Optimal monetary policy in an estimated DSGE Model for Hungary

Henrik Kucsera - Zoltán M. Jakab - Katalin Szilágyi - Balázs Világi
Magyar Nemzeti Bank

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Abstract

We explore the properties of optimal monetary policy in a medium-scale DSGE model for Hungary. Our framework is a two-sector, open economy, with imports as intermediate input to production, augmented with a wide range of nominal and real frictions. Our results suggest that "science of monetary policy" that is found robust in simple models, holds in this medium-scaled setting as well. That is, the welfare-maximizing policy that aims to eliminate distortions associated with nominal rigidities can be approximated by an inflation targeting rule. Adding exchange rate into the feedback rule only marginally improves the stabilization properties of the policy rule. However, a rule reacting to wage inflation can be significantly welfare-improving. These results may suggest that the distortions associated with sticky wage setting have more important welfare implications than those related to the price stickiness in product markets.

1 Introduction

1.1 Motivation

A theoretically appealing feature of dynamic stochastic general equilibrium (DSGE) models is the existence of a well-defined welfare measure (i.e. utility). Consequently, the models can be solved for the optimal (welfare-maximizing) policies. In this paper we explore optimal monetary policy in a medium-scaled DSGE model for Hungary. We apply the linear-quadratic (LQ) approximation to the optimal policy problem, as suggested by Benigno-Woodford (2005), and use the standardised algorithm proposed by Altissimo et al. (2005). While the solution to the optimal policy problem is a useful benchmark for monetary policy evaluation, it is not operational from a central bank's perspective. To make our results easier to interpret, we approximate the welfare-maximizing policy rule with a set of simple rules reacting only to observabe variables. We evaluate the overall performance and the stabiliztion properties of various simple monetary policy rules.

Motivation for this paper is twofold. The first is practical, by exploring optimal monetary policy and the simple feedback rules that can approximate it, our research aims to provide guidance to monetary policy in Hungary. Second, we believe that the way we model openness is more realistic than usually done in the literature. Our model is a two-sector open economy with imports as an intermediate input to production. Thus, from a theoretical point of view, our contribution is to examine how the way openness is modelled may change the findings on optimal policy that hold in more simple open-economy models.

1.2 Related literature

In economies with nominal rigidities, monetary policy has real effects and can therefore influence welfare. The classical insight by Clarida et al. (1999) stresses that sticky prices cause unintended fluctuations in producers' mark-ups in face of shocks hitting the economy (and, hence, producers' marginal costs). By moving the policy rate, monetary policy can offset variations in the marginal cost, and therefore eliminate incentives for producers to change prices.That is, monetary policy can assure that mark-ups are kept at their desired level and prices remain unchanged. If the only source of inefficiency is price stickiness, stabilizing prices constitutes an optimal policy. Moreover, a simple feedback rule with a strong enough reaction to inflation can approximate the optimal policy. As was shown by Erceg et al (2000) if nominal wages are also rigid, and cause efficiency losses in the labor market, too, then the otimal feedback rule should include a measure of wage inflation, too. Aoki (2000) showed in a two-sector closed-economy setup that if there are different degrees of price stickiness in the sectors of the economy, the sectoral relative price distortion should also be corrected for. The policy rate should react to inflation in both sectors with higher weight given to the sector with more sticky price setting.

Classical papers on the optimal monetary policy in an open economy reinforce the findings of simple closed-economy models. Clarida et al (2001, 2002) conclude that the optimal policy problem for the small open economy is isomorphic to the closed economy case. In particular, the small open economy dynamics can be reduced to a first order, two-equation dynamical system for domestic inflation and the output gap whose structure is identical to the one associated with the workhorse sticky price model of a closed economy. Again, the optimal policy should seek to stabilize domestic inflation (hence eliminate the distortion due to sticky domestic price setting), and the form of the intetest rate rule is not affected by the openness of the economy. A similar result is obtained in Gali-Monacelli (2005) where strict domestic inflation targeting turns out to be the optimal monetary policy, consequently outperforming a CPI inflation targeting rule.

As CPI is the average of domestic and import prices, and as such, is directly related to the nominal exchange rate, the striking implication of the papers above is that optimal monetary policy should not react to variations in the exchange rate. Put differently, the appropriate target variable is domestic, as opposed to CPI inflation. As this implication is in sharp contrast to communicated objectives of real-world central banks, many successive papers challenge on it. A higher elasticity of substitution between domestic and foreign goods, higher coefficient for risk aversion and incomplete exchange rate pass-through all help in justfying CPI targeting, or concern for exchage rate movements (see Sutherland (2005, 2006), De Paoli (2009)). Campolmi (2008) argues that the inclusion of sticky wages in an otherwise standard small open economy model also rationalizes CPI inflation targeting.

As to our knowledge, no paper is available on the properties of the welfare-maximizing policy of an empirically motivated, full-fledged small open economy model. Adolfson et al (2008) do a similar excercise based on the estimated DSGE model for Sweden, however, they purposely refrain from deriving the central bank objective from micro behavior, and simply assume a "sensible" loss function. The same is true for Batini et al. (2001) who study optimal simple rules in a DSGE model for the UK.

1.3 Outline of the paper

We describe the model in the next section. In Section 3, we lay out the policymaker's problem and solution method. We proceed by a comparative analysis of the optimal monetary policy rule to the estimated one. Further, we approximate optimal monetary policy by a set of simple feedback rules, and compare their stabilization properties to both the fully optimal and the empirical one. In the last section, we conclude and set the directions for further research.

