# Estimating Potential Output in a Modern Business Cycle Model

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**Abstract**

We estimate the potential level of output in postwar U.S. data using a modern business cycle model with imperfect competition and nominal rigidities. We show that a benchmark model is able to generate a measure of potential output that shows similarities with trend GDP, and therefore an output gap that is similar to traditional measures of the U.S. business cycle. These estimates are also robust to the specification of the monetary policy rule and the introduction of measurement errors in the estimation of the model. However, minor reinterpretations of the structural shocks in the model have significant effects on the estimated path of potential output.

**Keywords:** Efficient output; DSGE model; Bayesian estimation; Optimal monetary policy.

**JEL Classification:** E32, E52, C11.

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# 1 Introduction

The real business cycle literature (for example, Kydland and Prescott (1982); Long and Plosser (1983); King, Plosser, and Rebelo (1988)) demonstrated that business cycle fluctuations are not all inefficient. On the contrary, a large part of fluctuations may well be the efficient responses of firms and households to exogenous shifts in technology and preferences. This result greatly reduced the scope for economic policymakers to dampen business cycle fluctuations, at least in theory.

Modern monetary business cycle models (starting with Yun (1996); Goodfriend and King (1997); and Rotemberg and Woodford (1997)) extend the real business cycle framework to include imperfect competition and nominal rigidities. These models typically imply that economic policy should counteract fluctuations due to nominal rigidities, but accommodate fluctuations due to shifts in real factors (for example, technology and preferences). In other words, optimal policy should act to make the real economy mimic the core RBC model (Goodfriend and King (1997)).

Recent developments of the monetary business cycle model have demonstrated that medium-sized versions of the model that incorporate more frictions and shocks are able to well match the behavior of aggregate macroeconomic variables (Smets and Wouters (2007)). These models are now in use at many central banks for policy simulations and forecasting.

A strength of these models is that they are based on the optimizing behavior of private agents, and therefore can be used to quantify the welfare consequences of alternative policies. In particular, these models can be used to estimate the potential level of output, that is, the level of output that would appear in an equilibrium without nominal rigidities or imperfect competition. This is also the level of output towards which optimal monetary policy should try to steer the economy in order to maximize household welfare. Recently, much work has tried to estimate potential output, the potential rate of unemployment or the potential interest real interest rate.[1] Central banks in many countries also use these estimates to inform their policy decisions.

We contribute to this literature by estimating the potential level of output in postwar U.S. data using a monetary business cycle model with imperfect competition and nominal rigidities. We contrast two alternative measures of potential output that differ in the way they treat the current state of the economy. The first measure, advocated by Neiss and Nelson (2003), uses state variables in the hypothetical allocation where prices and wages have been flexible forever. This definition has been criticized by Woodford (2003), who instead suggests a second measure where state variables are taken from the allocation where prices and wages unexpectedly become flexible today, but are expected to remain flexible in the future. The latter definition is thus conditional on the current state of the economy, and following Adolfson, Laséen, Lindé, and Svensson (2008) we call this measure "conditional potential output," and the first measure "unconditional potential output."

We begin by estimating a benchmark specification of the model, similar to the models in Smets and Wouters (2007) and Justiniano and Primiceri (2008). We show that this speci-

[1]See, for instance, Neiss and Nelson (2003), Neiss and Nelson (2005), Edge, Kiley, and Laforte (2008), Sala, Söderström, and Trigari (2008), Justiniano and Primiceri (2008), or Coenen, Smets, and Vetlov (2008).

fication of the model produces time paths for potential output that follow actual output in booms and recessions, but tend to stay closer to the long-run trend. In this sense, these measures are similar to measures obtained using statistical techniques (for example, different detrending procedures or Kalman filter techniques). The estimated output gaps (the deviation of output from potential) therefore have much in common with traditional measures of the U.S. business cycle. We also show that the conditional potential output follows actual output more closely than does the unconditional measure, so the conditional output gap is less volatile than the unconditional gap. And while both measures are fairly precisely estimated, the uncertainty concerning the conditional output gap is smaller than that surrounding the unconditional gap.

We then focus on the robustness of the estimates. We study three small changes in the specification of the model. We first respecify the monetary policy rule; in the benchmark model monetary policy is set as a function of the rate of output growth, while in the alternative model policy instead responds to the output gap. We then estimate versions of the model where we allow for errors in the measurement of data. And finally, we reinterpret the shocks in the model, so that a shock interpreted in the benchmark model as time-variation in the wage markup instead is interpreted as a shock to the preference for leisure. We show that the first two changes do not have very large effects on the estimates of potential output. The structural interpretation of shocks, however, has radical implications for the estimated level of potential output and the output gap. These models therefore have very different implications for the desirable conduct of policy.

We conclude that as long as we are confident that inefficient movements in wage markups are responsible for most of the high-frequency fluctuations in the wedge between the marginal product of labor and the marginal rate of substitution between consumption and leisure, then our results suggest that measuring potential output is mainly a quantitative issue. The qualitative properties of the estimates are rather similar. But any doubts about the source of fluctuations in the labor wedge lead to great uncertainty about the potential level of output, the output gap, and therefore about the appropriate design of monetary policy.

## 2 A benchmark model

Our benchmark model is based on the specification of Smets and Wouters (2007), and is similar to many models used in the literature. This particular specification is taken from Justiniano, Primiceri, and Tambalotti (2008) but is essentially identical to that in Smets and Wouters (2007). The model is a monetary Dynamic Stochastic General Equilibrium (DSGE) framework with habit formation, investment adjustment costs, variable capital utilization, and nominal price and wage rigidities. The model also includes growth in the form of a non-stationary technology shock, as in Altig, Christiano, Eichenbaum, and Lindé (2005).

### 2.1 Households

The model is populated by a continuum of households, indexed by $j \in [0, 1]$. Each household consumes final goods, supplies labor to intermediate goods firms, saves in one-period nominal government bonds, and accumulates physical capital through investment. It transforms physical capital to effective capital by choosing the capital utilization rate, and then rents effective capital to firms.

Household $j$ chooses consumption $C_t(j)$, labor supply $L_t(j)$, bond holdings $B_t(j)$, the rate of capital utilization $\nu_t$, investment $I_t$, and physical capital $\bar{K}_t$ to maximize the intertemporal utility function

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \left[ \log\left(C_{t+s} - hC_{t+s-1}\right) - \varepsilon^L \frac{L_{t+s}(j)^{1+\omega}}{1+\omega} \right] \right\}, \tag{1}$$

where $\beta$ is a discount factor, $h$ measures the degree of habits in consumption preferences, $\omega$ is the Frisch elasticity of labor supply, and $\varepsilon_t^b$ is an exogenous shock to preferences. This preference shock is assumed to follow the autoregressive process

$$\log \varepsilon_t^b = (1-\rho_b)\log \varepsilon^b + \rho_b \log \varepsilon_{t-1}^b + \zeta_t^b, \qquad \zeta_t^b \sim i.i.d.N(0, \sigma_b^2), \tag{2}$$

where $\varepsilon^b = 1$ is the steady-state level of the preference shock.

The capital utilization rate $\nu_t$ transforms physical capital into effective capital according to

$$K_t = \nu_t \bar{K}_{t-1}, \tag{3}$$

which is rented to intermediate goods firms at the rental rate $r_t^k$. The cost of capital utilization per unit of physical capital is given by $\mathcal{A}(\nu_t)$, and we assume that $\nu_t = 1$ in steady state, $\mathcal{A}(1) = 0$, and $\mathcal{A}'(1)/\mathcal{A}''(1) = \eta_\nu$, as in Christiano et, Eichenbaum, and Evans (2005) and others.

Physical capital accumulates according to

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \varepsilon_t^i \left[ 1 - \mathcal{S}\left( \frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{4}$$

where $\delta$ is the rate of depreciation, $\varepsilon_t^i$ is an investment-specific technology shock with mean unity, and $\mathcal{S}(\cdot)$ is an adjustment cost function which satisfies $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and $\mathcal{S}''(\gamma_z) = \eta_k > 0$, where $\gamma_z$ is the steady-state growth rate. The investment-specific technology shock follows the process

$$\log \varepsilon_t^i = (1-\rho_i)\log \varepsilon^i + \rho_i \log \varepsilon_{t-1}^i + \zeta_t^i, \qquad \zeta_t^i \sim i.i.d.N(0, \sigma_i^2), \tag{5}$$

where $\varepsilon^i = 1$.

Let $P_t$ be the nominal price level, $R_t$ the one-period nominal (gross) interest rate, $W_t$ the nominal wage, $\Pi_t$ the household's lump-sum profits from ownership of firms, and $T_t$ lump-sum transfers. Household $j$'s budget constraint is then given by

$$P_tC_t + P_tI_t + B_t = T_t + R_{t-1}B_{t-1} + \Pi_t + W_t(j)L_t(j) + r_t^k \nu_t \bar{K}_{t-1} - P_t\mathcal{A}(\nu_t)\bar{K}_{t-1}. \tag{6}$$

### 2.2 Final goods producing firms

There is a perfectly competitive sector that combines a continuum of intermediate goods $Y_t(i)$ indexed by $i \in [0,1]$ into a final consumption good $Y_t$ according to the production function

$$Y_t = \left[ \int_0^1 Y_t(i)^{1/\varepsilon_t^p} di \right]^{\varepsilon_t^p}, \tag{7}$$

where $\varepsilon_t^p$ is a time-varying measure of substitutability among differentiated intermediate goods. This substitutability implies a time-varying (gross) markup of price over marginal cost equal to $\varepsilon_t^p$ that is assumed to follow the process

$$\log \varepsilon_t^p = \left(1 - \rho_p\right) \log \varepsilon^p + \rho_p \log \varepsilon_{t-1}^p + \zeta_t^p, \qquad \zeta_t^p \sim i.i.d.N(0, \sigma_p^2), \tag{8}$$

where $\varepsilon^p$ is the steady-state price markup.

Profit maximization by final goods producing firms yields the set of demand equations

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\varepsilon_t^p/(\varepsilon_t^p - 1)} Y_t, \tag{9}$$

for each $i$, where $P_t(i)$ is the price of intermediate good $i$ and $P_t$ is an aggregate price index given by

$$P_t = \left[ \int_0^1 P_t(i)^{1/(\varepsilon_t^p - 1)} di \right]^{\varepsilon_t^p - 1}. \tag{10}$$

### 2.3 Intermediate goods producing firms

Each intermediate good firm produces a differentiatied intermediate good $i$ using capital and labor goods according to the production function

$$Y_t(i) = \max \left\{ K_t(i)^{\alpha} \left[ Z_t L_t(i) \right]^{1-\alpha} - Z_t F, 0 \right\}, \tag{11}$$

where $Z_t$ is a labor-augmenting productivity factor, whose growth rate $\varepsilon_t^z = Z_t / Z_{t-1}$ follows a stationary exogenous process with steady-state value $\varepsilon^z$ which corresponds to the economy's steady-state (gross) growth rate $\gamma_z$, and $F$ is a fixed cost that ensures that profits are zero. The rate of technology growth is assumed to follow

$$\log \varepsilon_t^z = (1 - \rho_z) \log \varepsilon^z + \rho_z \log \varepsilon_{t-1}^z + \zeta_t^z, \qquad \zeta_t^z \sim i.i.d.N(0, \sigma_z^2). \tag{12}$$

Thus, technology is non-stationary in levels but stationary in growth rates. We assume that capital is perfectly mobile across firms and that there is a competitive rental market for capital.

