the previous section. Throughout, we use the benchmark calibration as outlined above, unless specified otherwise.

We find that the analysis of the previous subsection is confirmed in the quantitative experiments. The model with sticky prices and capital accumulation implies that Home relative equity and bond returns are highly correlated with each other and with both real exchange rate and human capital risk. In our benchmark calibration, real exchange rate risk is large and bonds are slightly better hedges for it than equities. Since the two assets are highly correlated, extreme positions are needed to achieve perfect risk sharing. Thus, large long positions in domestic bonds and large negative positions in domestic equities result, with both mainly driven by real exchange rate risk. It is worth noting that human capital hedging does indeed push the domestic equity portfolio towards equity home bias.

In Table 2, we also present the portfolios implied by models in the spirit of Engel and Matsumoto (2008a) ("sticky price"), Coeurdacier et al (2008)("Real Bonds + Cap. Acc."), and Heathcote and Perri (2008) ("Flexible Price"). The impulse responses of the sticky price model were discussed in the previous subsection [8]. And indeed, we find that the in the model without capital accumulation, equity home bias results from hedging human capital risk, while real exchange rate risk mainly affects bond positions. Nevertheless, bond positions are of a realistic size. In the model with capital accumulation and long term real bonds, real exchange rate risk is perfectly hedged by the bond position, leaving human capital hedging to lead to equity home bias. However, note that equities would - in the absence of trade in real bonds - still be good hedges for real exchange rate risk here, again due to the response

[8]The parameter values used in this case are different from the benchmark case in one respect. The persistence parameter for the monetary policy shocks is here set to $\rho_M = 0.9$. If monetary shocks are i.i.d as in the benchmark calibration, productivity shocks have a stronger influence on portfolio positions and we observe that equity positions continue to be more heavily affected by the motive to hedge real exchange rate risk

Table 2

Portfolios		Benchmark	Sticky Price*	Real Bonds + Cap. Acc.	Flexible Price
Share of Home Equity		**−3.17**	**0.67**	**0.92**	**1.99**
	due to:				
	HC	0.66	0.35	0.42	0.48
		(−1.00)	(−0.92)	(−1.00)	(−1.00)
	real ER	−4.33	−0.18	0	1.01
		(−0.68)	(−0.45)	(−0.69)	(−1.00)
Bonds/GDP		**26.13**	**1.34**	**18.29**	**0.00**
	due to:				
	HC	−1.53	−0.46	−1.07	0
		(−0.99)	(−0.35)	(−0.67)	(−0.63)
	real ER	27.66	1.80	19.36	0
		(−0.72)	(−0.96)	(−1.00)	(−0.63)
Corr (Equity,Bonds)		1.00	0.68	0.69	0.63

of investment. This is also true in the model with capital accumulation and no nominal rigidities. Without nominal rigidities, monetary shocks are pure noise and do not affect real quantities. Bonds are then not needed to achieve risk sharing and optimal bond positions are zero. Equity positions are then excessively home biased, as long positions in domestic equity are used both to hedge human capital and real exchange rate risk. Thus, the two classes of models that can generate robust and realistic portfolios are the sticky price model and the model with capital accumulation and trade in real bonds. In both cases, equity positions are relatively unaffected by the motive to hedge real exchange rate risk and equity home bias results from human capital hedging.

4.4 Robustness and Capital Adjustment Costs

Our benchmark model features trade in one period nominal bonds and equities, capital accumulation, sticky prices, local currency pricing and a monetary policy rule that responds to contemporaneous CPI inflation. However, the result that portfolios are excessive and sensitive to small changes in key parameters is robust to a change of most features of the model. In particular, it is robust to changing parameters within most of the plausible parameter space, different pricing regimes, monetary policy rules, trade in long term bonds or different types of shocks[9]. This should not come as a surprise. Above, we pointed out that there are two reasons for the instability of portfolios. Firstly, the response of investment implies that Home equities are good hedges for real exchange rate risk. Secondly, the presence of nominal risk implies that nominal bonds are not perfect hedges for real exchange rate risk. These two properties are often relatively little affected by all the model variations considered[10].

In table 3, we focus on two particular changes of interest. Firstly, we investigate whether the portfolio implications improve if we allowed long term bonds to be traded instead of one period bonds. In this variation, bonds pay one unit of local currency each period forever. These long term bonds now pay one unit of local currency ad infinitum. Relative bond returns then depend on the discounted infinite series of unexpected future nominal exchange rate appreciations:

$$\widehat{R}_t^{B,long} = \widehat{R}_{F,t}^{B,long} - \widehat{R}_{H,t}^{B,long} - \widehat{Z}_t + \widehat{Z}_{t-1} = -\left(1-\beta\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j Z_{t+j}\right],$$

It could be speculated that by matching the maturity of bonds with the maturity of the sources of risk, nominal bonds become even better hedges for real exchange rate risk and that equities are then no longer as affected by the motive to hedge real exchange rate risk. However, the table shows that this is not the case, for two reasons. Firstly, the hedging properties of equities remain unchanged. In particular, Home equities remain good hedges for real exchange rate risk. Secondly, bonds are now often worse at hedging real exchange rate risk than before. This is because nominal risk cumulates over time, as prices are flexible in the long run and as a result, long term nominal bonds are often less correlated with the

[9]Results for these variations are available on request.

[10]There are, however, two parameters that affect the association between nominal and real exchange rates and thus the hedging properties of nominal bonds, namely, the degree of price rigidity, θ, and the responsiveness of monetary policy, γ. It is worth noting, however, that only for very high and highly implausible values of these parameters do we observe that nominal and real exchange rates become close enough to ensure that real exchange rate hedging does not affect equity positions much. For high values of nominal rigidity, there is also a problem of real indeterminacy. See Carlstrom and Fuerst (2005) for a discussion in a closed economy context.

real exchange rate than short term bonds. More generally, the dynamics of the nominal and the real exchange rate can be more different than the initial response of the two variables, as can be seen in the impulse responses presented above. In general, since long term bonds exhibit lower correlations with the sources of risk, these model variations will imply bond positions that are larger in absolute size, sometimes implausibly so.

We also present results when we replace the monetary shock with investment efficiency shocks a la Fischer (2005). With these shocks, the law of motion for capital becomes:

$$K_{i,t+1} = (1-\delta) K_{i,t+1} + \chi_{i,t} I_{i,t},$$

where $\chi_{i,t}$ is a shock to investment efficiency the logarithm of which follows a first order autoregressive process[11]. This type of shocks merits special consideration for two reasons. Firstly, it has recently been shown that this shock can account for a substantial fraction of business cycle fluctuations (Fisher (2005), Justiniano and Primiceri (2006), Justiniano et al. (2007)). Secondly, Coeurdacier et al. (2008) have used this type of shocks to account for international portfolio composition. However, here, we find that allowing for investment efficiency shocks does not improve the performance of the model. This should not come as a surprise. Above, we pointed out that the failure of the model could be traced at least in part to the response of investment. Investment efficiency shocks do nothing to alleviate this problem and may exacerbate it. Due to the response of investment, domestic equities remain good hedges of real exchange rate risk here and they are, in general, now even better hedges than bonds. As a result, we now obtain excessive home bias, both for long and short term bonds.