2 The model economy

The model we use is a two-sector dynamic stochastic general equilibrium (DSGE) small-open-economy model for the Hungarian economy. The two sectors produce domestic and exported final goods. Following Christiano et al. (2005) and Smets and Wouters (SW, 2003), our model features different types of frictions, real and nominal rigidities which are necessary to replicate the empirical persistence of Hungarian data. The model incorporates external habit formation in consumption, Calvo-type price and wage rigidity complemented with indexation to past price and wage inflation, adjustment costs of investments, adjustment cost of capital, labor and import utilization and fixed cost in production. We follow the approach of McCallum and Nelson (2001), and consider imports as production input.

The detailed description is available in Jakab-Világi (2008),[1] here we just give a short description of the objectives and constraints that the agents in the economy solve. The log-linearized model equations are presented fully in the Appendix A.[2] Parametrization of the model is given in Appendix B.

2.1 Households

The economy is populated by a continuum of households indexed by $j \in (0,1)$. They are infinitely lived, and choose their consumption stream in the standard rational optimizing manner. Households have labor and capital income and they own the production sector of the economy. They have

[1] There are two major differences from the Jakab-Világi (2008) model. *Rule-of-thumb consumers* are assumed away. The reason is that their presence makes the problem of the social planner cumbersome (see Gali et al. (2004)), and much less operational from a central bank perspective. A general claim (see Adolfson et al. (2008)) that the approximation of household welfare is very model-dependent and aggregating utilities with heterogenous agents might be problematic. Even though Jakab - Világi (2008) found these type of agents helpful in matching the model to data, in this excercise we dropped them.

We didn't include the *learning mechanism of the 'underlying inflation'*. A special feature of the Jakab-Világi model is that agents' *'perception on underlying inflation'* is made endogenous by a real-time adaptive learning algorithm. As this percieved inflation term enters the price-setting mechanism (and serves as a nominal anchor for rule-of-thumb price setters), this 'underlying' inflation heavily influences long-term inflation developments. In fact, the speed of learning was interpreted as a measure for monetary policy credibility (or the lack thereof). Throughout optimal monetary policy calculations, we drop the learning mechanism. The reason is that the optimal policy calculations assume full commitment under timeless perspective which can also be thought of having a fully credible central bank. For remaining consistent to the assumptions implicit in the approach we take, we abstract from potential problems of imperfect commitment.

[2] Variables with no time index refer to steady state values, variables with asterisk denote foreign variables (or variables measured in foreign currency).

identical preferences and differ in only one respect: they supply differentiated labor to firms.

The expected utility function of household j is

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\eta_t^c) \left\{ u(H_t(j)) - (1+\eta_t^l) v(l_t(j)) \right\} \right], \tag{1}$$

where $H_t(j) = c_t(j) - hc_{t-1}$, where c_{t-1} is aggregate consumption at date $t-1$, parameter $h \in [0,1]$ measures the strength of external habit formation, $l_t(j)$ is the labor supply of household j, η_t^c and η_t^l are preference shocks. Furthermore, preferences are additively separable, with the usual constant relative risk aversion (CRRA) specification, that is $u(H) = H^{1-\sigma}/(1-\sigma)$, and $v(l) = l^{1+\varphi}/(1+\varphi)$, σ, $\varphi > 0$.

The budget constraint of household j is

$$\begin{aligned} & P_t c_t^o(j) + P_t I_t(j) + \frac{B_t(j)}{1+i_t} = \\ = \; & B_{t-1}(j) + X_t^w(j) + W_t(j) l_t(j) + P_t r_t^k u_t(j) k_{t-1}(j) - \Psi(u_t(j)) P_t k_{t-1}(j) \\ & + Div_t - T_t, \end{aligned} \tag{2}$$

where P_t is the consumer price index, $B_t(j)$ is the household's holding of riskless nominal bonds at the beginning of t, i_t is the one-period nominal interest rate, Div_t denotes dividends from firms. $k_t(j)$ is the stock of physical capital supplied, $u_t(j)$ is the utilization rate of capital. T_t denotes the lump-sum tax. $I_t(j)$ denotes investments in physical capital. $W_t(j)$ is the nominal wage paid to household j, X_t^w is a state-contingent security which eliminates the risk of heterogeneous labor supply and labor income.[3] The cost of changing capacity utilization is:

$$\Psi(u_t) = r^k \psi \left[\exp\left(\frac{u_t - 1}{\psi} \right) - 1 \right]. \tag{3}$$

This implies that in the steady state ($u = 1$) $\Psi(1) = 0$.

Physical capital accumulation:

$$k_t = (1-\delta) k_{t-1} + \left[1 - \Phi_I \left(\frac{(1+\eta_t^I) I_t}{I_{t-1}} \right) \right] I_t, \tag{4}$$

[3] Due to the existence of asset X_t^w, all households can insure themselves against idiosyncratic shocks to income. Consequently, incomes of all households are the same, and all households choose the same consumption allocation. We, therefore, drop index j from subsequent notations.

where function Φ_I represents investments adjustment costs, and η_t^I is an exogenous shock. It is assumed that

$$\Phi_I\left(\frac{(1+\eta_t^I)I_t}{I_{t-1}}\right) = \frac{\phi_{Inv}}{2}\left(\frac{(1+\eta_t^I)I_t}{I_{t-1}} - 1\right)^2, \quad \phi_I > 0. \tag{5}$$

This implies that $\Phi_I \geqslant 0$, and in the steady state $\Phi_I(1) = \Phi_I'(1) = 0$.