Cost minimization yields

$$MC_t(i) = \frac{W_t}{Z_t^{1-\alpha} \left( L_t(i)/K_t(i) \right)^{-\alpha}} \tag{13}$$

and

$$MC_t(i) = \frac{r_t^k}{Z_t^{1-\alpha}\left(K_t(i)/L_t(i)\right)^{\alpha-1}}, \tag{14}$$

implying that the marginal cost is common across firms and given by

$$MC_t = \left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]^{-1} (W_t/A_t)^{1-\alpha} \left(r_t^k\right)^\alpha. \tag{15}$$

Prices of intermediate goods are set in a staggered fashion, following Calvo (1983). Thus, only a fraction $1-\lambda_p$ of firms are able to reoptimize their price in any given period. The remaining fraction index their price to a combination of past inflation and steady-state inflation according to the rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}, \tag{16}$$

where $\pi_t = P_t/P_{t-1}$ is the rate of inflation and $\pi$ is the steady-state value of $\pi_t$. Firms that are able to set their price optimally instead choose their price $\tilde{P}_t$ to maximize the present value of future profits over the expected life-time of the price contract:

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} (\beta\lambda_p)^s \frac{\lambda_{t+s}}{\lambda_t} \left\{ \left[\prod\nolimits_{k=0}^{s} \pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p} P_t(i)\right] Y_{t+s}(i) - \left[W_{t+s}L_{t+s}(i) + r_{t+s}^k K_{t+s}(i)\right]\right\}\right\}, \tag{17}$$

where $\lambda_t$ is the marginal utility of consumption at time $t$.

The first order condition associated with this problem is

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} (\beta\lambda_p)^s \left[\frac{\lambda_{t+s}}{\lambda_t} Y_{t,t+s} \left(\prod\nolimits_{k=0}^{s} \pi_{t+k-1}^{\gamma_p}\pi^{1-\gamma_p} P_t^* - \varepsilon_{t+s}^p MC_{t+s}\right)\right]\right\} = 0, \tag{18}$$

where $MC_t$ is the nominal marginal cost. In the limiting case of full price flexibility ($\lambda_p = 0$) the optimal price is

$$P_t^* = \varepsilon_t^p MC_t, \tag{19}$$

for all $t$.

The evolution of the price index is

$$P_t = \left\{ (1-\lambda_p) \left(P_t^*\right)^{1/(\varepsilon_t^p-1)} + \lambda_p \left(\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}P_{t-1}\right)^{1/(\varepsilon_t^p-1)}\right\}^{\varepsilon_t^p-1}. \tag{20}$$

### 2.4 The labor market

As in Erceg, Henderson, and Levin (2000), each household is a monopolistic supplier of specialized labor $L_t(j)$, which is combined by perfectly competitive employment agencies into labor services $L_t$ according to the CES aggregator

$$L_t = \left[ \int_0^1 L_t(j)^{1/\varepsilon_t^w} dj \right]^{\varepsilon_t^w}, \tag{21}$$

where $\varepsilon_t^w$ is a time-varying measure of substitutability among labor varieties that translates into a time-varying (gross) markup of wages over the marginal rate of substitution between consumption and leisure equal to $\varepsilon_t^w$. This markup process is assumed to follow

$$\log \varepsilon_t^w = (1-\rho_w) \log \varepsilon^w + \rho_w \log \varepsilon_{t-1}^w + \zeta_t^w, \qquad \zeta_t^w \sim i.i.d.N(0, \sigma_w^2). \tag{22}$$

where $\varepsilon^w$ is the steady-state wage markup.[2]

Profit maximization by employment agencies yields the set of demand equations

$$L_t(j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\varepsilon_t^w/(\varepsilon_t^w - 1)} L_t, \tag{23}$$

for each $j$, where $W_t(j)$ is the wage received from employment agencies by the household supplying labor variety $j$, while $W_t$ is the aggregate wage index received from intermidiate firms by employment agencies, given by

$$W_t = \left[ \int_0^1 W_t(j)^{1/(\varepsilon_t^w - 1)} dj \right]^{\varepsilon_t^w - 1}. \tag{24}$$

In any given period, a fraction $1 - \lambda_p$ of households are able to set their wage optimally. The remaining fraction indexes their wage to past inflation according to

$$W_t(j) = W_{t-1}(j) \left( \pi_{t-1} \varepsilon_{t-1}^z \right)^{\gamma_w} (\pi \gamma_z)^{1-\gamma_w}. \tag{25}$$

The optimizing households instead choose an optimal wage $W_t^*$ to maximize

$$-\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \lambda_w)^s \varepsilon_{t+s}^b \left[ \varepsilon^L \frac{L_{t+s}(j)^{1+\omega}}{1+\omega} \right] \right\}, \tag{26}$$

subject to the labor demand function.

The first order condition associated with this problem is

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \lambda_w)^s \left[ \lambda_{t+s} L_{t,t+s} \left( \Pi_{t,t+s}^w \frac{W_t^*}{P_{t+s}} - \varepsilon_{t+s}^w \varepsilon_{t+s}^b \varepsilon^L \frac{L_{t,t+s}^{\omega}}{\lambda_{t+s}} \right) \right] \right\} = 0, \tag{27}$$

[2]It is well known that the wage markup shock is observationally equivalent to a shock to the preference for leisure, a "labor supply shock" (see, for example, Chari, Kehoe, and McGrattan, (2007); Smets and Wouters (2007); or Shimer (2009)). However, the two interpretations have different implications for potential output. In the benchmark model we use the specification with a wage markup shock; in Subsection 5.4 we instead explore an interpretation with labor supply shocks.

with

$$\Pi^w_{t,t+s} = \begin{cases} 1 \quad for \quad s=0 \\ \prod_{k=1}^{s}\left[\left(\pi_{t+k-1}\varepsilon^z_{t+k-1}\right)^{\gamma_w}\left(\pi\gamma_z\right)^{1-\gamma_w}\right] \quad for \quad s=1,2,...,\infty \end{cases} \tag{28}$$

The limiting case of full wage flexibility ($\lambda_w = 0$) implies that

$$\frac{W_t^*}{P_t} = \varepsilon_t^w \varepsilon_t^b \varepsilon^L \frac{L_t^\omega}{\lambda_t} \tag{29}$$

for all $t$.

The evolution of the wage index is

$$W_t = \left\{(1-\lambda_w)\left(W_t^*\right)^{1/(\varepsilon_t^w-1)} + \lambda_w\left[\left(\pi_{t-1}\varepsilon^z_{t-1}\right)^{\gamma_w}\left(\pi\gamma_z\right)^{1-\gamma_w} W_{t-1}\right]^{1/(\varepsilon_t^w-1)}\right\}^{\varepsilon_t^w-1}. \tag{30}$$

## 2.5 Government

The government sets public spending $G_t$ according to

$$G_t = \left[1 - \frac{1}{\varepsilon_t^g}\right] Y_t, \tag{31}$$

where $\varepsilon_t^g$ is a spending shock with mean unity that follows the process

$$\log \varepsilon_t^g = (1-\rho_z)\log \varepsilon^g + \rho_z \log \varepsilon^g_{t-1} + \zeta_t^g, \qquad \zeta_t^g \sim i.i.d.N(0,\sigma_g^2). \tag{32}$$

Monetary policy is set follwing the interest rate rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_s}\left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi}\left(\frac{\Delta \log Y_t}{\gamma_z}\right)^{r_y}\right]^{1-\rho_s}\varepsilon_t^r, \tag{33}$$

where $\varepsilon_t^r$ is a monetary policy shock with mean unity that follows

$$\log \varepsilon_t^r = (1-\rho_r)\log \varepsilon^r + \rho_r \log \varepsilon^r_{t-1} + \zeta_t^r, \qquad \zeta_t^r \sim i.i.d.N(0,\sigma_r^2). \tag{34}$$

We specify the monetary policy rule in terms of output growth rather than the output gap, defined as the deviation of output from potential. Thus, in the benchmark model we implicitly assume that the central bank is unable to observe potential output, or is unwilling to let monetary policy depend on its estimate of potential output. As a consequence, our estimates of the benchmark model are independent of the definition of potential output. In Subsection 5.1 we instead estimate a version of the model where monetary policy responds to the output gap rather than output growth.

### 2.6 Market clearing

Finally, to close the model, the resource constraint implies that output is equal to the sum of consumption, investment, government spending, and the capital utilization costs:

$$Y_t = C_t + I_t + G_t + a(\nu_t)\bar{K}_{t-1}. \tag{35}$$

### 2.7 Model summary

The complete model consists of 15 equations in the 15 endogenous variables. In addition there are seven exogenous shocks: to household preferences, labor-augmenting technology, investment-specific technology, government spending, price and wage markups, and monetary policy. In the model output, the capital stock, investment, consumption, government spending, and the real wage all share the common trend introduced by the non-stationary technology shock. Therefore, the model is rewritten on stationary form by normalizing these variables by the non-stationary technology shock, and then log-linearized around its steady state. The log-linearized model is shown in Appendix A.[3]

## 3 Estimation results

We estimate the log-linearized version of the model on quarterly U.S. data from 1960Q1 to 2005Q1 for seven variables: (1) output growth: the quarterly growth rate of per capita real GDP; (2) consumption growth: the quarterly growth rate of per capita real personal consumption expenditures of nondurables; (3) investment growth: the quarterly growth rate of per capita real investment; (4) employment: hours of all persons in the non-farm business sector divided by population, multiplied by the ratio of total employment to employment in the non-farm business sector; (5) real wage growth: the quarterly growth rate of compensation per hour in the non-farm business sector; (6) inflation: the quarterly growth rate of the GDP deflator; and (7) the nominal interest rate: the quarterly average of the federal funds rate.[4]

We use growth rates for the non-stationary variables (output, consumption, investment, and the real wage, which are non-stationary also in the theoretical model) and we write the measurement equation of the Kalman filter to match the seven observable series with their model counterparts. Thus, the state-space form of the model is characterized by the state equation

$$\mathbf{X}_t = \mathbf{A}(\boldsymbol{\theta})\mathbf{X}_{t-1} + \mathbf{B}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t, \tag{36}$$

where $\mathbf{X}_t$ is a vector of state variables, $\boldsymbol{\varepsilon}_t$ is a vector of structural shocks, and $\boldsymbol{\theta}$ is a vector

[3] In addition, we supplement the log-linearized model with a block of equations that determines the allocation with flexible wages and prices. While this block is not needed in the estimation of the benchmark model with output growth in the monetary policy rule, it is required to estimate the model with the output gap in the policy rule.

[4] All data were obtained from the FRED data base of the Federal Reserve Bank of St. Louis.

of structural parameters; and the measurement equation

$$\mathbf{Y}_t = \mathbf{C}\mathbf{X}_t, \tag{37}$$

where $\mathbf{Y}_t$ is a vector of observable variables, that is,

$$\mathbf{Y}_t = [\Delta \log Y_t,\ \Delta \log C_t,\ \Delta \log I_t,\ L_t,\ \Delta \log W_t,\ \pi_t,\ R_t]\,. \tag{38}$$

In the benchmark estimation we assume that all variables are observable, so the observation equation does not contain measurement errors. In Subsection 5.2 below we instead estimate a version of the model where we introduce measurement errors.