The previous experiments suggest that the response of investment lies at the heart of the difficulty of the models to generate realistic and robust portfolios. One logical conclusion would be to reduce the size of the response of investment. In table 4, we introduce quadratic adjustment costs to capital to the benchmark model, as well as the model with long term

[11] We follow Coeurdacier et al. (2008) and specify the persistence of the process to be $\rho_\zeta = 0.91$.

Table 3

Portfolios		Benchmark	Long Bonds	Investment Shocks	Inv. Shock + Long Bonds
Share of Home Equity		**−3.17**	**1.42**.	**1.41**	**1.74**
	due to:				
	HC	0.66	0.39	0.38	0.38
		(−1.00)	(−1.00)	(−1.00)	(−1.00)
	real ER	−4.33	0.53	0.53	0.86
		(−0.68)	(−0.68)	(−0.71)	(−0.71)
Bonds/GDP		**26.13**	**−7.08**	**2.60**	**−5.38**
	due to:				
	HC	−1.53	0.41	−0.11	0.03
		(−0.99)	(−0.65)	(0.49)	(−0.78)
	real ER	27.66	−7.50	2.71	−5.41
		(−0.72)	(0.17)	(−0.31)	(−0.20)
Corr (Equity.Bonds)		1.00	0.62	−0.41	0.78

bonds or investment efficiency shocks. The law of motion for investment in the benchmark case then becomes:

$$K_{i,t+1} = (1-\delta)\, K_{i,t} + I_{i,t+1} - \frac{\lambda}{2}\left(\frac{I_{i,t+1}}{K_{i,t}}\right)^2 K_{i,t},$$

where λ is the quadratic adjustment cost parameter[12]. Indeed, the table shows that the performance of the model improves, sometimes substantially. As expected, the reason can be traced to the behaviour of investment. The existence of adjustment costs implies that investment responds less strongly to the shocks. As a result, equity and bond returns are more differentiated. In particular, relative equity returns respond negatively and relative labour income positively to a relative Home productivity shock. This implies that relative bond and equity returns become more differentiated and in particular, Home equity is no longer a good hedge for real exchange rate risk. However, it does remain a good hedge for human capital risk, as reducing the response of investment affects both Home relative dividends as well as Home relative labour income. The reduction in Home investment reduces the initial investment outlay, and also the ultimate output and thus the fall in

[12]Following Cummins et al. (1994), we set $\lambda = 2$, but we also note that moderate deviations from this value only have a limited effect on the nature of the results.

Table 4

Portfolios		Benchmark	Long Bonds	Investment Shocks	Inv. Shock + Long Bonds
Share of Home Equity		**0.58**	**1.17**.	**1.10**	**1.24**
	due to:				
	HC	0.39	0.35	0.34	0.34
		(−1.00)	(−1.00)	(−1.00)	(−1.00)
	real ER	−0.31	0.32	0.26	0.40
		(−0.36)	(−0.36)	(−0.44)	(−0.56)
Bonds/GDP		**2.40**	**−3.36**	**2.20**	**−4.46**
	due to:				
	HC	−0.18	0.25	−0.16	0.13
		(−0.71)	(−0.42)	(0.15)	(−0.30)
	real ER	2.58	−3.61	2.36	−4.59
		(−0.88)	(0.72)	(−0.87)	(−0.65)
Corr (Equity.Bonds)		0.76	0.35	−0.06	0.27

relative prices. But smaller investment demand also implies reduced demand for Home labour. The benchmark model augmented with capital adjustment costs then becomes qualitatively similar to the sticky price model. The versions with investment shocks or long bonds continue to exhibit excessive home bias, though at a much reduced rate.

5 Empirical Part (Preliminary and Incomplete)

The models discussed above imply that equity positions should be determined by the covariance between relative equity return innovations and human capital and real exchange rate risk, conditional on bond returns, while bond portfolios should depend on the covariance between relative bond return innovations and human capital and real exchange rate risk, conditional on relative equity return innovations. In this section, we use data on G 7 countries to calculate the empirical counterparts to the relevant covariances in order to compute the implied portfolios. Relative labour income, real exchange rates and relative equity and bond returns are constructed using national accounts and financial market data, as in Coeurdacier and Gourinchas (2009). The data series are quarterly and run from 1970 to 2008 and are taken from Coeurdacier and Gourinchas (2009) to which we refer for further details.

Table 6

Labour and Consumption Shares

	Canada	France	Germany	Italy	Japan	UK	US	EU	Average
Labour Share	0.69	0.71	0.73	0.61	0.63	0.68	0.73	0.69	0.68
Naive Labour Share	0.60	0.61	0.59	0.49	0.57	0.63	0.62	0.57	0.59
Consumption Share	0.71	0.66	0.63	0.65	0.60	0.70	0.71	0.65	0.67
Share of World Output	0.04	0.08	0.11	0.08	0.16	0.08	0.45	0.26	1

Compared to Coeurdacier and Gourinchas (2009), there are three key differences, due to the dynamic nature of our approach. Firstly, our measure of real exchange rate risk comprises both current and future unexpected real exchange rate fluctuations. Similarly, we use innovations to equity and bond returns rather than bond and equity returns themselves. Finally, our reduced form expressions for portfolios are slightly different, due to the dynamic nature and the possibility of investment. We find that, based on the estimated moments, equity home bias can be rationalised in the US, Japan, and the UK. In all countries, equity positions are mainly affected by the human capital hedging motive and not by the real exchange rate motive. Our results also suggest two empirical failures of the models. Firstly, equity return innovations are not as volatile as in the data and too highly correlated with both sources of risk. Secondly, real exchange rate risk is too large in our models.

In Table 6, we summarise the labour, consumption, and output shares for the G7 countries. The table shows both a measure of the labour share that includes a fraction of mixed surplus and a measure which only includes compensation of employees. It is worth noting that the difference between the two measures is quite sizable, a point emphasized by Gollin (2002).

5.1 Innovations and Risk Loadings

In order to to estimate the relevant covariances, we need to construct innovations to bond, equity and human capital returns, as well as a measure of real exchange rate risk.