Households choose the paths of consumption, bond holdings, investment and capacity utilization to maximize (1) subject to constraints (2)-(5). The log-linearized first order conditions are presented in the Appendix A.

2.1.1 Wage setting

A household supplying type j of labor belongs to a trade-union representing the interest of households of type j. The union determines the labor supply and the nominal wage of its members. Union j sets $W_t(j)$, the nominal wage level of type j of labor. The composite labor input of the economy is a CES aggregate of different types of labor,

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\theta_w - 1}{\theta_w}}\, dj\right)^{\frac{\theta^w}{\theta^w - 1}},$$

where $\theta^w > 1$ is elasticity of substitution between different types of labor. The demand for labor supplied by union j is given by

$$l_t(j) = \left(\frac{W_t}{W_t(j)}\right)^{\theta^w} l_t,$$

where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta^w}\, dj\right)^{\frac{1}{1-\theta^w}}.$$

There is sticky wage setting. Every union at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1 - \gamma_w$. Unions, which do not optimize at the given date follow a rule of thumb. Wage setting scheme of the rule-of-thumb wage setters is:

$$W_T(j) = W_t(j)(1 + \pi_{T,t}^{Iw}) = W_t(j)(1 + \pi_T^{Iw})(1 + \pi_{T-1}^{Iw}) \cdots (1 + \pi_{t+1}^{Iw}),$$

where

$$1 + \pi_t^{Iw} = \left(1 + \pi_{t-1}^w\right)^{\vartheta_w}, \tag{6}$$

$$1 + \pi_t^w = \frac{w_t}{w_{t-1}}(1 + \pi_t) \tag{7}$$

and π_t^w is the nominal wage inflation, π_t is consumer price inflation, w_t is the real wage, and ϑ_w represents the degree of indexation to past wage inflation. The optimization excercise is described in details in Jakab-Világi (2008), the log-linearized equation characterizing the solution to the optimal wage setting problem (i.e. the wage Phillips curve) is given in Appendix A.

2.2 Firms

Domestic and exported final goods are produced in different sectors. The structure of the sectors are identical, however the imported input and capital requirement of production in the exports sector is higher (or put differently, the domestic sector is more labor-intensive). This assumption is necessary two reproduce the empirical co-movement of exports and imports. Price formation mechanisms are similar in both sectors, they are captured by the sticky-price model of Calvo, however the exporters sets their prices in foreign currency (local currency pricing).

Production has a hierarchical structure: at the first stage labor and imported inputs are transformed into an intermediate input in a perfectly competitive industry. At the second stage the intermediate input and capital are used to produce differentiated goods in a monopolistically competitive industry. Finally, a homogenous final good is produced from the differentiated goods in a perfectly competitive environment. There are two sectors in the economy: a domestic production sector and exports sector, labeled by d and x, respectively.

2.2.1 Competitive producers of final goods

Final good y_t^s in sector s $(s = d, x)$ is produced in a competitive market by a constant-returns-to-scale technology from a continuum of differentiated intermediate goods $y_t^s(i)$, $i \in [0, 1]$. The technology is represented by the following CES production function:

$$y_t^s = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}}, \tag{8}$$

where $\theta > 1$ measures the degree of the elasticity of substitution. As a consequence, the price index P_t^s is given by

$$P_t^s = \left(\int_0^1 P_t^s(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}, \tag{9}$$

where $P_t^s(i)$ denotes the prices of differentiated goods $y_t^s(i)$, and the demand for $y_t^s(i)$ is determined by

$$y_t^s(i) = \left(\frac{P_t^s}{P_t^s(i)}\right)^{\theta} y_t^s. \tag{10}$$

Monopolistically competitive producers of intermediate goods

The continuum of goods $y_t^s(i)$ is produced in a monopolistically competitive market. Each $y_t^s(i)$ is made by an individual firm, and they apply the same CES technology. Firm i uses technology

$$y_t^s(i) = A_t \left(\bar{\alpha}_s^{\frac{1}{\varrho}} \bar{k}_{t-1}^s(i)^{\frac{\varrho-1}{\varrho}} + (1-\bar{\alpha}_s)^{\frac{1}{\varrho}} z_t^s(i)^{\frac{\varrho-1}{\varrho}}\right)^{\frac{\varrho}{\varrho-1}} - \bar{f}_s, \tag{11}$$

where $\bar{k}_t^s(i)$ is the firm's effective utilization of physical capital, $\bar{k}_t^s(i) = u_t k_t^s(i)$, where u_t is the degree of capacity utilization given from the households' decision, $k_t^s(i)$ the firm's utilization of the homogenous capital good. $z_t^s(i)$ is the firm's utilization of a composite intermediate input good z_t^s. Variable A_t is a uniform exogenous productivity factor and $\bar{f}_s$ is uniform real fixed cost of the industry. The parameter $0 < \varrho$ measures the elasticity of substitution between $\bar{k}_t^s$ and z_t^s and $0 < \bar{\alpha}_s < 1$ measures the steady-state share of capital in marginal cost of production. Good $z_t^s(i)$ is composed by composite labor and imported inputs. Solution of firms' cost minimization problem and the implied marginal costs in the two sectors are desribed in detail in Jakab-Világi, and log-linearized equations are presented in the Appendix.