The model contains 18 structural parameters, not including the parameters that characterize the seven exogenous shocks. We calibrate four parameters using standard values: the discount factor $\beta$ is set to 0.99, the capital depreciation rate $\delta$ to 0.025, the capital share $\alpha$ in the Cobb-Douglas production function is set to 0.33, and the average ratio of government spending to output $g/y$ to 0.2.

We estimate the remaining 14 structural parameters: the steady-state growth rate, $\gamma_z$; the elasticity of the utilization rate to the rental rate of capital, $\eta_\nu$;[5] the elasticity of the investment adjustment cost function, $\eta_k$; the habit parameter $h$ and the labor supply elasticity $\omega$; the steady-state price and markups $\varepsilon^p$ and $\varepsilon^w$; the wage and price rigidity parameters $\lambda_w$ and $\lambda_p$; the wage and price indexing parameters $\gamma_w$ and $\gamma_p$; and the Taylor rule parameters $r_\pi$, $r_y$, and $\rho_s$. In addition, we estimate the autoregressive parameters of the exogenous disturbances, as well as the standard deviations of the shocks.[6]

We estimate the model with Bayesian methods (see An and Schorfheide, 2007, for an overview). Letting $\boldsymbol{\theta}$ denote the vector of structural parameters to be estimated and $\mathbf{Y}$ the data sample, we combine the likelihood function, $L(\boldsymbol{\theta}, \mathbf{Y})$, with priors for the parameters to be estimated, $p(\boldsymbol{\theta})$, to obtain the posterior distribution: $L(\boldsymbol{\theta}, \mathbf{Y})p(\boldsymbol{\theta})$. Draws from the posterior distribution are generated with the Random-Walk Metropolis-Hastings algorithm.

Tables 1 and 2 report the prior distribution of the parameters along with the median and the 5th and 95th percentiles of the posterior distribution.

# 4 Potential output in the benchmark model

The aim of our study is to estimate the path of potential output in post-war U.S. data. We define potential output as the level of output in the counterfactual model without nominal rigidities and without shocks to monetary policy and to the price and wage markups. Following Woodford (2003, Ch. 5) and Edge (2003), we distinguish between two different measures of potential output. The first is derived from the allocation where prices and wages have been flexible forever, and thus uses the state variables from this allocation.[7] Following Adolf-

[5]Following Smets and Wouters (2007), we define $\psi_\nu$ such that $\eta_\nu = (1 - \psi_\nu)/\psi_\nu$ and estimate $\psi_\nu$.

[6]We normalize the shocks before estimation. In particular, . . . [to be completed]

[7]In our model, the state variables are the physical stock of capital, lagged consumption, and lagged investment.

son, Laséen, Lindé, and Svensson (2008) we call this the "unconditional potential output." The second measure instead uses the state variables in the allocation with sticky prices and wages. This measure, which we call "conditional potential output," is thus taken from an allocation where prices and wages unexpectedly become flexible today and are expected to remain flexible in the future.

The existing literature focuses on the unconditional measure.[8] Neiss and Nelson (2003) and Coenen, Smets, and Vetlov (2008) motivate this choice by the fact that the conditional potential output depends not only on the efficient shocks, but also on past shocks to monetary policy (and other inefficient shocks), through their effect on the current state variables. Therefore, if monetary policy is set as a function of the conditional output gap, a mistake in monetary policy today is not fully offset in the future, as it has affected both actual and potential output, and therefore has had a small effect on the output gap. Woodford (2003, Ch. 5) instead argues that the conditional potential output is more closely related to the efficient level, which depends on the current state of the economy.[9] Another argument in favor of the conditional measure is that monetary policy that is determined by the unconditional output gap depends not only on the current state of the economy and the current shocks, but also on the entire path of historical shocks, as these affect the state variables in the flexible price/wage economy. That is, the current state variables and shocks are not sufficient to determine potential output, and therefore the appropriate monetary policy.

Figure 1 shows the path of U.S. GDP since 1960 along with our two estimates of potential output. Over this 45-year period, the three series move closely together, with booms and recessions reflected in both actual and potential output. For instance, the recessions in the 1970s and 1980s saw a decrease in actual as well as in the two measures of potential output from the long-run trend, and all three variables increased relative to trend during the boom in the late 1990s. Both measures of potential output move less than actual output in recessions and booms, so actual output falls relative potential in recessions, and output grows faster than potential in booms. The conditional measure is more closely related to actual output than is the unconditional measure, which is natural as the conditional measure uses the actual state variables. The unconditional measure is therefore above the conditional measure during the 1970s when output was below trend, but below the conditional measure when output was above trend in the 1990s.

To understand further the difference between the two measures, Figure 2 shows the capital stock in the models with flexible and with sticky prices and wages. The capital stock in the hypothetical flexible price/wage model is much larger than in the model with sticky prices and wages throughout the sample period, and the difference was more than ten percent in the 1980s. Thus, a monetary policy that tried to attain the unconditional potential output

[8]Examples include Neiss and Nelson (2003, 2005), Edge, Kiley, and Laforte (2008), Sala, Söderström, and Trigari (2008), Justiniano and Primiceri (2008), Coenen, Smets, and Vetlov (2008), and Basu and Fernald (2009).

[9]The efficient allocation has perfect competition, so price and wage markups are zero. In our model where monopolistic profits in the intermediate goods sector are zero due to fixed costs, there is a constant distance between the efficient and the conditional potential levels of output. Thus fluctuations in the efficient and the conditional potential output are identical. This is no longer true with positive profits, as the steady-state ratios between output, consumption and investment then depend on the average markup.

would have been seriously constrained by the actual capital stock which was not consistent with such a high level of output.

Figure 3 shows the levels of actual and potential output expressed as deviations from the trend given by the non-stationary technology process. Again we see how conditional potential output moves much more in line with actual output than does the unconditional measure. The unconditional measure is also more volatile than the conditional measure and actual output: the standard deviations of the unconditional potential output is 3.85 percentage points, compared with 2.72 and 2.93, respectively, for the conditional potential and the actual level of output. The correlation between actual output and the unconditional measure is 0.62, while that with the conditional potential output is 0.87. Thus, measured as conditional potential output, a large fraction of the volatility of U.S. GDP is accounted for by efficient fluctuations.[10]

Figure 4 then shows the estimated output gaps, that is the percent deviation of actual output from potential. As conditional potential moves closely with output, the conditional output gap is typically smaller and less volatile than the unconditional gap. The two measures are closely related, however, and move together over time. (The correlation coefficient is around 0.95.) Again we see that the output gaps tend to fall in recessions (as in the mid-1970s and early 1980s) and increase in booms (as during the late 1990s). The largest troughs in the two gaps is in 1975 and in the recession following the Volcker disinflation in 1982–83. Thus, our model interprets the recession in the early 1980s as a large drop in actual output below potential. This is of course in line with the usual interpretation of that recession as being caused by disinflationary monetary policy. However, earlier estimates of potential output instead implied large drops in both actual and potential output and a positive output gap through the 1980s; see, for instance, Walsh (2005) or Chari, Kehoe, and McGrattan (2009).[11] A puzzling feature of the output gaps is that they were negative also throughout the 1970s. Thus, our model does not interpret the 1970s as a period where potential output fell below actual, leading to high inflation. Instead, a historical decomposition suggests that this period was characterized by large positive markup shocks, leading to high inflation and negative output gaps.[12]

Figure 4 also shows probability intervals around the output gap estimates, in the form of the 5th and 95th percentiles over 2,000 draws from the posterior distribution of parameters. The uncertainty surrounding the unconditional gap is reasonably large, with the width of the 90 percent probability interval ranging from 0.65 percentage points in the late 1980s to 3.5 percentage points in the beginning of the sample and 2 percentage points at the end. Thus, the uncertainty is larger at the beginning and the end of the sample than in the middle. The conditional gap is estimated with much more precision, with the width of the probability interval typically ranging from 0.5 to 0.8 percentage points. And there are no

[10]A decomposition of the theoretical variance of output reveals that efficient shocks account for around 45 percent of output fluctuations, and inefficient shocks for the remaining 55 percent.

[11]Sala, Söderström, and Trigari (2008) and Justiniano and Primiceri (2008) instead obtain negative output gaps in the Volcker recession.

[12]This interpretation could be sensitive to the specification of the monetary policy rule. In future versions we plan to estimate versions of the model where policy rule coefficients are allowed to differ before and after 1979–80, or with a time-varying inflation target.

signs of uncertainty being much larger at the beginning or end of the sample.

Finally, we compare the estimated output gaps with three traditional measures of the U.S. business cycle: recessions dated by the NBER, the deviation of output from a Hodrick-Prescott trend, and the deviation from potential as calculated by the Congressional Budget Office. Figure 5 shows the unconditional output gap along with these traditional gaps, and Figure 6 shows the conditional gap. The NBER recessions are the shaded areas.

Some features of the estimated output gaps are similar to the more traditional measures of the business cycle. Both estimates match well the business cycles dated by the NBER: in each of the NBER recessions, the estimated output gaps are falling sharply, that is, actual GDP is falling relative to potential GDP. The gaps calculated using the CBO estimate of potential output or a trend calculated with the Hodrick-Prescott filter are more volatile and have shorter periods of expansions and contractions. But also according to these measures, the 1960s and the period from the mid 1990s until the early 2000s were mainly characterized by expansions, while the 1970s and 1980s were largely contractions. The turning points of the estimated and the traditional gaps also largely coincide.

Our estimate of the unconditional output gap is also similar to those obtained using a model with search frictions on the labor market and equilibrium unemployment, as shown by Sala, Söderström, and Trigari (2008), or those estimated by Justiniano and Primiceri (2008) using a similar model. These paper do not estimate the conditional output gap, however.

# 5 Robustness

We have so far focused on the estimates of potential output coming from a benchmark model, which is similar to many models used in the literature. But our estimates are clearly sensitive to various choices concerning the exact model specification or the treatment of data in the estimation. In this section we therefore study how the estimates change when we alter some of these choices.

## 5.1 Specifying monetary policy in terms of the output gap

The benchmark model assumed that monetary policy was set as a function of the observed rate of output growth. With this specification of the policy rule, the estimated rule does not depend on the definition of potential output. However, the optimal policy in this class of models typically does not aim at stabilizing fluctuations in output growth, but rather at stabilizing output around potential output. We therefore reestimate the model assuming that the central bank is able to infer the level of potential output, and sets the interest rate as a function of the output gap.

Figure 7 compares the estimated path of the unconditional output gap in these two specifications of the model.[13] Qualitatively the gaps are very similar in the two models, although with the output gap in the policy rule, the estimated gap is slightly less volatile with smaller peaks and troughs.

[13]We have so far only estimated the model with the unconditional output gap in the policy rule.

Thus, the output gap estimate is fairly robust to this particular respecification of the monetary policy rule.

### 5.2 Introducing measurement errors

As a second robustness exercise, we reestimate the model allowing for errors in the measurement of the data used for estimation. We estimate seven different versions of the benchmark model, introducing errors in the measurement of one data series at a time: GDP, investment, consumption, the real wage, employment, the interest rate and inflation. The measurement equation of the Kalman filter is then respecified as

$$\mathbf{Y}_t = \mathbf{C}\mathbf{X}_t + \eta_t, \tag{39}$$

where $\eta_t$ is a vector of measurement errors, assumed to be normally i.i.d. with diagonal covariance matrix $\mathbf{\Sigma}_\eta$.