Denote by $r^W_{i,t+1}$ the log of the gross simple return on human capital in country i between t and $t+1$. Following Campbell (1996), under the assumption that the dividend price ratio on human wealth is stationary, we can write:

$$r^W_{i,t+1} \equiv \log\left(LI_{i,t+1} + V^W_{i,t+1}\right) - \log\left(V^W_{i,t}\right) = k + \zeta_{i,t} - \rho\zeta_{i,t+1} + \Delta \log LI_{i,t+1},$$

where $V^W_{n,t}$ measures nonfinancial wealth, $\zeta_{i,t} = \log\left(LI_{i,t}/V^W_{i,t}\right)$ is the log-dividend price ratio, and $\rho^{-1} = 1 + \exp(\zeta) = 1 + \left(LI_i/V^W_i\right) = \left(LI_i + V^W_i\right)/V^W_i$. As in Coeurdacier and Gourinchas (2009), we will use $\rho = 0.98$, while k is an unimportant constant. Solving this equation forward and imposing that $\lim_{t\to\infty} \rho^t \left(r^W_{i,t} - \Delta \log LI_{i,t}\right) = 0$, we obtain (up to a constant):

$$\zeta_{i,t} = \sum_{j=0}^{\infty} \rho^j \left(r^W_{i,t+j} - \Delta \log LI_{i,t+1+j}\right)$$

This expression states that the ratio of labour income to the value of human capital (the equivalent of a dividend-price ratio for human capital) is high today either when future human capital returns are high, or when future nonfinancial income growth is low. We do not estimate future expected returns to human capital. Instead, following Coeurdacier and Gourinchas (2009), we assume that the conditional expected return on human capital equals the conditional expected return on equities $\left(E_t r^W_{i,t+j} = E_t r^E_{i,t+j}\right)$. We then obtain the following expression:

$$r^W_{i,t+1} - E_t\left[r^W_{i,t+1}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \left(\Delta \log LI_{i,t+1+j}\right) - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r^W_{i,t+j+1}$$

This expression states that the innovation to the return on human capital depends upon innovations to the path of future expected labour income growth , as well as innovations to the path of future expected equity returns proxying for future expected human capital returns. Human capital return innovations today are high, if current and future innovations to expected labour income growth are high, or if innovations to future expected human capital returns are low. Finally, we obtain innovations to the relative expected human capital return by subtracting this expression for country i from the equivalent expression for the rest of the world, assuming that ρ is the same for all countries:

$$r^W_{i,t+1} - E_t\left[r^W_{i,t+1}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j\left(\Delta\log li_{i,t+1+j}\right) - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r^E_{i,t+j+1}.$$

We also need to obtain a measure for real exchange rate risk. Coeurdacier and Gourinchas (2009) use a static model and correspondingly their measure of real exchange rate risk is simply related to contemporaneous real exchange rate fluctuations. By contrast, due to the dynamic nature of our approach, our measure of real exchange rate risk implies that innovations to current and future expected real exchange rate changes need to be taken into account. Using a similar procedure as above, we then arrive at the following equation:

$$r^Q_{i,t+1} - E_t\left[r^Q_{i,t+1}\right] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j\left(\Delta\log Q_{i,t+1+j}\right),$$

,where $r^Q_{i,t+1} - E_t\left[r^Q_{i,t+1}\right]$ is the relevant measure of real exchange rate risk.

We then estimate the following vector autoregression:

$$g_{t+1} = Ag_t + \varepsilon_{t+1},$$

where $z'_t = \left[\begin{array}{ccccc} r^E_t & \Delta li_t & r^B_t & \Delta q & x_t \end{array}\right]$. x_t represents other controls that help to predict the (relative) growth rate of labour income, bond and equity returns, and the real exchange rate. Here, we follow Coeurdacier and Gourinchas (2009) and use relative log consumption expenditure and the nxa variable from Gourinchas and Rey (2007). We then take the estimates from the VAR, $\widehat{A}$ and $\widehat{\varepsilon}$, to construct the innovations to human capital returns and real exchange rate risk. Human capital and real exchange rate risk are obtained as:

$$
\begin{aligned}
r_{i,t+1}^{W} - E_t\left[r_{i,t+1}^{W}\right] &= \left(e_2^{'} - \rho e_1^{'}\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \\
r_{i,t+1}^{Q} - E_t\left[r_{i,t+1}^{Q}\right] &= e_4^{'}\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1}
\end{aligned}
$$

while equity and bond return innovations are:

$$
\begin{aligned}
r_{i,t+1}^{E} - E_t\left[r_{i,t+1}^{E}\right] &= e_1^{'}\widehat{\varepsilon}_{t+1} \\
r_{i,t+1}^{B} - E_t\left[r_{i,t+1}^{B}\right] &= e_3^{'}\widehat{\varepsilon}_{t+1},
\end{aligned}
$$

where e_i is a unit vector whose ith element is equal to one, while all other elements are equal to zero.

In Table 7, we present simple measures of volatility and comovement for asset returns and measures of risk for the US. As noted above, it is the *conditional* measures which are of importance for portfolios. However, these measures still contain some interesting information. Firstly, equity return innovationas are more volatile than bond return innovations by a factor of more than two, and the volatility of equity returns is of the same magnitude as the volatility of human capital and real exchange rate risk. Equity return innovations are also quite highly correlated with bond return innovations and real exchange rate risk, while the correlation with human capital is somewhat lower. Bond return innovations in turn are quite highly correlated with both human capital and real exchange rate risk.

5.1.1 Estimating the Loading on Human Capital Risk

We are now in a position to estimate the relevant covariance ratios. In order to do that, we follow Warnock and van Wincoop (2008) and Coeurdacier and Gourinchas (2009) and use a regression based approach. To that end, we run the following regression:

$$
\widetilde{r}_{i,t+1}^{W} = k + \beta_{w,b}^{i}\widetilde{r}_{i,t+1}^{B} + \beta_{w,e}^{i}\widetilde{r}_{i,t+1}^{E} + \varepsilon_{i,t}^{w},
$$

Table 7

Volatility and Correlation of Returns and Measures of Risk for the US

	Equity Returns	Human Capital Risk	Bond Returns	Real ER Risk
Variance	0.0048	0.0052	0.0019	0.0054
Correlation with:				
Equity Returns	1	0.12	0.46	0.45
Human Capital Risk	0.12	1	0.69	0.15
Bond Returns	0.46	0.69	1	0.66
Real ER Risk	0.45	0.15	0.66	1

where $\widetilde{r}^j_{i,t+1} = r^j_{i,t+1} - E_t r^j_{i,t+1}$ is he return innovation and $\varepsilon^j_{i,t}$ is attributed both to the measurement error in the construction of real exchange rate risk and to the fluctuations in real exchange rate risk not spanned by relative bond and equity returns (which is zero in our models due to effectively complete markets). Note that the difference to Coeurdacier and Gourinchas (2009) is that we use relative equity and bond return innovations rather than simple bond and equity returns in this regression.