Price setting Let us consider how monopolistically competitive firms set their prices. To simplify the exposition, we discuss the general features of price setting of domestic and export producers.[4] It is assumed that prices are sticky: as in the model of Calvo (1983), each intermediate good producer at a given date changes its price in a rational, optimizing, forward-looking way with a constant probability of $1-\gamma^s$ $(s = d, x)$. Those firms which do not optimize at the given date follow a rule of thumb. Rule of thumb price setters increase their prices by past rate of inflation, as in Yun (1996). Formally, if firm i does not optimize at date t

$$Ps_t^s(i) = P_{t-1}^s(i)\Pi_{t-1}^{Is} = P_{t-1}^s(i)\left(\Pi_{t-1}^s\right)^{\vartheta_s}$$

[4] The price setting problem of domestic and export producers are very similar. Note however that we allow for differences in both the Calvo parameter and the indexation coefficient. Note also that exporters set prices in foreign currency.

where $\Pi_{t-1}^s = P_{t-1}^s / P_{t-2}^s$, ϑ_s measures the degree of indexation to past inflation. The above formula implies if a given firm does not optimize between $t+1$ and T its price at date T is given by

$$P_T^s(i) = P_t^s(i)\Pi_{T,t}^{Is} = P_t^s(i)\Pi_T^{Is}\Pi_{T-1}^{Is}\cdots\Pi_t^{Is}. \tag{12}$$

If $P_t^s(i)$ is the chosen price of a firm at date t, then its profit at date T will be

$$V_T(P_t^s(i)) = y_T^s(i)\left(P_T^s(i) - MC_T^s\right) - \bar{f}.$$

If firm i sets its price optimally at date t it solves the following maximization problem.

$$\max_{P_t^s(i)} = \mathrm{E}_t\left[\sum_{T=t}^{\infty}(\beta\gamma)^{T-t} D_{T,t} V_T(P_t^s(i))\right], \tag{13}$$

where $D_{T,t}$ is the stochastic discount factor,

$$D_{T,t} = \beta^{T-t}\frac{\lambda_T/P_T}{\lambda_t/P_t},$$

and λ_t is the marginal utility of consumption of households, who own the firms. That is, firms seek to maximize (13) subject to constraints (10) and (12). Solution of firms' price setting problem is desribed in detail in Jakab-Világi, while the log-linearized Phillips-curves for both sectors are in Appendix A.

2.2.2 Competitive producers of the composite intermediate input

The composite intermediate input is produced in a competitive industry by the following CES technology,

$$z_t^s = \left(\bar{a}_s^{\frac{1}{\varrho_z}}\left(m_t^s\right)^{\frac{\varrho_z-1}{\varrho_z}} + (1-\bar{a}_s)^{\frac{1}{\varrho_z}}\left(l_t^s\right)^{\frac{\varrho_z-1}{\varrho_z}}\right)^{\frac{\varrho_z}{\varrho_z-1}} - z_t^s\Phi_z\left(z_t^s\right). \tag{14}$$

where l_t^s is labor and m_t^s is the imported input good m_t^s. Furthermore $0 < \varrho_z$ and $0 < \bar{a}_s < 1$, and the adjustment cost function

$$\Phi_{zs}\left(z_t^s\right) = \frac{\phi_z}{2z^s}\left(\frac{z_t^s}{z^s} - 1\right)^2, \quad \phi_z > 0.$$

Properties of this function are $\Phi'_{zs} > 0$, $\Phi_{zs}\left(z^s\right) = \Phi'_{zs}\left(z^s\right) = 0$. z^s is the steady state level of the composite input. The price of composite input W_t^{zs} is equal to the marginal cost of its production. Solution of cost minimization problem is desribed in detail in Jakab-Világi, and presented in the Appendix.

2.3 Government

2.3.1 Fiscal authority

Government spends on domestic final good, and finances its purchases from lump-sum taxes levied on households:

$$T_t = P_t(1 + \eta_t^G)G.$$

2.3.2 Monetary authority

The central bank follows an inflation targeting regime, and sets the nominal interest rate according to the following rule:

$$(1 + i_t)/(1 + r) = \left(\frac{1 + i_{t-1}}{1 + r}\right)^{\varsigma^i} \left((1 + \pi_t)^{\varsigma^\pi} de_t^{\varsigma^e}\right)^{1-\varsigma^i} (1 + \eta_t^i), \tag{15}$$

where i_t is the policy rate, r is the steady state real interest rate, ς^i is the parameter for interest smoothing, de_t is the change in the nominal exchange rate.

2.4 Rest of the world

2.4.1 Trade flows

In terms of trade flows, the rest of the world is represented by an ad hoc export demand equation:

$$\frac{x_t}{x_{t-1}^{h_X}} = (1 + \eta_t^x)x^* \left(P^{x*}\right)^{-\theta^{x*}}, \tag{16}$$

where h_X is a habit parameter, θ^x is the price elasticity, and η_t^x is a shock to foreign demand, x^*.

Also, the price of imports (P_t^{m*}) in foreign currency is set exogenously abroad:

$$P_t^{m*} = (1 + \eta_t^{Pm*})P^{m*}.$$

2.4.2 Financial flows

In terms of credit flows, the rest of the world is represented by an ad hoc upward-sloping credit supply curve (á lá Schmitt-Grohe–Uribe):

$$\frac{1 + i_t^*}{1 + r} = \exp(-\nu(b_t - b))(1 + \eta_t^{pr}), \tag{17}$$

where i_t^* is the foreign-currency denominated interest on domestic assets, b_t is the net foreign asset position of the economy (given in terms of foreign currency), and η_t^{pr} is a risk-premium shock.