Figure 8 shows the estimated output gaps in the seven models with measurement errors. The estimated gaps are again similar to those in the benchmark model, although the size of business cycle fluctuations differ slightly. For instance, when we allow for measurement errors in GDP, the model shifts the estimated output gap up in the 1960s and 1970s and down from the early 1980s until the end of the sample. This pattern is more pronounced for the unconditional output gap. Therefore, the model implies a larger positive output gap in the 1960s but a smaller positive gap in the late 1990s and early 2000s. But the fluctuations in the different output gaps are still very similar.

### 5.3 A range of estimates

The robustness exercises so far have had small qualitative effects on the estimated gaps. To evaluate the quantitative effects, Figures 9 and 10 show the unconditional and conditional gaps for all different models estimated so far. It is clear that even if the different gaps move in similar cycles over time, there is considerable uncertainty about the exact size of the gap.

This is especially obvious for the unconditional gap in Figure 9. The different models produce a wide range of gap estimates, where the largest gap is often several percentage points above the smallest gap. The uncertainty would be even larger if we also took into account the uncertainty from the posterior distribution of parameters.

Again, the uncertainty concerning the unconditional gap is larger at the beginning and the end of the sample. In the middle of the sample, the range of estimates is between one and three percentage points wide. In the beginning of the sample, the range is above four percentage points, and in the end of the sample it's above six percentage points. Of course, these different estimates have very different implications for a central bank that wants to set monetary policy to stabilize output around potential.

Figure 10 reveals, however, that uncertainty concerning the conditional output gap is smaller than for the unconditional gap. The range of estimates is typically around one percentage point wide, and the largest range, which is at the end of the sample, is 2.3 percentage points. Again, the conditional output gap is estimated with more precision than the unconditional gap.

### 5.4 Labor supply shocks or wage markup shocks

It is well known that the shocks to the wage markup in our benchmark model are observationally equivalent to shocks to the preference for leisure, or labor supply shocks; see, for instance, Chari, Kehoe, and McGrattan (2007), Smets and Wouters (2007), or Shimer (2009). However, the two shocks have very different implications for potential output: as wage markup shocks do not affect the efficient allocation, they are "inefficient" shocks that drive output away from potential and therefore should be counteracted by monetary policy. Labor supply shocks, on the other hand, are shocks to preferences which are "efficient" shocks that affect potential output and therefore should be accommodated by monetary policy.

To evaluate the sensitivity of the output gaps to these alternative interpretations of the model, we finally estimate a version of the model without wage markup shocks (so the elasticity of substitution across the differentiated labor types, $\lambda_w$ is constant) and instead introduce a shock to the preference for leisure, or a labor supply shock. Then household utility is given by

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \left[ \log \left( C_{t+s} - hC_{t+s-1} \right) - \varepsilon_{t+s}^{\psi} \frac{L_{t+s}(j)^{1+\omega}}{1+\omega} \right] \right\}, \tag{40}$$

where $\varepsilon_t^{\psi}$ has mean unity and is assumed to follow the process

$$\log \varepsilon_t^{\psi} = \left(1 - \rho_{\psi}\right) \log \varepsilon_t^{\psi} + \rho_{\psi} \log \varepsilon_{t-1}^{\psi} + \zeta_t^{\psi}, \qquad \zeta_t^{\psi} \sim i.i.d.N(0, \sigma_{\psi}^2). \tag{41}$$

It is easily verified that the log-linearized versions of the two models are observationally equivalent. Letting $\widehat{x}_t$ denote the log deviation of the (stationary) variable $X_t$ from steady state, the aggregate wage in the benchmark model can be written on log-linearized form as

$$\begin{aligned} \widehat{w}_t \;\; &= \;\; \gamma_b \left[ \widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \varepsilon_t^z \right] + \gamma_o \left[ \omega \widehat{l}_t - \widehat{\lambda}_t + \varepsilon_t^b \right] \\ &\quad + \gamma_f \left[ \widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t + \varepsilon_{t+1}^z \right] + \varepsilon_t^w, \end{aligned} \tag{42}$$

where $\gamma_b$, $\gamma_o$ and $\gamma_f$ are convolutions of various structural parameters. In the model with labor supply shocks, the aggregate wage instead follows

$$\begin{aligned} \widehat{w}_t \;\; &= \;\; \gamma_b \left[ \widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \varepsilon_t^z \right] + \gamma_o \left[ \omega \widehat{l}_t - \widehat{\lambda}_t + \varepsilon_t^b + \omega \varepsilon_t^{\psi} \right] \\ &\quad + \gamma_f \left[ \widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t + \varepsilon_{t+1}^z \right]. \end{aligned} \tag{43}$$

Thus, the two shocks are related as $\varepsilon_t^w = \gamma_o \omega \varepsilon_t^{\psi}$, and estimating one model is equivalent to estimating the other, up to the scale factor $\gamma_o \omega$ that only affects the estimated standard deviations of the two shocks.

In the benchmark model, the monetary policy rule is specified in terms of output growth. Therefore, the two interpretations are observationally equivalent also in the reduced form of the model, and normalizing the labor supply shock in the same way as the wage markup shock, the parameter estimates are identical.[14]

Figure 11 shows the actual and estimated potential output in the model with labor supply shocks. Figure 12 shows the estimated output gaps in this model, along with the gaps in the benchmark model with wage markup shocks. As expected, potential output is more volatile when we interpret the shock as a labor supply shock. However, the quantitative effects are striking: also the measure of potential output that is conditional on the current state variables is sometimes up to twice as large as actual output. The output gap is therefore completely dominated by movements in potential output. While the two measures of the output gap in the benchmark model have standard deviations of 3.1 and 1.5 percent for the unconditional and conditional gaps, respectively, the gaps in the model with labor supply shock are more than ten times as volatile, with standard deviations of 37 and 27 percent, respectively.

As the two models are observationally equivalent, their empirical fit is identical. There is therefore no way to determine empirically whether one model is preferred to the other. But the two interpretations of the model have very different implications for monetary policy.[15]

# 6 Concluding remarks

Our first estimates of potential output and the output gap were comforting. We showed that a benchmark model was able to produce sensible estimates of potential output, and also estimates of the output gap that have much in common with traditional interpretations of the U.S. business cycle. Our favorite measure of the output gap, that is conditional on the current state of the economy, was also very precisely estimated. This was true when we took into account uncertainty about parameters, as well as more general uncertainty about model specification.[16]

However, the estimates were then shown to be very sensitive to the structural interpretation of different shocks. We illustrated this by comparing estimates where we interpret shocks to the wage equation as originating in movements in wage markups or in shifts in the preference for leisure. These shocks both shift the wedge between the marginal product of labor and the marginal rate of substitution between consumption and leisure. But in our specific model framework, they are observationally equivalent, and can therefore not be distinguished empirically.[17]

[14]The observational equivalence no longer holds if the monetary policy rule is expressed in terms of the output gap. The equivalence also breaks down in the model with search and matching frictions estimated by Sala, Söderström, and Trigari (2008).

[15]Justiniano and Primiceri (2008) also study the sensitivity of their output gap estimates to this alternative interpretation of the shocks. Like our model, their benchmark model only has wage markup shocks, but these are i.i.d., rather than following an AR(1) process as in our model. Their alternative model introduces labor supply shocks that follow an AR(1) process in addition to the i.i.d. wage markup shocks, and they show that the estimated output gap is very similar in these two models. Presumably this is because the wage markup shock picks up the high-frequency fluctuations also in the alternative model. In our model, the labor supply shock is forced to pick up also the high-frequency fluctuations, and these translate into volatility in potential output.

[16]Future versions of this paper will extend this analysis along several lines. For instance, we intend to introduce breaks in the coefficients in the monetary policy rule, as well as time-variation in the inflation target.

[17]This equivalence is a key ingredient of Chari, Kehoe, and McGrattan's (2009) critique of the New Keyne-

As long as we are willing to believe that inefficient movements in wage markups are responsible for most of the high-frequency fluctuations in the "labor wedge," then our results suggest that measuring potential output is mainly a quantitative issue. The uncertainty faced could then be handled using appropriate techniques, for instance, Bayesian model averaging. But any doubts about the source of fluctuations in the labor wedge lead to great uncertainty about the potential level of output, the output gap, and therefore about the appropriate design of monetary policy.

Several authors (e.g., Chari, Kehoe, and McGrattan (2007) and Shimer (2009)) have questioned both of these interpretations of fluctuations in the labor wedge. Shimer (2009) suggests that the presence of search frictions on the labor market combined with real wage rigidities could be a more promising explanation for the time-varying labor wedge. The models developed by Gertler and Trigari (2008) and Gertler, Sala, and Trigari (2008) would seem to be suitable laboratories for pursuing this line of research. In future work we plan to go down this path.

sian model.

# A Steady state and log-linearized model

For the non-stationary variables $Y_t$, $K_t$, $Z_t$, $I_t$, $G_t$, and $W_t$, let $\widehat{x}_t$ denote the log deviation of the detrended variable from steady-state, that is

$$\widehat{x}_t \equiv \log\left(\frac{X_t/Z_t}{X/Z}\right). \tag{A1}$$

For the stationary variables, we instead let $\widehat{x}_t$ denote the log deviation from steady state:

$$\widehat{x}_t \equiv \log\left(\frac{X_t}{X}\right). \tag{A2}$$

The steady state and the log-linearized model is then given by the following system of equations for the endogenous variables.

## A.1 Steady state

[To be completed]

## A.2 Log-linearized model

Production function

$$\widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\,\widehat{l}_t \tag{A3}$$

Resource constraint

$$\widehat{y}_t = c_y\widehat{c}_t + i_y\widehat{i}_t + g_y\widehat{g}_t + z_y\widehat{z}_t, \tag{A4}$$

where $c_y = (C/Z)/(Y/Z)$, $i_y = (I/Z)/(Y/Z)$, $g_y = (G/Z)/(Y/Z)$, and $z_y = (r^k K/Z)/(Y/Z)$.

Effective capital

$$\widehat{k}_t + \varepsilon_t^z = \widehat{z}_t + \widehat{\bar{k}}_{t-1} \tag{A5}$$

Physical capital dynamics

$$\widehat{\bar{k}}_t = \frac{1-\delta}{\gamma_z}\left[\widehat{\bar{k}}_{t-1} - \varepsilon_t^z\right] + \left(1 - \frac{1-\delta}{\gamma_z}\right)\left[\widehat{i}_t + \varepsilon_t^i\right] \tag{A6}$$

Consumption Euler equation

$$\widehat{\lambda}_t = \mathrm{E}_t\widehat{\lambda}_{t+1} + [\widehat{r}_t - \mathrm{E}_t\widehat{\pi}_{t+1}] - \mathrm{E}_t\varepsilon_{t+1}^z \tag{A7}$$

Marginal utility

$$\left(1-\frac{h}{\gamma_z}\right)\left(1-\frac{\beta h}{\gamma_z}\right)\widehat{\lambda}_t = \frac{h}{\gamma_z}\left[\widehat{c}_{t-1}-\varepsilon_t^z\right]-\left(1+\frac{\beta h^2}{\gamma_z^2}\right)\widehat{c}_t + \frac{\beta h}{\gamma_z}\mathrm{E}_t\left[\widehat{c}_{t+1}+\varepsilon_{t+1}^z\right]+\left(1-\frac{h}{\gamma_z}\right)\left[\varepsilon_t^b-\frac{\beta h}{\gamma_z}\mathrm{E}_t\varepsilon_{t+1}^b\right] \tag{A8}$$

Capital utilization

$$\widehat{z}_t = \eta_\nu \widehat{r}_t^k \tag{A9}$$

Investment

$$\widehat{i}_t = \frac{1}{1+\beta}\left[\widehat{i}_{t-1}-\varepsilon_t^z\right]+\frac{1}{\eta_k\gamma_z^2(1+\beta)}\left[\widehat{q}_t^k+\varepsilon_t^i\right]+\frac{\beta}{1+\beta}\mathrm{E}_t\left[\widehat{i}_{t+1}+\varepsilon_{t+1}^z\right] \tag{A10}$$

Capital renting

$$\widehat{p}_t^w+\widehat{y}_t-\widehat{k}_t=\widehat{r}_t^k, \tag{A11}$$

where $\widehat{p}_t^w$ is marginal cost.