The results are presented in table 8. We see that the coefficient on bond returns is always significantly positive and quite large. In fact, in most countries in the sample, the coefficient is larger than one in a statistical sense. For equities, the picture is more nuanced. In Japan, the UK and the US, equity returns are significantly negatively correlated with human capital returns, indicating that equities could hedge human capital risk. In Canada, Italy and the EU the coefficient is insignificantly different from zero, while it is significantly positive in a statistical sense in France and Germany, but still quite low. These results imply that long positions in domestic equity can in fact hedge human capital risk.

Finally, Benigno and Ristico (2009) argue in favour of computing covariance and variance ratios directly from the estimated residual covariance matrix of the VAR. We have done this and results are virtually identical to those of the above regression.

Table 8

Loadings of human capital risk on bond and equity return innovations

	Canada	France	Germany	Italy	Japan	UK	US	EU
Bonds	1.08	0.89	1.28	1.14	1.10	1.22	1.35	1.29
	0.08	0.06	0.07	0.08	0.16	0.08	0.11	0.07
Equity	−0.01	0.13	0.09	−0.02	−0.34	−0.09	−0.26	0.03
	0.04	0.02	0.03	0.03	0.08	0.03	0.04	0.04
R2	0.66	0.75	0.78	0.64	0.26	0.68	0.53	0.78
Obs.	136	136	136	136	136	136	136	136

5.1.2 Estimating the Loading on Real Exchange Rate Risk

In order to estimate the loadings on real exchange rate risk, we run:

$$\widetilde{r}^{Q}_{i,t+1} = k + \beta^{i}_{q,b}\widetilde{r}^{B}_{i,t+1} + \beta^{i}_{q,e}\widetilde{r}^{E}_{i,t+1} + \varepsilon^{q}_{i,t},$$

where $\varepsilon^{q}_{i,t}$ is attributed both to the measurement error in the construction of real exchange rate risk and to the fluctuations in real exchange rate risk not spanned by relative bond and equity returns (which is zero in our models due to effectively complete markets). Note that there are now two differences between our regression and Coeurdacier and Gourinchas (2009). Firstly, we use a measure of real exchange rate risk that takes into account current and future real exchange rates. Secondly, as above, we use return innovations rather than returns themselves.

The results are presented in table 9. The loadings of real exchange rate risk on bonds are always significant and quite large. What is more, they are statistically insignificantly different from one in France, the US and the EU, while they are always close to one in an economic sense. The loadings of equity vary quite widely, but they are generally fairly small and sometimes insignificantly different from zero. However, in the US, the coefficient is positive and modestly large. Again, the estimates from VAR based calculations are virtually identical.

Table 9

Loadings of real exchange rate risk on bond and equity return innovations

	Canada	France	Germany	Italy	Japan	UK	US	EU
Bonds	1.14	1.12	1.03	0.86	1.16	1.09	0.97	1.08
	0.06	0.11	0.07	0.09	0.06	0.05	0.12	0.10
Equity	−0.04	−0.07	0.14	0.01	0.07	−0.03	0.20	0.10
	0.03	0.04	0.04	0.03	0.03	0.02	0.08	0.06
R2	0.79	0.47	0.68	0.42	0.84	0.81	0.46	0.54
Obs.	136	136	136	136	136	136	136	136

Table 9

Estimate Equity Portfolios and Home Bias

	Canada	France	Germany	Italy	Japan	UK	US	EU
Output Share	0.04	0.08	0.11	0.08	0.16	0.08	0.45	0.26
Home Equity Share	0.06	−0.20	−0.11	0.10	0.65	0.25	0.84	0.21
Home Bias	0.02	−0.28	−0.22	0.02	0.48	0.17	0.38	−0.05
due to Human Capital	0.02	−0.28	−0.22	0.02	0.48	0.17	0.38	−0.05
due to Real ER	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

5.2 Implied Bond and Equity Positions

In order to compare our empirical results with the model predictions, we need to adapt our reduced form expression for bond and equity portfolios to allow for countries of different size. We then have:

$$S^* = \widetilde{\lambda} + \left(1 - \widetilde{\lambda}\right) \left(\frac{-\left(1-\beta\right) C^* cov_{\widehat{R}_t^B} \left(\widehat{R}_t^Q, \widehat{R}_t^S\right) - W^* L_H cov_{\widehat{R}_t^B} \left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{D^* var_{\widehat{R}_t^B} \left(\widehat{R}_t^S\right)} \right),$$

where $\widetilde{\lambda} = \frac{C_H}{C_H + C_F} = \frac{D_H}{D_H + D_F} = \frac{Y_H}{Y_H + Y_F}$ is the steady state ratio of consumption, dividends and output between the Home country and the rest of the world. Table 6 presents the implied equity portfolios:

The estimated covariances imply significant home bias in the UK, Japan, and the US. In all cases, the effect of real exchange rate fluctuations on equity positions is very small, implying that home bias is virtually entirely determined by a motive to hedge movements in real exchange rates.

6 Conclusion

This paper shows that the response of investment makes it difficult for open economy macro models to generate robust and equity home biased portfolios. A strong response of investment implies that domestic equity is a good hedge for both human capital and real exchange rate risk. In many settings, relative equity and bond returns are highly correlated and due to nominal risk, equity is often a better hedge for real exchange rate risk than bonds. As a result, asset positions are usually excessive and very sensitive to parameter changes. We find that one way to improve the portfolio implications of the model is to restrict the response of investment, e.g. by the introduction of quadratic adjustment costs. This change differentiates the behaviour of equity and bond returns and often implies that holding domestic equities is no longer as effective in hedging real exchange rate risk. Several open questions remain, however. Firstly, as in many standard macroeconomic models, the behaviour of equity and bond returns and exchange rates in these models does not quite fit the data. Equity returns are not volatile enough and the volatilty of bond returns relative to equity returns is too large. It would therefore be interesting to explore whether applying recent insights of the asset pricing literature, e.g. the use of alternative classes of utility functions, such as those featuring external habits or non time separability, can improve the return implications of this class of models also. Similarly, introducing learning, as in Adam et al. (2009) may be a promising way to generate additional volatility in equity returns. Further empirical work is clearly needed, too. We currently envisage two routes. Firstly, some further investigation of the comovement between asset returns and human capital and exchange rate risk could help to clarify the results presented in the final section of this paper. Thus, we can use national accounts data, as in Baxter and Jermann (1997), or use returns to the firm rather than equity returns. What is more, above we highlighted that impulse response functions can provide some useful intuition about the hedging properties of different assets. It thus only seems appropriate to investigate whether impulse responses in the data suggest that hedging of real exchange rate and human capital risk can be important drivers of asset positions. Finally, one key advantage of the recent literature on the structure of international portfolios is to allow for more sophisticated asset menus, including bonds. Above, we highlighted that asset positions do in fact depend significantly on the existence of other assets, but there is currently not a systematic approach to compare the implications of these models for bonds with data on bond holdings.