Also, arbitrage condition between domestic and foreign assets is given by the uncovered interest rate parity condition:

$$\frac{1+i_t}{1+i_t^*} = \mathrm{E}_t \frac{e_{t+1}}{e_t} \tag{18}$$

2.5 Equlibrium conditions and identities

In the market for domestic goods, equlibrium holds:

$$y_t^d = c_t + I_t + (1+\eta_t^G)G + \Psi\left(u_t\right) k_{t-1}^d, \tag{19}$$

where the last term refers to output loss due to real costs of changing capacity utilization.

In the market for exported goods, equlibrium implies:

$$y_t^x = x_t + \Psi\left(u_t\right) k_{t-1}^x, \tag{20}$$

In the international credit market, net foreign assets evolve according to:

$$b_t = (1+i_{t-1}^*)b_{t-1} + P_t^{x*}x_t - P_t^{m*}m_t \tag{21}$$

"Domestic part" of the real exchange rate is given by the following identity:

$$q_t = \frac{e_t}{P_t} \tag{22}$$

2.6 Parametrization

The exact parametrization of the model is described in Table 1-3 in Appendix B. Throughout the simulations, the posterior means of the parameters were chosen. While most parameters fall close to usual-in-the-literature values, two qualifications stand out.

First, the estimated Calvo and indexation parameters suggest that price setting is most flexible in the exports sector with both the probability of resetting prices optimally and the degree of indexation to past inflation for rule-of-thumb price setters being the highest.[5] Domestic product market

[5] Note that indexation to past inflation somewhat relaxes the rigidity of prices. That is, it provides price setters who cannot fully optimize with the opportunity to partially free-ride on those who can. In this way, for a given Calvo parameter, prices will be much less sticky in a sector with higher degree of indexation.

comes close second, while nominal rigidities are most pervasive in the labor market.

Monetary policy is characterised by a simple feedback rule reactig to domestic inflation and the exchange rate.[6] The coefficient of inflation is less than 1.4, less than the standard baseline estimate of 1.5 (Clarida-Gali-Gertler). While this is an empirical characterisation of the Hungarian monetary policy, it can be interpreted as a warning that the empirically motivated policy rule is possible too "dovish". Note also that the cofficient on the nominal rate suggests that monetary policy doesn't target the nominal exchange rate directly.

2.7 The role of monetary policy in the model

The instrument of monetary policy is the nominal interest rate, i.e. the cost of borrowing in domestic currency. However, by moving the instrument, monetary policy has influence on the economy through two channels. The first is the usual *aggregate demand channel* also present in a closed economy. In a simple closed economy setup, changes in the cost of borrowing affect households' consumption decisions (and, hence, aggegate demand) through the Euler equation. With the appropriate monetary policy response, the central bank can manipulate demand, and, consequently, "close the output gap" for shocks hitting the economy.

In an open economy, the *exchange rate channel* adds to the monetary transmission mechanism. A change in the policy rate alters the nominal exchange rate via the uncovered interest parity condition, and, in our setup, induces supply-side reactions as well. First, a nominal appreciation, say, lowers the cost of imported inputs, and hence, the marginal cost of production in domestic currency. Second, with export prices sticky in foreign currency, it lowers the profitability of the export sector (or, increases the marginal cost of exports in terms of foreign currency). Note that neither of these "supply-side" effects are present in the simple small open-economy models with imports as consumption goods and one sector with producer currency pricing. Therefore we are primarily interested in whether deviations from these simple modelling assumptions towards what we believe to be a more realsitic setup, change the normative implications for monetary policy.

[6] Note that in the model domestic inflation coincides with CPI inflation. The empirical estimate of the feedback coefficients suggests that the monetary policy hardly cares about the nominal exchange rate, once the indirect effects of nominal exchange rate movements (working through the marginal cost of production) are controlled for.

Optimal policy and optimal simple rules

2.8 Optimal policy

We define optimal policy as the one under commitment that maximizes households' lifetime utility subject to competitive equilibrium conditions, given the exogenous stochastic processes of the shocks and the past values of the endogenous variables. As common in the optimal policy literature and following Woodford, we assume a particular recursive formulation of the policy commitment. This imposes that the policy rule that is found optimal in the latter periods is also adhered to in the initial one (where private sector expectations are already formed), due to time-consistency and credibility reasons. The policy rule that satisfies this property is called optimal from a timeless perspective.

A general assumption in optimal monetary policy calculations is that the fiscal policy can eliminate first-order distortions due to monopolistic competition by properly subsidising goods and labor markets. While theoretically appealing, we refrain this unrealistic assumption. The reason for this is that our primary objective is practical guidance to monetary policy.

Finally, an optimal policy for hoseholds' preferences given by (1) implies very aggressive policy responses, or put differently, implausibly large fluctuations in the policy instrument.[7] However, this is not what we see in practice. Real-world policymakers seem to prefer more gradual and persistent movements in the nominal interest rate, presumably because of financial stability considerations. Our baseline model above doesn't take this motive into account. We can correct for this by including an interest-smoothing motive into the utility function.[8] That is, the period utility function associated with

[7] Large swings in the policy rate would also imply that the policy rate hits the zero lower bound. Imposing some restriction on the variability of the policy rate rules out this possibilty.