Tobin's q

$$\widehat{q}_t^z = \frac{\beta(1-\delta)}{\gamma_z}\mathrm{E}_t\widehat{q}_{t+1}^k+\left[1-\frac{\beta(1-\delta)}{\gamma_z}\right]\mathrm{E}_t\widehat{r}_{t+1}^z-\left[\widehat{r}_t-\mathrm{E}_t\widehat{\pi}_{t+1}\right] \tag{A12}$$

Labor demand

$$\widehat{w}_t=\widehat{p}_t^w+\widehat{y}_t-\widehat{l}_t \tag{A13}$$

Aggregate wage

$$\begin{aligned}\widehat{w}_t &= \gamma_b\left[\widehat{w}_{t-1}-\widehat{\pi}_t+\gamma\widehat{\pi}_{t-1}-\varepsilon_t^z\right]+\gamma_o\left[\omega\widehat{l}_t-\widehat{\lambda}_t+\varepsilon_t^b\right] \\ &\quad +\gamma_f\left[\widehat{w}_{t+1}+\widehat{\pi}_{t+1}-\gamma\widehat{\pi}_t+\varepsilon_{t+1}^z\right]+\varepsilon_t^w\end{aligned} \tag{A14}$$

where

$$\gamma_b=\phi^{-1},\quad \gamma_o=(\varsigma/\tau)\phi^{-1},\quad \gamma_f=\beta\phi^{-1},$$

and

$$\phi=1+\varsigma/\tau+\beta,\quad \varsigma=(1-\lambda)(1-\lambda\beta)\lambda^{-1},\quad \tau=1+\omega\varepsilon^w/\left(\varepsilon^w-1\right).$$

Phillips curve

$$\widehat{\pi}_t=\iota_b\widehat{\pi}_{t-1}+\iota_o\left[\widehat{p}_t^w+\varepsilon_t^p\right]+\iota_f\mathrm{E}_t\widehat{\pi}_{t+1}, \tag{A15}$$

where

$$\iota_b = \gamma^p \left(\phi^p\right)^{-1}, \quad \iota_o = \varsigma^p \left(\phi^p\right)^{-1}, \quad \iota_f = \beta \left(\phi^p\right)^{-1},$$

and

$$\phi^p = 1 + \beta\gamma^p, \quad \varsigma^p = \left(1 - \lambda^p\right)\left(1 - \lambda^p \beta\right)\left(\lambda^p\right)^{-1}.$$

Monetary policy rule

$$\widehat{r}_t = \rho_s \widehat{r}_{t-1} + \left(1 - \rho_s\right)\left[r_\pi \widehat{\pi}_t + r_y \left(\widehat{y}_t - \widehat{y}_{t-1} + \varepsilon_t^z\right)\right] + \varepsilon_t^r \tag{A16}$$

Government spending

$$\widehat{g}_t = \widehat{y}_t + \frac{1 - g_y}{g_y} \varepsilon_t^g \tag{A17}$$

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Table 1: Prior and posterior distributions of structural parameters in the benchmark model

| | | Prior distribution | Posterior distribution Median | 5% | 95% |
|---|---|---|---|---|---|
| Steady-state growth rate | $\gamma_z$ | Uniform (1,1.5) | 1.0034 | 1.0021 | 1.0046 |
| Utilization rate elasticity | $\psi_\nu$ | Beta (0.5,0.1) | 0.859 | 0.789 | 0.914 |
| Capital adjustment cost elasticity | $\eta_k$ | Normal (4,1.5) | 2.365 | 1.458 | 3.714 |
| Habit parameter | $h$ | Beta (0.5,0.1) | 0.818 | 0.768 | 0.852 |
| Labor supply elasticity | $\omega$ | Gamma (2,0.75) | 3.304 | 2.132 | 4.830 |
| Calvo wage parameter | $\lambda_w$ | Beta (0.75,0.1) | 0.827 | 0.766 | 0.873 |
| Calvo price parameter | $\lambda_p$ | Beta (0.66,0.1) | 0.549 | 0.498 | 0.603 |
| Wage indexing parameter | $\gamma_w$ | Uniform (0,1) | 0.963 | 0.890 | 0.996 |
| Price indexing parameter | $\gamma_p$ | Uniform (0,1) | 0.049 | 0.005 | 0.138 |
| Steady-state wage markup | $\varepsilon^w$ | Normal (1.15,0.05) | 1.141 | 1.061 | 1.226 |
| Steady-state price markup | $\varepsilon^p$ | Normal (1.15,0.05) | 1.344 | 1.289 | 1.398 |
| Taylor rule response to inflation | $r_\pi$ | Normal (1.7,0.3) | 1.925 | 1.694 | 2.185 |
| Taylor rule response to output | $r_y$ | Gamma (0.125,0.1) | 0.440 | 0.264 | 0.632 |
| Taylor rule inertia | $\rho_s$ | Beta (0.75,0.1) | 0.784 | 0.735 | 0.820 |

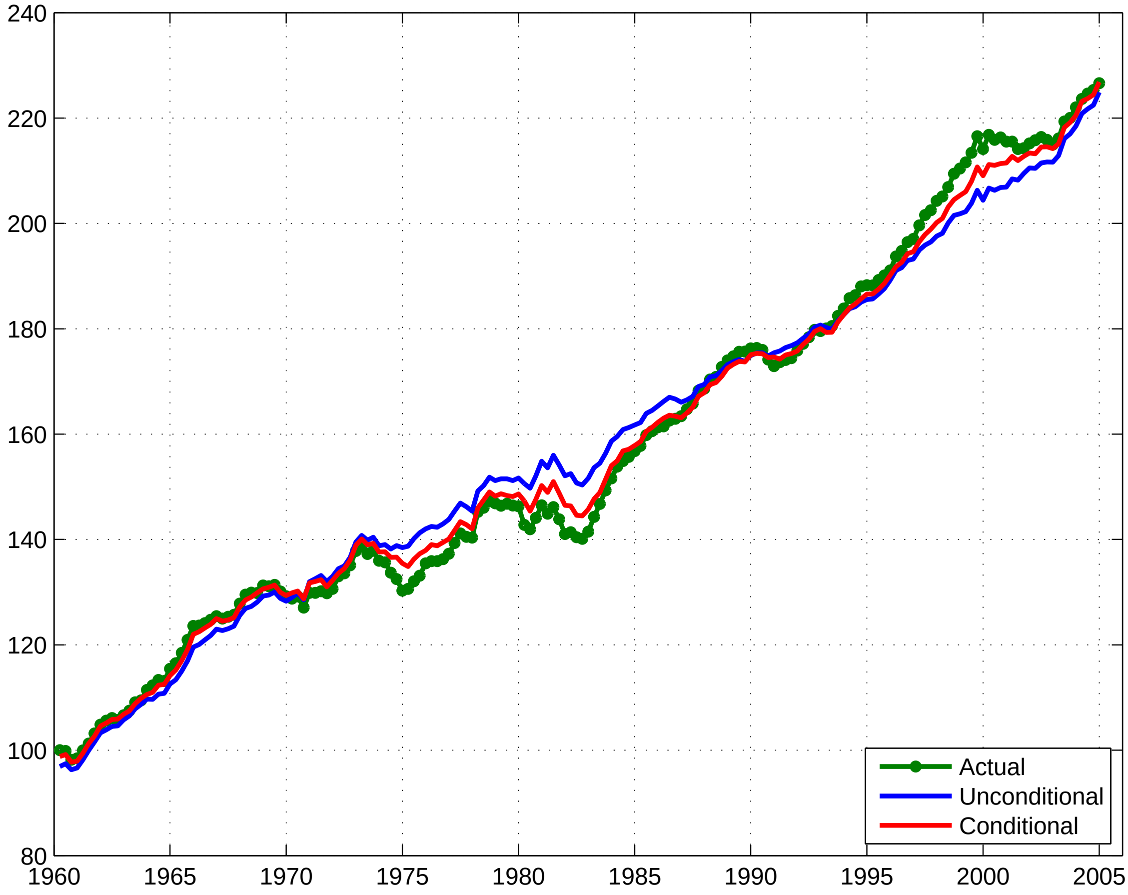
This table reports the prior and posterior distribution of the estimated structural parameters. For the uniform distribution, the two numbers in parentheses are the lower and upper bounds; for the other distributions the two numbers are the mean and the standard deviation of the distribution.

Table 2: Prior and posterior distributions of shock parameters in the benchmark model

| | | Prior distribution | Posterior distribution | | |
|---|---|---|---|---|---|
| | | | Median | 5% | 95% |
| *(a) Autoregressive parameters* | | | | | |
| Productivity growth rate | $\rho_z$ | Beta (0.5,0.15) | 0.149 | 0.082 | 0.236 |
| Preferences | $\rho_b$ | Beta (0.5,0.15) | 0.380 | 0.255 | 0.551 |
| Investment-specific technology | $\rho_i$ | Beta (0.5,0.15) | 0.861 | 0.797 | 0.917 |
| Wage markup | $\rho_w$ | Beta (0.5,0.15) | 0.271 | 0.171 | 0.383 |
| Price markup | $\rho_p$ | Beta (0.5,0.15) | 0.949 | 0.901 | 0.978 |
| Government spending | $\rho_g$ | Beta (0.5,0.15) | 0.989 | 0.982 | 0.994 |
| Monetary policy | $\rho_r$ | Beta (0.5,0.15) | 0.290 | 0.189 | 0.391 |
| *(b) Standard deviations* | | | | | |
| Productivity growth rate | $\sigma_z$ | IGamma (0.15,0.15) | 1.053 | 0.967 | 1.151 |
| Preferences | $\sigma_b$ | IGamma (0.15,0.15) | 0.852 | 0.469 | 1.366 |
| Investment-specific technology | $\sigma_i$ | IGamma (0.15,0.15) | 0.122 | 0.089 | 0.168 |
| Wage markup | $\sigma_w$ | IGamma (0.15,0.15) | 0.227 | 0.197 | 0.260 |
| Price markup | $\sigma_p$ | IGamma (0.15,0.15) | 0.091 | 0.076 | 0.109 |
| Government spending | $\sigma_g$ | IGamma (0.15,0.15) | 0.353 | 0.324 | 0.389 |
| Monetary policy | $\sigma_r$ | IGamma (0.15,0.15) | 0.232 | 0.211 | 0.257 |