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7 Appendix

7.1 Stochastic Discount Factors

The Euler equations for investment in Home equity are:

$$\beta E_t\left[\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}\right)\left(\frac{D_{H,t+1}+P^S_{H,t+1}}{P^S_{H,t}}\right)\right] = 1 \tag{42}$$

$$\beta E_t\left[\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\left(\frac{P^F_{C,t}}{P^F_{C,t+1}}\right)\left(\frac{D_{H,t+1}+P^S_{H,t+1}}{P^S_{H,t}}\frac{Z_t}{Z_{t+1}}\right)\right] = 1 \tag{43}$$

Linearising:

$$\begin{aligned}
0 &= E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)+\left(\widehat{P}^H_{C,t}-\widehat{P}^H_{C,t+1}\right)+(1-\beta)\,\widehat{D}_{H,t+1}+\beta\widehat{P}^S_{H,t+1}-\widehat{P}^S_{H,t}\right]\\
0 &= E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)+\left(\widehat{P}^F_{C,t}-\widehat{P}^F_{C,t+1}\right)+(1-\beta)\,\widehat{D}_{H,t+1}+\beta\widehat{P}^S_{H,t+1}-\widehat{P}^S_{H,t}+\widehat{Z}_t-\widehat{Z}_{t+1}\right]
\end{aligned}$$

which imply:

$$\begin{aligned}
& E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)-\left(\widehat{P}^H_{C,t+1}-\widehat{P}^H_{C,t}\right)\right] && (44)\\
=\ & E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)-\left(\widehat{P}^F_{C,t+1}-\widehat{P}^F_{C,t}\right)\right]+\widehat{Z}_t-\widehat{Z}_{t+1} && (45)
\end{aligned}$$

Thus, in a first order approximation, the Home and Foreign stochastic discount factors are the same, once they are expressed in the same units.

7.2 Budget Constraint

The budget constraint for the Home agent is:

$$\begin{aligned}
& S^H_{H,t}P^S_{H,t}+S^H_{F,t}P^S_{F,t}Z_t+B^H_{H,t}P^B_{H,t}+B^H_{F,t}P^B_{F,t}Z_t\\
=\ & S^H_{H,t-1}\left(P^S_{H,t}+D_{H,t}\right)+S^H_{F,t-1}\left(P^S_{F,t}+D_{F,t}\right)Z_t+B^H_{H,t-1}\\
& +B^H_{F,t-1}Z_t+L_{H,t}W_{H,t}-P^H_{C,t}C_{H,t} && (46)
\end{aligned}$$

The net foreign asset position is then given by assets held abroad minus domestic assets held by foreign agents:

$$NFA_{H,t}=S^H_{F,t}P^S_{F,t}Z_t+B^H_{F,t}Z_t-S^F_{H,t}P^S_{H,t}-B^F_{H,t}P^B_{H,t} \tag{47}$$

From the asset market clearing conditions:

$$S_{H,t}^{F} = 1 - S_{H,t}^{H} \qquad B_{H,t}^{F} = -B_{H,t}^{H} \tag{48}$$

which gives:

$$NFA_{H,t} = \left(S_{H,t}^{H} - 1\right) P_{H,t}^{S} + S_{F,t}^{H} Z_t P_{F,t}^{S} + B_{F,t}^{H} Z_t + B_{H,t}^{H} P_{H,t}^{B} \tag{49}$$

Define the portfolio excess return as:

$$\begin{aligned} \xi_{H,t} &= \left(S_{H,t-1}^{H} - 1\right)\left(P_{H,t}^{S} + D_{H,t}\right) + S_{F,t-1}^{H}\left(P_{F,t}^{S} + D_{F,t}\right) Z_t \\ &\quad + B_{H,t-1}^{H} + B_{F,t-1}^{H} Z_t - NFA_{H,t-1} R_{H,t}^{S}, \end{aligned} \tag{50}$$

i.e. the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in the home equity. Note that $R_H^S = \frac{P_{H,t}^S + D_{H,t}}{P_{H,t-1}^S}$, $R_F^S \frac{Z_t}{Z_{t-1}} = \frac{P_{F,t}^S + D_{F,t}}{P_{F,t-1}^S} \frac{Z_t}{Z_{t-1}}$. Then,

$$\begin{aligned} \xi_{H,t} &= \left(S_{H,t-1}^{H} - 1\right) P_{H,t-1}^{S} R_{H,t}^{S} + S_{F,t-1}^{H} P_{F,t-1}^{S} Z_{t-1} R_{F,t}^{S} \frac{Z_t}{Z_{t-1}} \\ &\quad + B_{H,t-1}^{H} P_{H,t-1}^{B} R_{H,t}^{B} + B_{F,t-1}^{H} Z_{t-1} P_{F,t-1}^{B} R_{F,t}^{B} \frac{Z_t}{Z_{t-1}} - NFA_{H,t-1} R_{H,t}^{S}. \end{aligned} \tag{51}$$

Substitute for net foreign assets above to get:

$$\begin{aligned} \xi_{H,t} &= S_{F,t-1}^{H} P_{F,t-1}^{S} Z_{t-1}\left(R_{F,t}^{S} \frac{Z_t}{Z_{t-1}} - R_{H,t}^{S}\right) + B_{H,t-1}^{H} P_{H,t-1}^{B}\left(R_{H,t}^{B} - R_{H,t}^{S}\right) \\ &\quad + B_{F,t-1}^{H} Z_{t-1} P_{F,t-1}^{B}\left(R_{F,t}^{B} \frac{Z_t}{Z_{t-1}} - R_{H,t}^{S}\right) \end{aligned} \tag{52}$$

Rewriting the original budget constraint:

$$\begin{aligned} NFA_{H,t} + P_{H,t}^{S} &= S_{H,t-1}^{H} P_{H,t-1}^{S} R_{H,t}^{S} + S_{F,t-1}^{H} Z_{t-1} P_{F,t-1}^{S} R_{F,t}^{S} \frac{Z_t}{Z_{t-1}} \\ &\quad + B_{H,t-1}^{H} P_{H,t-1}^{B} R_{H,t}^{B} + B_{F,t-1}^{H} Z_{t-1} P_{F,t-1}^{B} R_{F,t}^{B} \frac{Z_t}{Z_{t-1}} + L_{H,t} W_{H,t} - P_{C,t}^{H} C_{H,t} \end{aligned} \tag{53}$$