[8] It can be shown that this is equivalent to having an additively separable money-in-the-utility term in househlds' utility. If households's utility takes the form

$$(1+\eta_t^c)\left[\frac{H_t^{1-\sigma}}{(1-\sigma)} - \frac{(1+\eta_t^l)l_t^{1+\varphi}}{(1+\varphi)} - \varepsilon^M\left(\frac{M_t}{P_t} - \overline{m}\right)^2\right],$$

where M_t is the nominal money stock, and $\overline{m}$ is the targeted level of real money balances, then plugging the first order condition on real money holdings $(U'_{M/P}/U'_C = (1+i)/i)$ back to the utility function leads to the expression of V above. Note that our calibrated value for ε^M implies that the term associatd with real money balances has a negligibe weight in the utility function.

policymakers revealed preferences takes the form:

$$V_t = (1+\eta_t^c)\left[\frac{H_t^{1-\sigma}}{(1-\sigma)} - \frac{(1+\eta_t^l)l_t^{1+\varphi}}{(1+\varphi)} - \frac{H_t^{-\sigma}}{2\varepsilon^M}\left(\frac{1+i_t-1/\beta}{1+i_t}\right)^2\right], \quad (23)$$

where the first two terms are identical to the terms in the households' utility function, while the last one is responsible for smoothing out interest rate fluctuations by "punishing" any deviation of the interest factor $(1+i_t)$ from its staedy-state value $(1/\beta)$.

The policymaker thus seeks to maximize

$$W_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t V_t, \quad (24)$$

given private sector decisions, market equilibrium conditions and appropriate initial ("precommited") values for forward-looking variables and Lagrange multipliers. The solution of the problem gives the policy instrument as a function of state variables and shocks, that can be obtained using numerical methods. We apply a standardized algorithm provided by Altissimo et al. (2006) to derive the optimal instrument rule.

2.9 Optimal simple rules

The optimal instrument rule derived in the last subsection represents a usuful benchmark for policy evaluation. However, as it is a function of a wide range of unobservables, it is hardly operational from a central bank perspective. From a practical perspective it is often more convenient to refer to optimal simple rules that approximate the welfare-maximizing policy, but, similarly to the estimated empirical rule, react to observable variables only. Moreover, simple rules are more directly comparable to the esimated one, as we can easily compare the feedback coefficients if the rules are in similar functional forms.

We consider rules of the form:

$$\frac{(1+i_t)}{1+r} = \left(\frac{1+i_{t-1}}{1+r}\right)^{\varsigma^i} \left((1+\pi_t)^{\varsigma^\pi} de_t^{\varsigma^e} (1+\pi_t^w)^{\varsigma^{\pi w}}\right)^{1-\varsigma^i}, \quad (25)$$

and search for optimised feedback coefficients, that is we solve for

$$\widetilde{\varsigma} = \arg\max_{\varsigma} W_0,$$

where $\varsigma = [\varsigma^i, \varsigma^\pi, \varsigma^e, \varsigma^{\pi w}]$ represents the vector of feedback coefficients in (25).

From the literature on optimal monetary policy (see Section 1.2) we know that monetary policy should ideally seek to eliminate distortions generated by nominal rigidities. There are three sources of such microeconomic distortions (stickiness in domestic and export price setting, and stickiness in wage setting) in the model, thus three inefficiency wedges the monetary policy should try to close. Intuitively, optimal simple rules should target variables that are intimately related to the wedges. Domestic inflation, wage inflation and the exchange rate are natural candidates for such target variables. A feedback rule reacting to domestic price inflation seeks to eliminate unintended markup fluctuations arising from price stickiness in the domestic sector, and the reaction to wage inflation is supposed to reduce the inefficiency wedges in the labor market. With reacting to the exchange rate, the monetary policy can reduce the inefficiency wedge for exporters. Of course reacting to these wedges may easily yield conflicting implications for the setting of a single instrument. On the other hand, the opposite can also hold: a change in the instrument aimed to deal with a particular kind of inefficiency, can have positive effects on some other.

2.10 Calibration issues

Before we turn to the comparative analysis, we need to restrict some parameters in order to get comparable results.

First, we calibrated ε^M, the term in the policymaker's objective function (23) that governs the interest smoothing motive. We set the parameter to ensure that the variance of the change in the policy rate under the optimal rule is set to match the empirical value. That is, ε^M was chosen to make the following equation hold:

$$Var(i_t/i_{t-1})_{empirical} = Var(i_t/i_{t-1})_{optimal},$$

where the RHS is a function of ε^M. The intuition behind is that the desire for smoothing the policy rate is a revealed preference of the policymaker, that is reflected in the empirical variance. To pounce on the importance of this motive, we take actual decisions as a benchmark, and calibrate the preference parameter to match the empirical variance of the policy rate.

Second, somewhat similarly, for the optimal simple rules we restricted the parameter of interest rate smoothing to its empirical (estimated) counterpart, that is

$$\varsigma^i_{empirical} = \varsigma^i_{osr},$$

where osr refers to all proposed forms of the optimal simple rules. By doing so, we took the revealed smoothing motive as given, and concentrated only on the feedback parameters.

3 Comparative analysis of monetary policy rules

3.1 Scope of the excercise

We complete the following comparative excercises.

- First, we explore the performance of the fully optimal rule (that is, the solution to the policymaker's problem), and compare it to the estimated one. By doing so, we seek for systematic differences in stabilization properties. Stabilization is captured by the implied volatilty of selected nominal and real variables. The impulse responses to different types of shocks under the different policy rules can help us to understand the mechanisms at work.

- Next, we include the optimal simple rules into the comparative excercise. We examine, on the one hand, how sucessful the simple rules are in replicating the performance of the fully optimal one, and, on the other hand, the systematic differences from the empirical rule. The overall performance of the rules is reflected in the value of the policymaker's objective, W_0 in (24). To have a comparable welfare measure, we calculate the share of steady state share of consumption necessary to compensate for following a particular type of rule. Also, the potential utility gain (in terms of steady state consumption equivalent) of including the exchange rate or the measure of wage inflation into the feedback rule allows us to assess the relative welfare costs of different types of nominal rigidities.