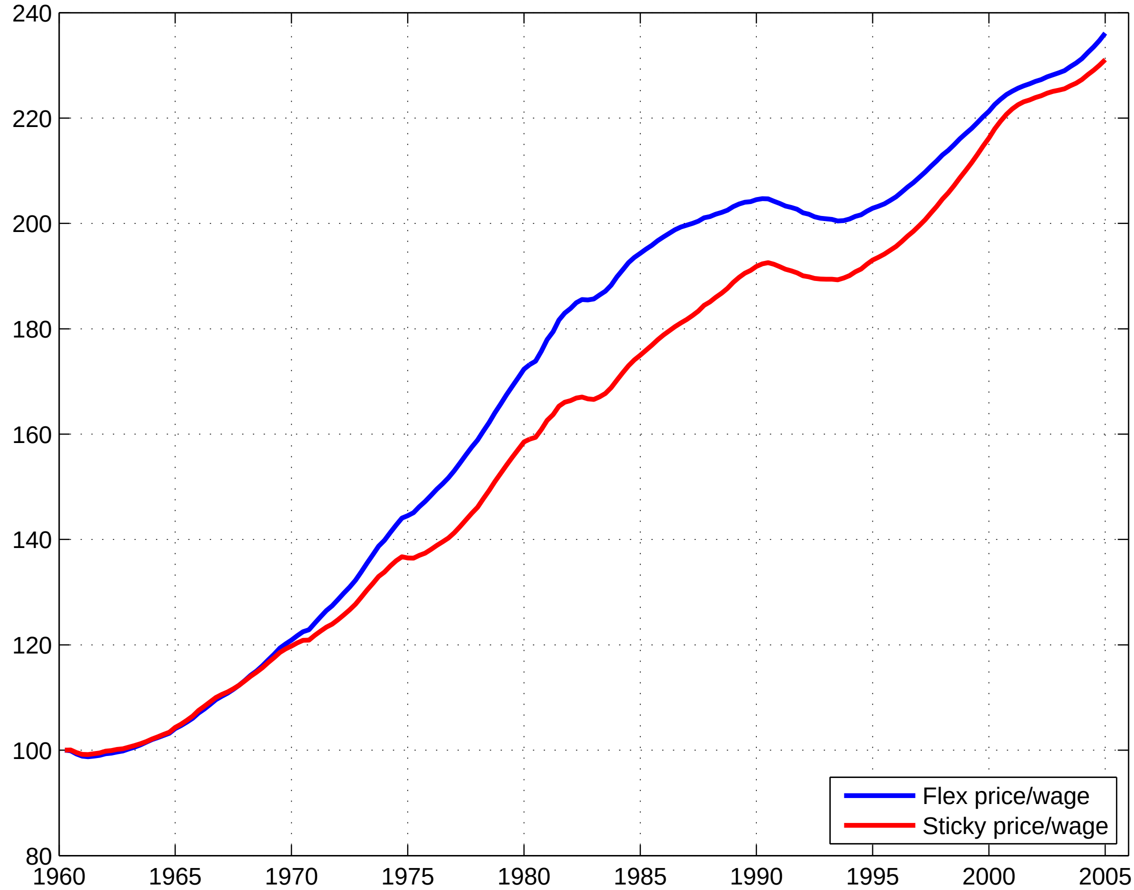
This table reports the prior and posterior distribution of the estimated parameters of the exogenous shock processes. The two numbers in parentheses are the mean and the standard deviation of the distribution.

Figure 1: Actual and potential output



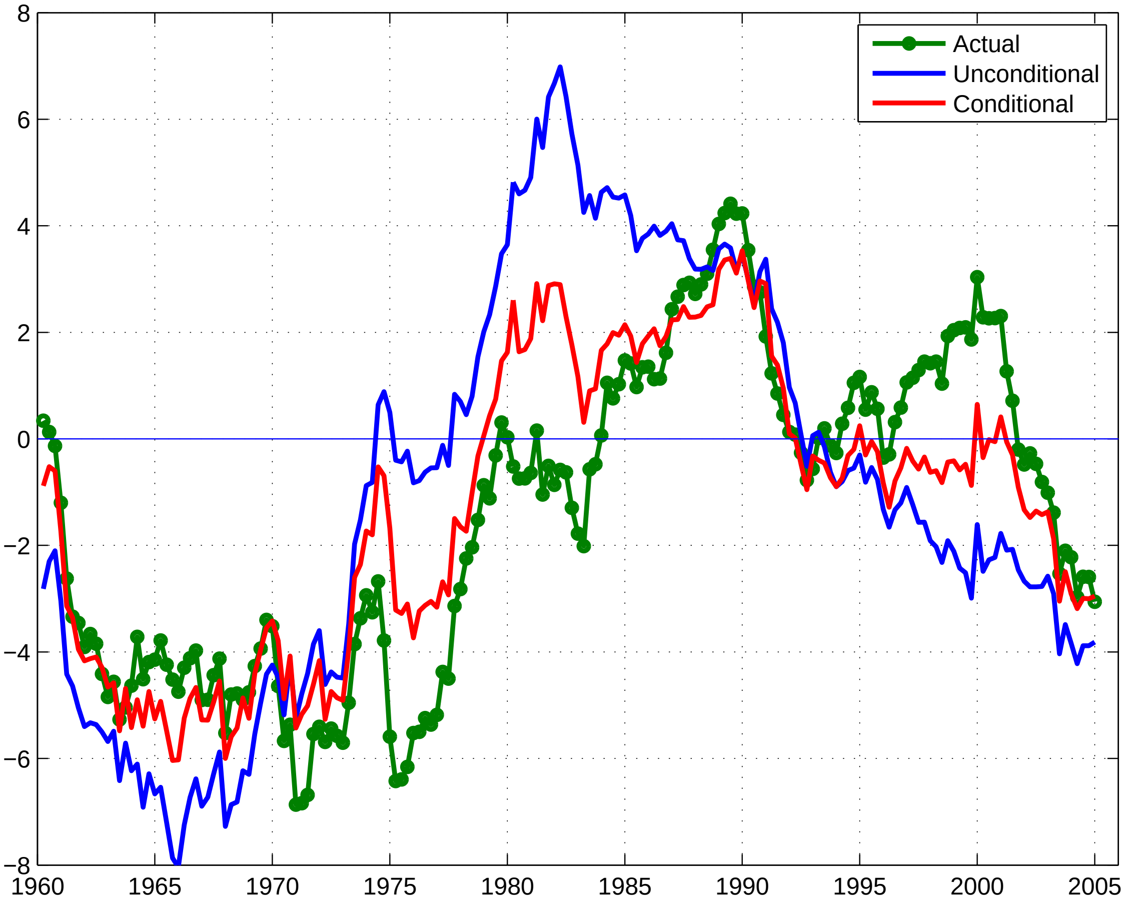

This figure shows the actual level of output and the estimated potential levels of output in the benchmark model. Actual output is normalized to 100 in 1960:Q1.

Figure 2: Capital stock in models with flexible and sticky prices and wages



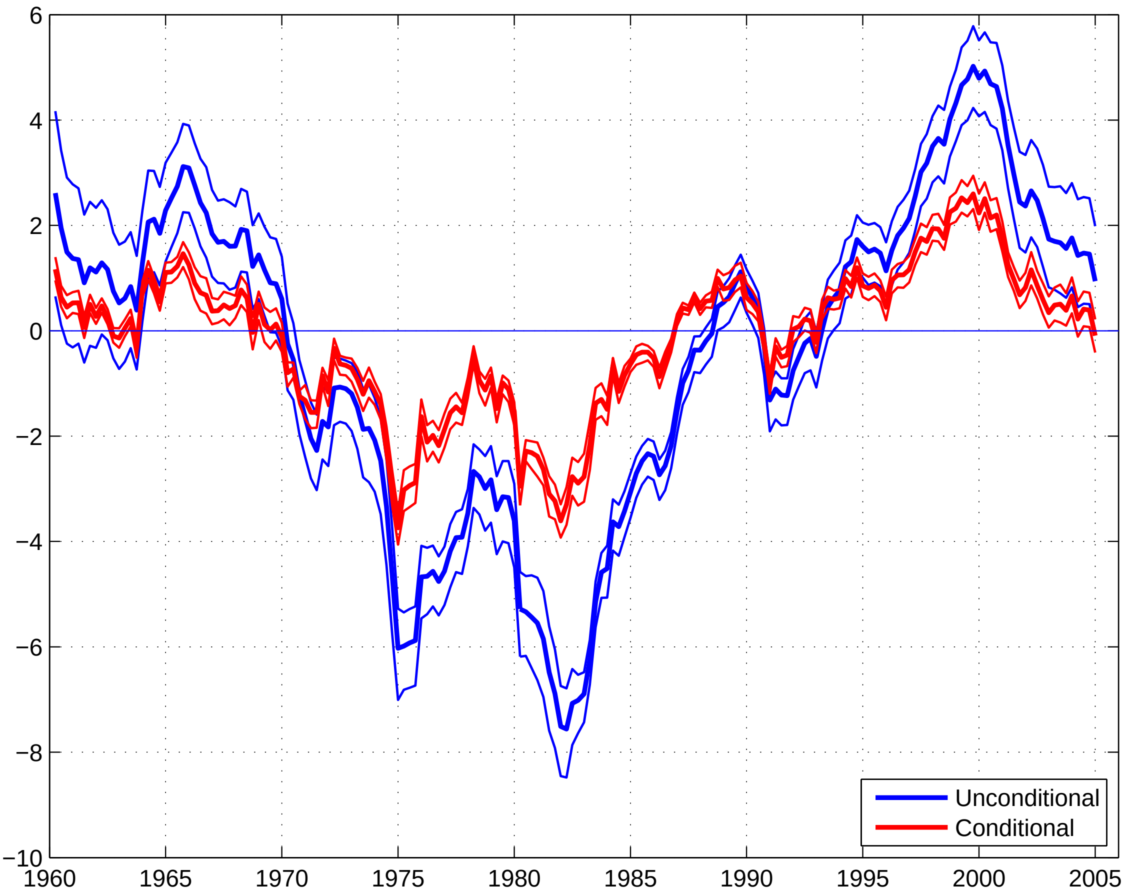

This figure shows the physical capital stock in the models with flexible wages and prices and with sticky wages and prices. The series is normalized to 100 in 1960:Q1.