Using the expressions for the portfolio excess return and net foreign assets, we obtain:

$$NFA_{H,t} = NFA_{H,t-1} R_{H,t}^{S} + P_{H,t-1}^{S} R_{H,t}^{S} + \xi_{H,t} + L_{H,t} W_{H,t} - P_{C,t}^{H} C_{H,t} - P_{H,t}^{S} \tag{54}$$

From $R^S_{H,t} = \frac{D_{H,t}+P^S_{H,t}}{P^S_{H,t-1}}$, $D_{H,t} = \Pi_{H,t} + R^K_{H,t}K_{H,t} - I_{H,t}P^H_{I,t}$ and $\Pi_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - R^K_{H,t}K_{H,t} - W_{H,t}L_{H,t}$

$$NFA_{H,t} = NFA_{H,t-1}R^S_{H,t} + Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t + \xi_{H,t} - P^H_{C,t}C_{H,t} - I_{H,t}P^H_{I,t} \tag{55}$$

Remembering that next exports were defined as: $NX_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - I_{H,t}P^H_{I,t} - P^H_{C,t}C_{H,t}$, we have:

$$NFA_{H,t} = NFA_{H,t-1}R^S_{H,t} + \xi_{H,t} + NX_{H,t} \tag{56}$$

or, in linear form:

$$\widehat{NFA}_{H,t} = \widehat{NFA}_{H,t-1}\frac{1}{\beta} + \widehat{\xi}_{H,t} + \widehat{NX}_{H,t}, \tag{57}$$

Linearising the expression for the portfolio excess return and using (57), we have:

$$\widehat{\xi}_{H,t} = \left(S^H_H - 1\right)\frac{P^S}{\beta}\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + B\left(\widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right), \tag{58}$$

We can then write the budget constraint in linearised form as:

$$\begin{aligned}\widehat{NFA}_{H,t} &= \widehat{NFA}_{H,t-1}\frac{1}{\beta} + (S-1)\frac{P^S}{\beta}\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) \\ &\quad + B\left(\widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + \widehat{NX}_{H,t}\end{aligned} \tag{59}$$

Taking expectations at time t and rewriting:

$$\begin{aligned}\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= E_t\left[\widehat{NFA}_{H,t} - \widehat{NX}_{H,t} - \frac{P^S}{\beta}(S-1)\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right)\right] \\ &\quad - BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right]\end{aligned} \tag{60}$$

Iterating forward, imposing $T \to \infty$,$lim_{T\to\infty}E_t\left[NFA_{t+T}\right] = 0$ and using $E_t\left[\widehat{R}^S_{H,t+\tau} - \widehat{R}^S_{F,t+\tau} + \widehat{Z}_{t+\tau} - \widehat{Z}_{t+\tau-1}\right] = 0, \tau > 1$, we get:

$$\begin{aligned}\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= \sum_{j=0}^{T} -\beta^j E_t\left[\widehat{NX}_{H,t+j}\right] - \frac{P^S}{\beta}(S-1)E_t\left[\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \\ &\quad - BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right]\end{aligned} \tag{61}$$

Rearranging and using the expressions for linearised relative bond and equity returns, we have:

$$\sum_{j=0}^{T} -\beta^j E_t\left[NX_{H,t+j}\right] = \frac{1}{\beta}NFA_{H,t-1} + D\left(S-1\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t\left[-\widehat{Z}_t\right], \qquad (62)$$

where $\widetilde{E}_t\left[X_t\right] = E_t\left[X_t\right] - E_{t-1}\left[X_t\right]$.
This budget constraint holds if and only if:

$$\sum_{j=0}^{T} -\beta^j \widetilde{E}_t\left[NX_{H,t+j}\right] = D\left(S-1\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t\left[-\widehat{Z}_t\right] \qquad (63)$$

and

$$\sum_{j=0}^{T} -\beta^j E_{t-1}\left[NX_{H,t+j}\right] = \frac{1}{\beta}NFA_{H,t-1} \qquad (64)$$

Using, $\widehat{NX}_{H,t} = \widehat{D}_{H,t}D + \left(\widehat{W}_{H,t} + \widehat{L}_{H,t}\right)WL - C\left(\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right)$ and rewriting, we then have:or, rewriting:

$$\begin{aligned} & C\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right] \\ = \; & WL\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right] + SD\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\widehat{D}_{H,t+j}\right] \\ & + \left(1-S\right)D\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t\left[-\widehat{Z}_t\right] \end{aligned} \qquad (65)$$

The analogous expression for the foreign country is, in terms of Home currency:

$$
\begin{aligned}
& C\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right] \\
= \; & WL\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right]+SD\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(1-S\right)D\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\widehat{D}_{H,t+j}\right]-B\widetilde{E}_{t}\left[-\widehat{Z}_{t}\right]
\end{aligned}
\tag{66}
$$

Deducting the foreign budget constraint from the Home one, we obtain:

$$
\begin{aligned}
& C\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\left(\widehat{P}^{H}_{C,t}+\widehat{C}_{H,t}\right)-\left(\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
= \; & WL\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}-\left(\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(2S-1\right)D\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{H,t+j}-\left(\widehat{D}_{F,t+j}-\widehat{Z}_{t+j}\right)\right)\right] \\
& +2B\widetilde{E}_{t}\left[-\widehat{Z}_{t}\right]
\end{aligned}
\tag{67}
$$

As mentioned above, with two assets and two sources of uncertainty, markets are complete to a first order approximation. It can then be shown that the Backus Smith Kollmann condition holds in linearised form:

$$
-\sigma\left(\widehat{C}_{H,t}-\widehat{C}_{F,t}\right)=\widehat{P}^{H}_{C,t}-\widehat{Z}_{t}-\widehat{P}^{F}_{C,t}=-\widehat{Q}_{t}
\tag{68}
$$

Consumption expenditure can then be expressed solely as a function of the real exchange rate:

$$
\begin{aligned}
& \left(\widehat{P}^{H}_{C,t}+\widehat{C}_{H,t}\right)-\left(\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right) \\
= \; & \left(1-\frac{1}{\sigma}\right)\left(\widehat{P}^{H}_{C,t}-\widehat{Z}_{t}-\widehat{P}^{F}_{C,t}\right)=-\left(1-\frac{1}{\sigma}\right)\widehat{Q}_{t}
\end{aligned}
\tag{69}
$$