- The next excercise consists of comparing optimized policy rules with different feedback variables. The comparative analysis of simple rules allow us to judge the stabilization properties of different policy rules when monetary policy reacts to variables other than domestic inflation. Moreover, the optimised relative weights on the feedback parameters are interesting in themselves.

- Finally, we carry out sensitivity analysis on some structural parameters of the economy. Specifically, we study how the optimised coefficients for the simple feedback rules change if the openness (proxied by the steady state share of imports) of the economy is different.

3.2 Set of policy rules compared

We analyse the performance and stabilization properties of the following rules:

1. the estmated empirical rule (denoted hereafter by EMP),
2. the fully optimal policy rule (LQ),
3. the simple rule with the optimised coefficient reacting to dometic inflation only ($OSR1$),
4. the simple rule with optimised coefficients reacting to domestic inflation and the exchange rate ($OSR2$),
5. the simple rule with optimised coefficients reacting to domestic inflation and wage inflation ($OSR3$).

A summary of the rules is given in the next table.

Table 1: Summary of the monetary policy rules analysed

Name	Specification	Methodology
Empirical	$\frac{1+i_t}{1+r} = \left(\frac{1+i_{t-1}}{1+r}\right)^{\varsigma^i} \left((1+\pi_t)^{\varsigma^\pi} de_t^{\varsigma^e}\right)^{1-\varsigma^i}$	Bayesian estimation of $\varsigma^i, \varsigma^\pi, \varsigma^e$
Fully optimal		Solving for policymaker's problem
OSR1	$\frac{1+i_t}{1+r} = \left(\frac{1+i_{t-1}}{1+r}\right)^{\varsigma^i} \left((1+\pi_t)^{\varsigma^\pi}\right)^{1-\varsigma^i}$	Optimising on ς^π, restriction on ς^i
OSR2	$\frac{1+i_t}{1+r} = \left(\frac{1+i_{t-1}}{1+r}\right)^{\varsigma^i} \left((1+\pi_t)^{\varsigma^\pi} de_t^{\varsigma^e}\right)^{1-\varsigma^i}$	Optimising on ς^π, ς^e, restriction on ς^i
OSR3	$\frac{1+i_t}{1+r} = \left(\frac{1+i_{t-1}}{1+r}\right)^{\varsigma^i} \left((1+\pi_t)^{\varsigma^\pi} (1+\pi_t^w)^{\varsigma^{\pi w}}\right)^{1-\varsigma^i}$	Optimising on ς^π, $\varsigma^{\pi w}$, restriction on ς^i

3.3 Results

3.3.1 Comparing optimal policy with the empirical rule

Our first excercise consists of comparing the fully optimal policy to the estimated rule. First, volatilities of selected variables provide an indication of their stabilization performance. These are reported in Table 2.

Table 2: Volatility of selected variables under optimal monetary policy and the empirical rule

Variable	**LQ**	**EMP**
domestic inflation (π)	0.310	0.761
wage inflation (π^w)	0.0491	0.574
real exchange rate (q)	107.179	144.695
nominal exchange rate (de)	28.922	40.556
domestic output $\left(y^d\right)$	35.446	37.631
export output (y^x)	9.979	10.046
consumption (c)	39.844	31.456

The figures in the table suggest that compared to the estimated rule, optimal monetary policy smoothes out the paths of the nominal variables. This is a general finding for all nominal variables (domestic and wage inflation, nominal exchange rate), but applies most strikingly to wage inflation. In Appendix C, we plotted impulse responses for selected shocks (productivity, government consumption, import prices, and export demand) under the optimal and the empirical rule.[9] The figures reinforce that for all the shocks plotted, the nominal variables return to their steady state values more rapidly than real variables. This is quite in line with intuition: the best that monetary policy can achieve under a time-consistent rule is to eliminate relative price distortions (inefficiencies generated by nominal rigidities). Stabilizing nominal variables (domestic prices and wages) has exactly this effect, as it indicates that monetary policy has successfully eliminated all incentives for adjusting prices, i.e. closed the inefficiency wedge ("output gap" for the domestic price setters, and the "marginal rate of substitution gap" for wage setting unions). Note that the stabilization perfomance of the optimal policy is most signifcant for wage inflation.

The empirical rule generates excess smoothness in real variables, particularly in consumption. This is a natural consequence of the fact that the estimated policy rule is quite "dovish". In face of shocks hitting the economy, the empirical rule with modest feedback coefficients, doesn't imply aggressive movements in the nominal rate, and hence only very subdued movements in the real rate. As a consequence, the path of consumption is smoothed out, too. Note that with flexible prices, the real interest rate would strongly ad-

[9] The shocks were chosen to represent an evenly distributed sample of the sources of exogenous variation: there are two shocks with domestic and two with foreign origin, two hitting the demand side of the economy (government spending, export demand), and two affecting the suplly side (productivity, import prices). In the full-fledged model of the economy, there are much more shocks that could be analysed, though we believe that our selection of them gives a balanced picture on the performance of different rules.

just to shocks, and consumption would be more volatile, and that is exactly the allocation the optimal policy seeks to replicate.