Figure 3: Actual and potential output deviation from non-stationary technology process



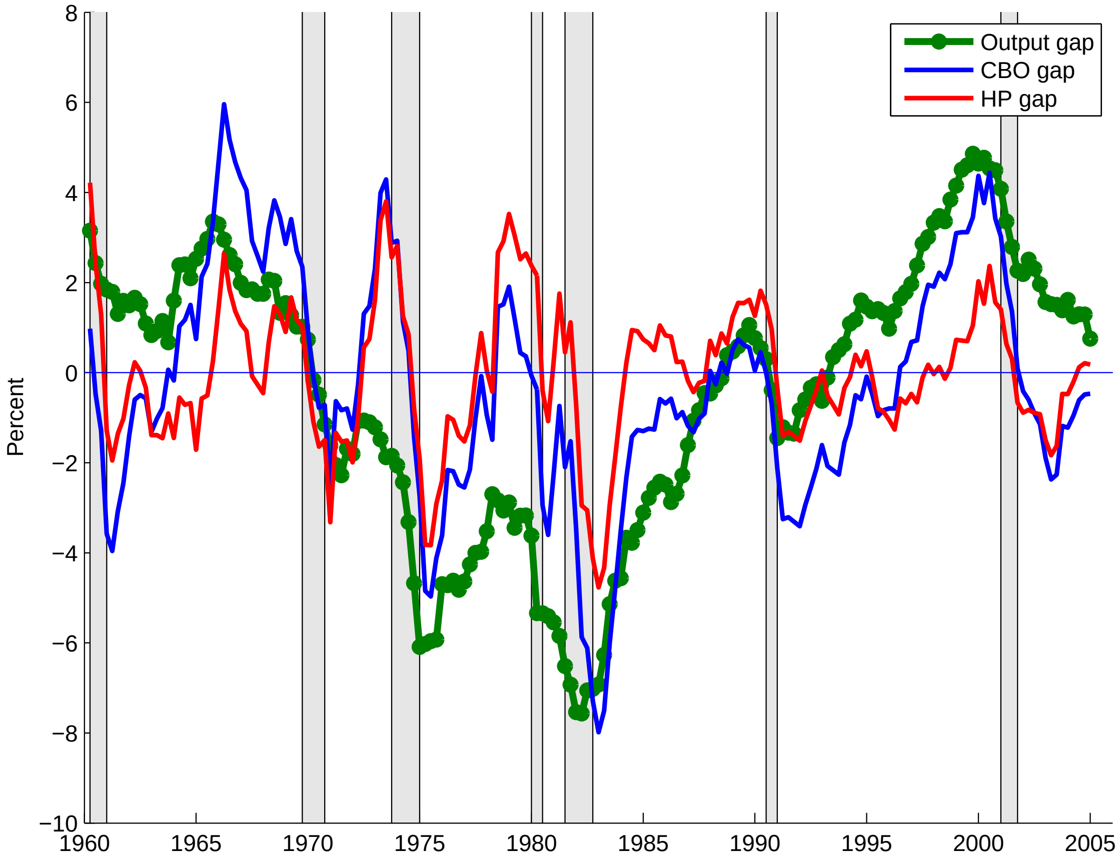

This figure shows the percent deviations of actual output and the estimated potential output from the non-stationary technology process in the benchmark model.

Figure 4: Estimated output gaps



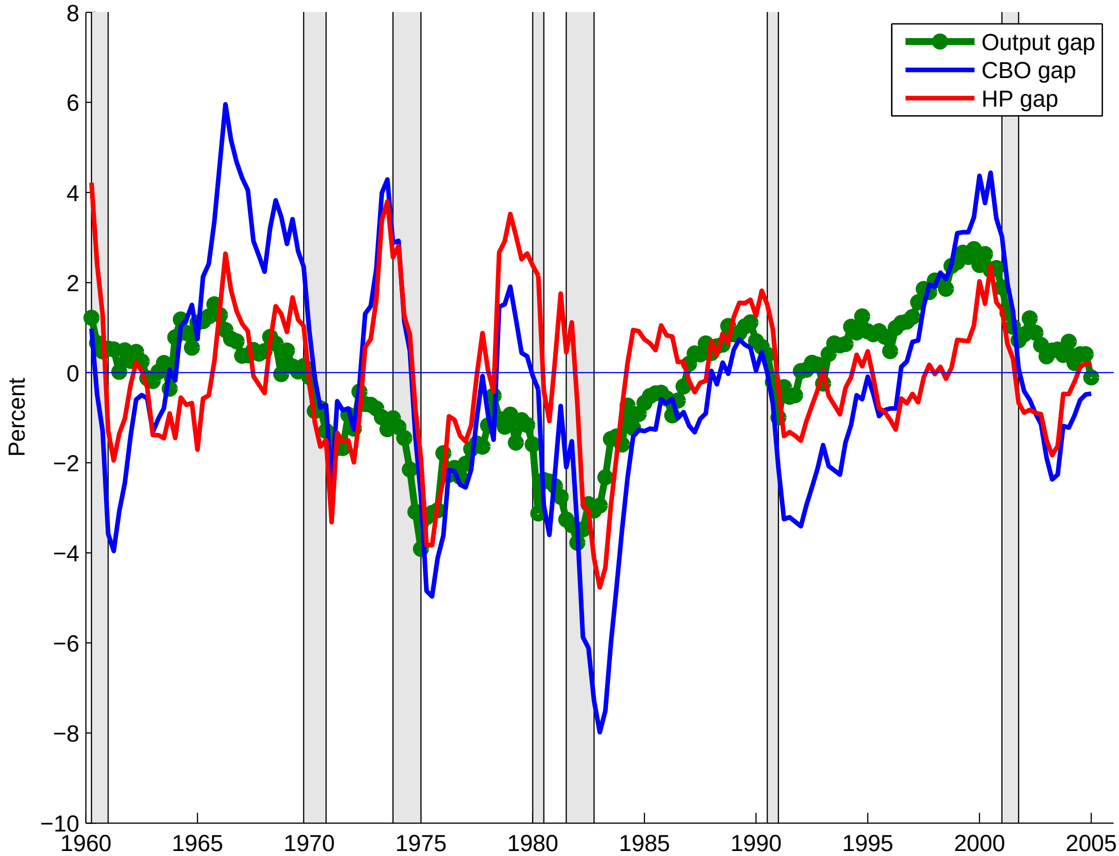

This figure shows the estimated output gaps in the benchmark model, along with the 5th and 95th percentiles from 2,000 draws from the posterior distribution of parameters.

Figure 5: Unconditional output gap and two traditional measures of the business cycle



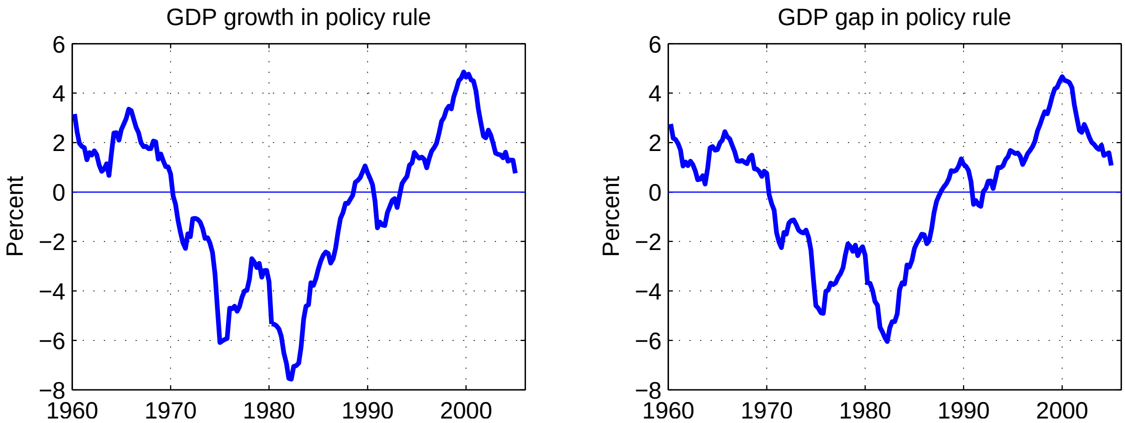

This figure shows the estimated unconditional output gap from the benchmark model, along with the deviation of GDP from potential output as calculated by the Congressional Budget Office and calculated with the Hodrick-Prescott filter.

Figure 6: Conditional output gap and two traditional measures of the business cycle



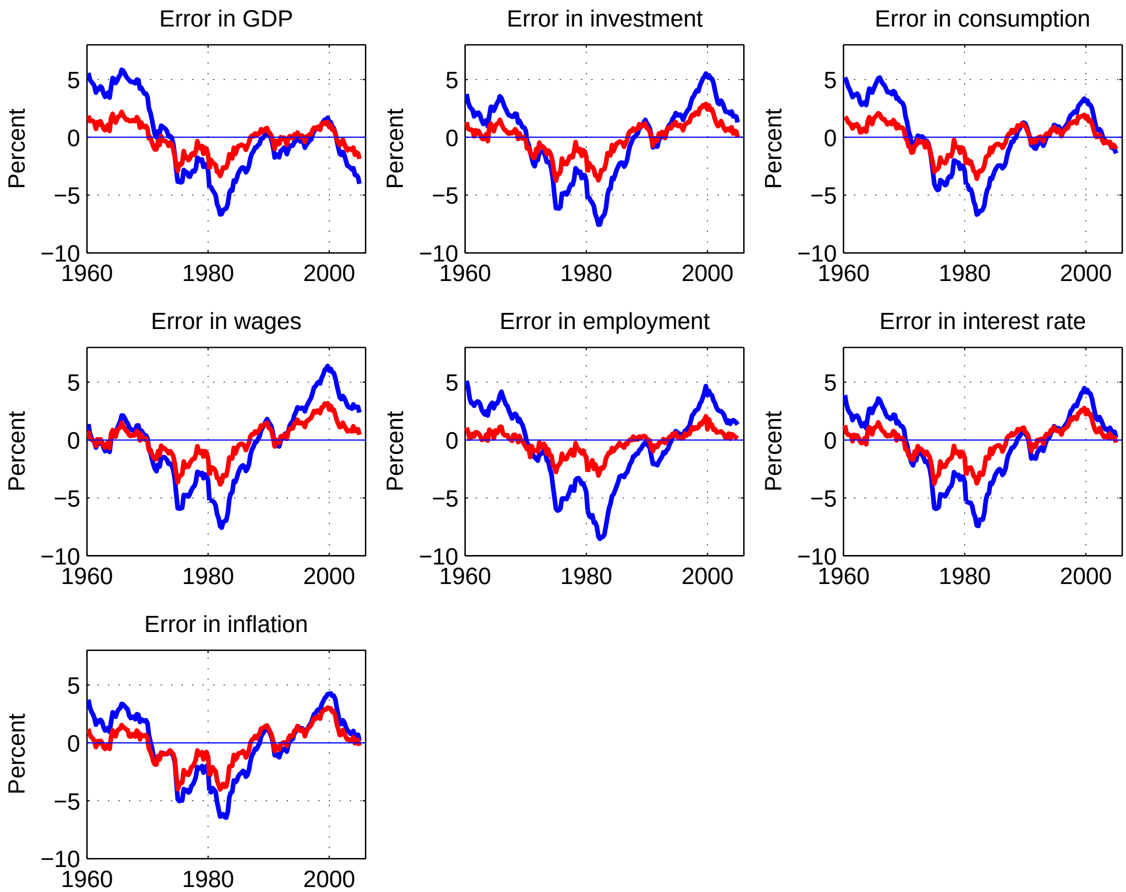

This figure shows the estimated conditional output gap from the benchmark model, along with the deviation of GDP from potential output as calculated by the Congressional Budget Office and calculated with the Hodrick-Prescott filter.