Now define the return on human capital in country i as

$$R^W_{H,t} = \frac{W_{H,t}L_{H,t} + P^W_{H,t}}{P^W_{H,t-1},}, \tag{70}$$

where $P^W_{H,t} = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{C_{H,t+j}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+j}} W_{H,t+j}L_{H,t+j}$ is the present value of labour income in Home. Linearising, the expression for returns, we have:

$$R^W_{H,t} = (1-\beta)\sum_{j=0}^{\infty}\beta^j\left(-\sigma\left(\widehat{C}_{H,t+j} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j} + \widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right) \tag{71}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\widehat{R}^W_{H,t} - \widehat{\widetilde{R}}^W_{F,t} = (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right) \tag{72}$$

Now we can write the budget constraint as:

$$C\widehat{R}^Q_t = \frac{WL}{1-\beta}\widehat{R}^W_t + (2S-1)\frac{D}{1-\beta}\widehat{R}^S_t + 2B\widehat{R}^B_t, \tag{73}$$

where $R^W_t = \widehat{R}^W_{H,t} - \widehat{\widetilde{R}}^W_{F,t}, R^S_t = \widehat{R}^S_{H,t} - \widehat{\widetilde{R}}^S_{F,t},\ \widehat{R}^B_t = \widehat{R}^B_{H,t} - \widehat{\widetilde{R}}^B_{F,t} = \widetilde{E}_t\left[-\widehat{Z}_t\right]$ and $\widehat{R}^Q_t = -\left(1-\frac{1}{\sigma}\right)\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{Q}_{t+j}\right]$

Now project this equation on relative bond returns $\widehat{R}^B_t$:

$$CP\left[\widehat{R}^Q_t|\widehat{R}^B_t\right] = \frac{WL}{1-\beta}P\left[\widehat{R}^W_t|\widehat{R}^B_t\right] + \frac{D}{1-\beta}(2S-1)\,P\left[\widehat{R}^S_t|\widehat{R}^B_t\right] + 2B\widehat{R}^B_t, \tag{74}$$

where $P\left[\widehat{X}_t|\widehat{Y}_t\right]$ is the projection of X_t on Y_t. Subtracting this equation from the one before, we have:

$$\begin{aligned} & C\left(\widehat{R}^Q_t - P\left[\widehat{R}^Q_t|\widehat{R}^B_t\right]\right) \\ = \;& \frac{WL}{1-\beta}\left(\widehat{R}^W_t - P\left[\widehat{R}^W_t|\widehat{R}^B_t\right]\right) + \frac{D}{1-\beta}(2S-1)\left(\widehat{R}^S_t - P\left[\widehat{R}^S_t|\widehat{R}^B_t\right]\right) \end{aligned} \tag{75}$$

Rearranging, we have:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\,C\widetilde{E}_t\left(\widehat{R}_t^Q - P\left[\widehat{R}_t^Q|\widehat{R}_t^B\right]\right) - WL\left(\widehat{R}_t^W - P\left[\widehat{R}_t^W|\widehat{R}_t^B\right]\right)}{D\left(\widehat{R}_t^S - P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}\right) \tag{76}$$

Multiplying the numerator and the denominator by $\left(\widehat{R}_t^S - \left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)$, and taking unconditional expectations, we have:

$$S = \frac{1}{2}\left(1 + \frac{(1-\beta)\,Ccov_{R_t^B}\left(\widehat{R}_t^Q, \widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)} - \frac{WLcov_{R_t^B}\left(\widehat{R}_t^W, \widehat{R}_t^S\right)}{Dvar_{R_t^B}\left(\widehat{R}_t^S\right)}\right),$$

where $cov_{R_t^B}\left(X_t, Y_t\right)$ is the covariance of X_t and Y_t conditional on relative bond returns R_t^B.

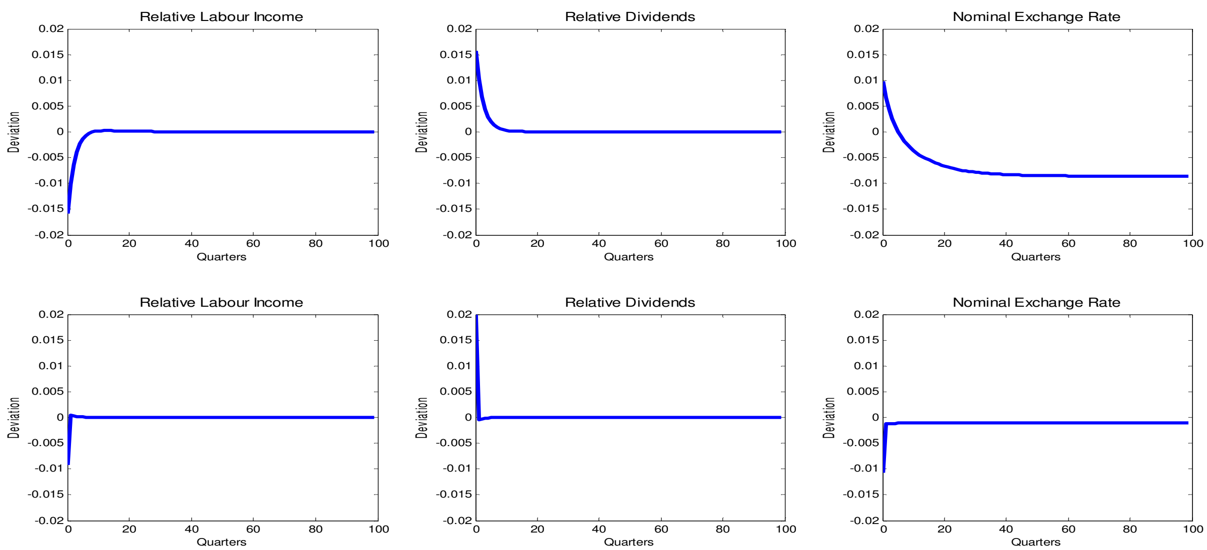

Figure 1: Impulse Responses to Relative Productivity Shocks (top row) and Relative Monetary Shocks (bottom row) in Sticky Price Model