Figure 7: Unconditional output gap under two specifications of the monetary policy rule



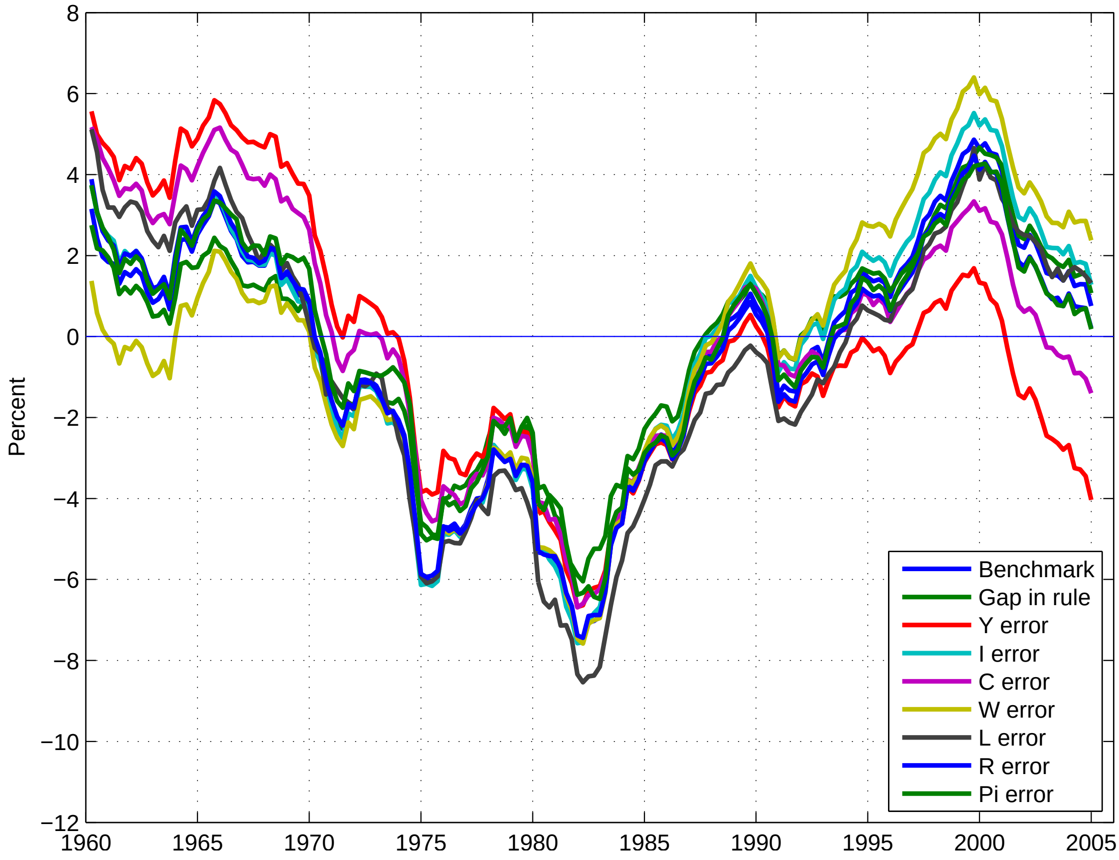

This figure shows the estimated unconditional output gap in the benchmark model with output growth in the monetary policy rule, and in a model with the unconditional output gap in the monetary policy rule.

Figure 8: Estimated output gaps in models with measurement errors



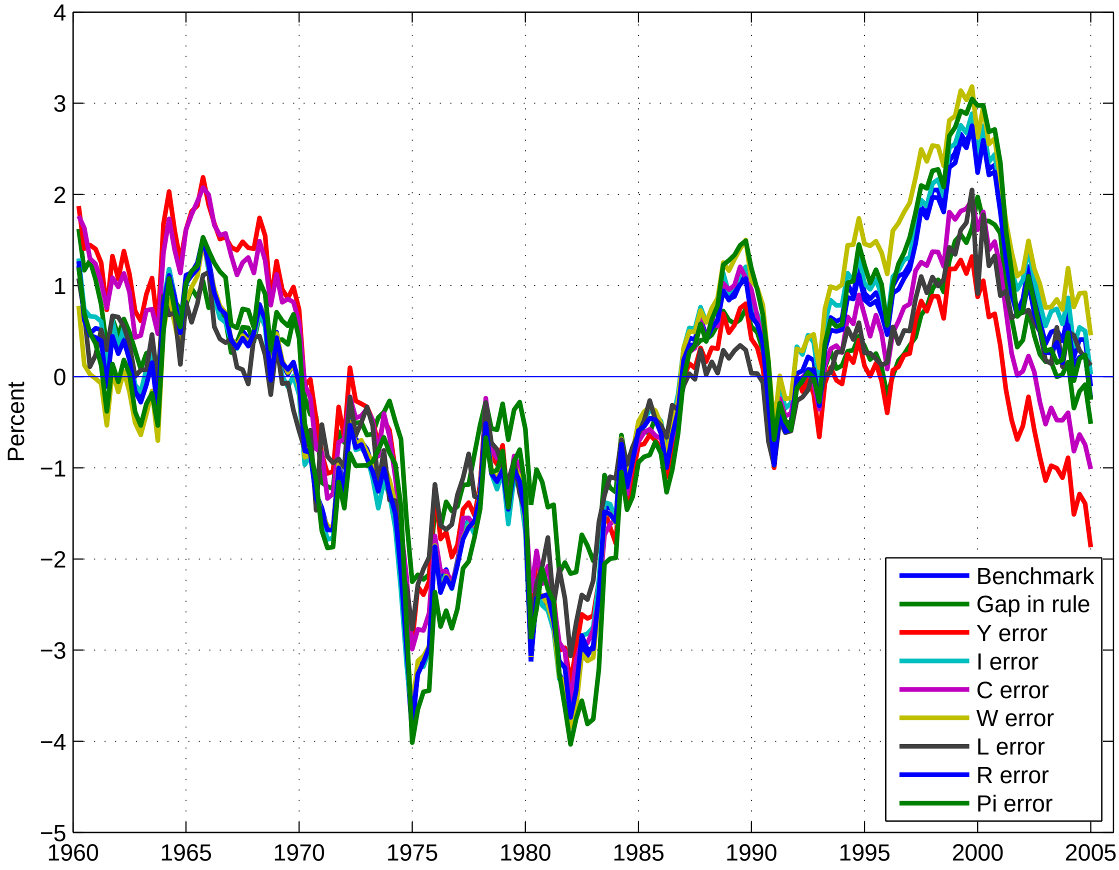

This figure shows the estimated unconditional and conditional output gaps in versions of the benchmark model estimated with measurement errors in one observable variable at a time.

Figure 9: Unconditional output gap estimates in different models



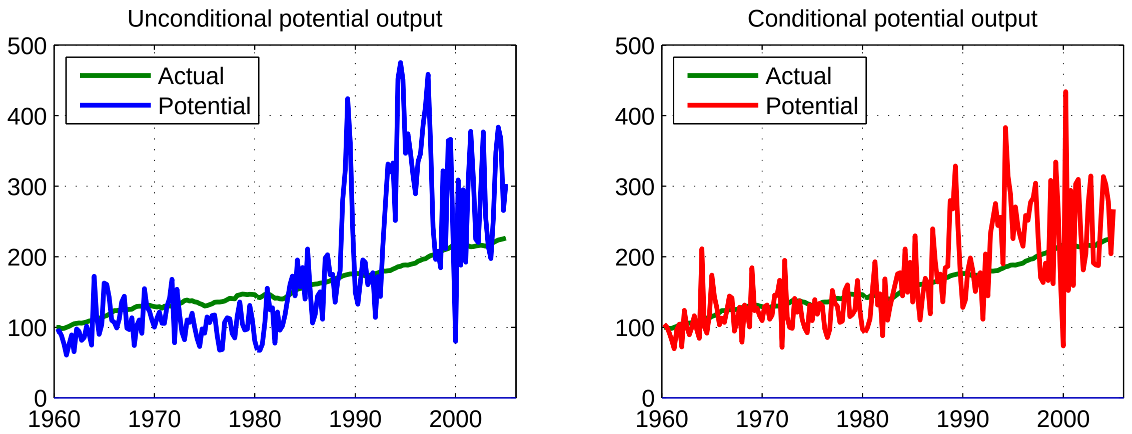

This figure shows the estimated unconditional output gap in nine different versions of the benchmark model.

Figure 10: Conditional output gap estimates in different models



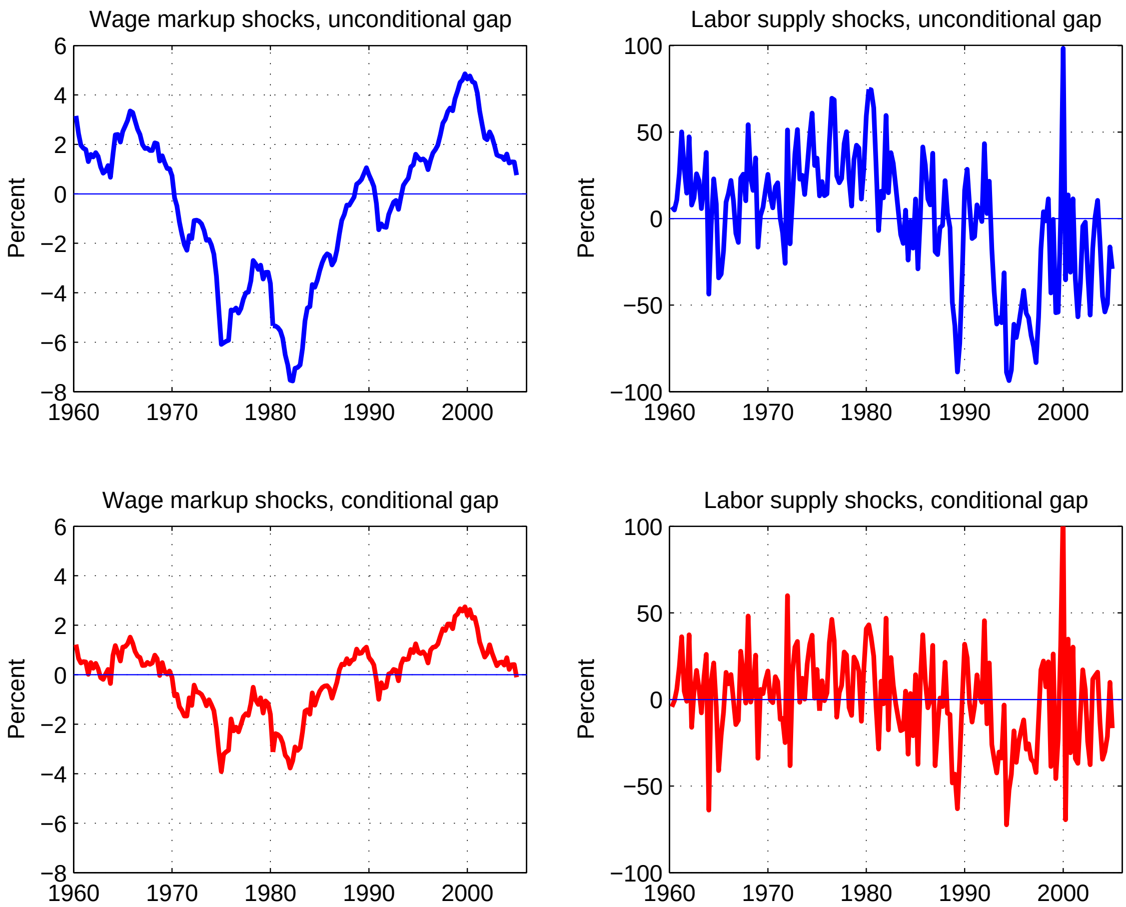

This figure shows the estimated conditional output gap in nine different versions of the benchmark model.

Figure 11: Actual and potential output in model with labor supply shocks




This figure shows the actual level of output and the estimated potential levels of output in a model with labor supply shocks. Actual output is normalized to 100 in 1960:Q1.

Figure 12: Output gaps under different interpretations of the wage shock




This figure shows the estimated output gaps in the benchmark model with wage markup shocks and a model with labor supply shocks.