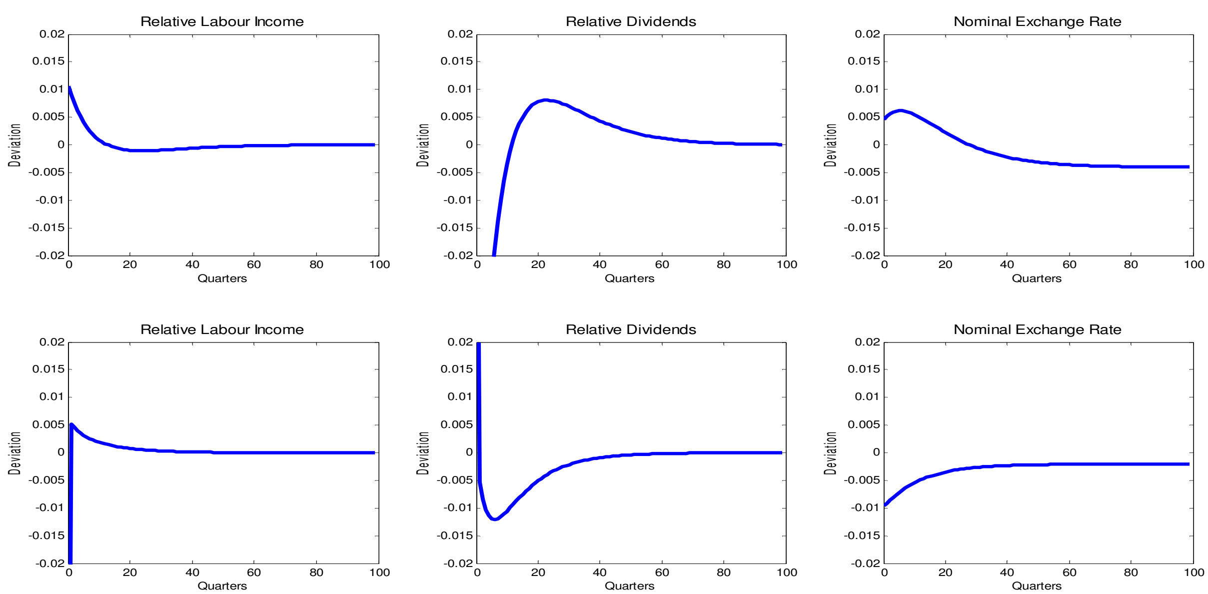

Figure 2: Impulse Responses to Relative Productivity Shocks (top row) and Relative Monetary Shocks (bottom row) in Benchmark Model

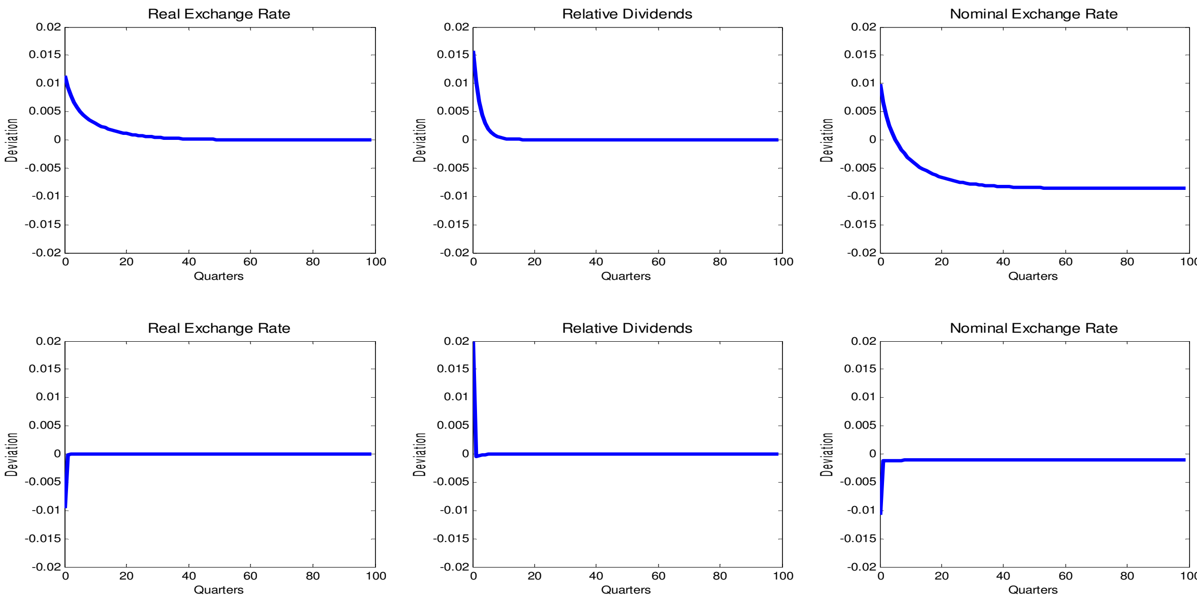

Figure 3: Impulse Responses to Relative Productivity Shocks (top row) and Relative Monetary Shocks (bottom row) in Sticky Price Model

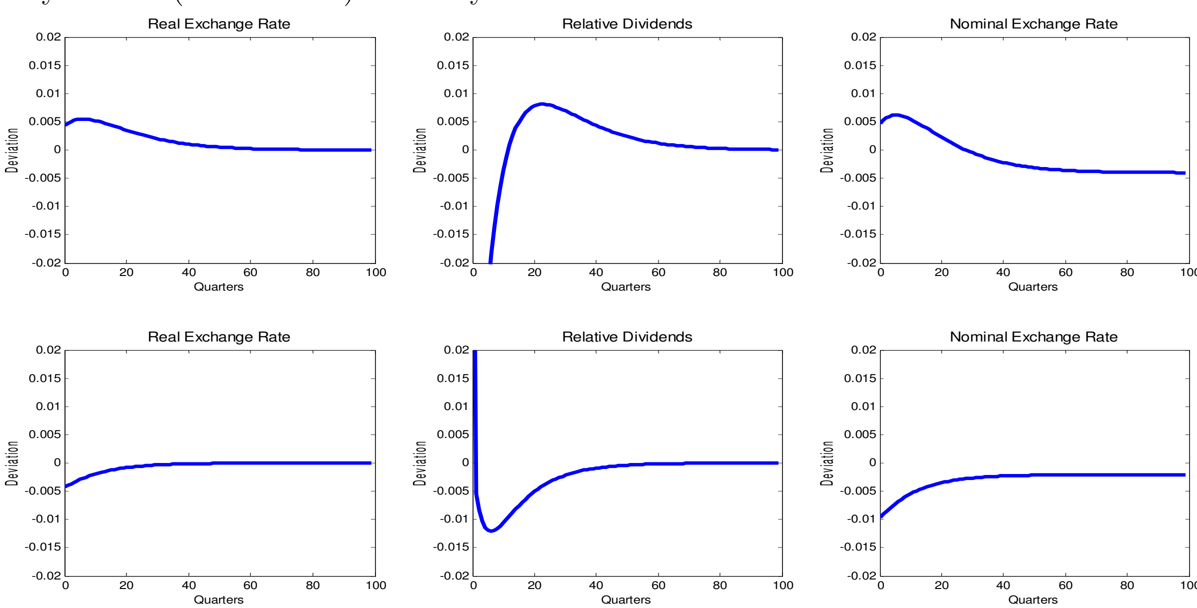

Figure 4: Impulse Responses to Relative Productivity Shocks (top row) and Relative Monetary Shocks (bottom row) in Benchmark Model