3.3.2 Comparing optimal policy, optimal simpe rules and the empirical rule

First, we would like to know how close the simple rules can get to the fully optimal one. To assess welfare implications of alternative monetary policy rules, we recall the consumption equivalent welfare measure originally introduced by Lucas in his famous 1987 volume. In our case it would mean the percentage difference of the (steady state) level of consumption that would make the representative consumer indifferent (in terms of utility) in between the different macroeconomic outcomes created by alternative monetary policy rules. This measure for different policy rules are presented in the next table.

Table 3: Welfare gain from following different policy rules (in terms of steady state consumption)

Welfare gain relative to	**LQ**	**EMP**	**OSR1**	**OSR2**	**OSR3**
LQ policy	0	-0.1	-0.07	-0.07	-0.04
Empirical rule	0.1	0	0.03	0.03	0.06

The first line of the table tells us that the welfare consequences of following suboptimal monetary policies are huge. That is, shifting to the LQ policy from the empirical one creates a welfare gain amounting to 10 percent of steady state consumption. This looks like a dramatic and somewhat implausible difference. However, the corresponding figures in the opimal monetary policy literature are generally calculated in models with much less frictions. With the possible sources of inefficiencies increasing, the advantage of the fully optimal policy is expected to grow as well. Moreover, we restricted our analysis to very simple rules (with at most two feedback variables) that clearly cannot handle the full range of possible shocks.

Turning to the optimal simple rules, the welfare analysis also indicates that switching from the empirical to an optimal simpe rule can significantly improve welfare. This is true even in the case where the functional form and the feedback variables of the policy rule remain unchanged ($OSR1$, $OSR2$). Another interesting implication of the analysis is that including the nominal exchange rate into the policy rule doesn't have a significant effect. However, targeting also nominal wage inflation does imply sizeable welfare gains, in line with our prior belief that nominal rigidities in wage setting cause severe inefficiencies in the labor market.

A more easily interpretable measure of the relative performance of different policy rules, is the amount of volatility they imply for selected variables in percent of the implied volatility under the empirical rule. This is shown in the next table.

Table 4: Volatility of selected variables under different monetary policy rules (in % of the volatility under the empirical rule)

Variable	**LQ**	**OSR1**	**OSR2**	**OSR3**
domestic inflation (π)	$40,70\%$	$40,74\%$	$39,55\%$	$37,71\%$
wage inflation (π^w)	$8,50\%$	$29,79\%$	$27,53\%$	$27,70\%$
real exchange rate (q)	$74,00\%$	$91,53\%$	$85,06\%$	$93,14\%$
nominal exchange rate (de)	$71,30\%$	$88,99\%$	$70,66\%$	$93,70\%$
domestic output $\left(y^d\right)$	$94,10\%$	$97,29\%$	$97,32\%$	$97,41\%$
export output (y^x)	$99,30\%$	$102,66\%$	$101,90\%$	$102,79\%$
consumption (c)	$126,60\%$	$127,36\%$	$128,93\%$	$132,52\%$

The figures above reinforce our finding from the previous section: the empirical rule seems to oversmooth real variables (most notably, consumption), while significant welfare improvements can be achieved by reducing he volatiliy of nominal variables. The impulse responses (presented in Appendix C) to selected shocks under the optimal simple and the empirical rule are also in line with this result.

3.3.3 Comparing different optimal simple rules

The differences in terms of welfare gains and stabilization performance between the optimal simple rules are reflected in the optimised feedback coefficients, too. These are reported in the next table.

Table 5: Optimised feedback coefficients in the empirical and different optimal simple rules

Optimised feedback parameter to	**EMP**	**OSR1**	**OSR2**	**OSR3**
inflation (π)	1.375	4.77	5.08	2.8
exchange rate (de)	0.025	0	0.27	0
wage inflation (π^w)	0	0	0	9.12

Note first, that all the optimal simple rules give a much higher feedback coefficient to domestic inflation. This also implies that all the optimal simple rules involve much more variation in the domestic real interest rate, and thus a more volatile consumption path. Second, if the nominal exchange rate is included in the policy rule, it gets a much lower optimised coefficient than domestic inflation. Third, the policy rule reacting also to wage inflation gives a much higher weigth on the measure of wage inflation than on domestic price inflation. This is in line with our intuition that monetary policy should ideally care more about the stickier "sector" and attach higher weights to reacting to inflation that is supposed to generate more inefficiencies.

3.3.4 Comparing optimal monetary policy with different degrees of openness

In our last comparative excercise, we do a sensitivity analysis on openness. We compare our baseline model with a hypothetical economy that is very similar to Hungary, but is less open. Otherwise, the thought experiment leaves all the estimated structural parameters of the economy unchanged. We proxy openness by the steady state share of imports in production.[10]

Stabilisation properties of the fully optimal policy in the more and the less open economy are reported in th next table.

Table 6: Volatility of selected variables under the optimal monetary policy in the baseline and the "moderately open" economy

Variable	**Baseline**	**"Moderately open"**
domestic inflation (π)	0.310	0.335
wage inflation (π^w)	0.049	0.075
real exchange rate (q)	107.179	142.488
nominal exchange rate (de)	28.922	37.166
domestic output $\left(y^d\right)$	35.446	33.626
export output (y^x)	9.979	15.686
consumption (c)	39.844	22.266

The table tells us that by increasing openness, optimal monetary policy stabilizes nominal variables by more and domestic real variables by less. However, from the figures above it is not clear whether it is the difference in the transmission mechanism or in the welfare implications of the same amount of fluctuations (and hence, the policymaker's objective) that drives the result. Examining the optimised feedback coefficients can give a hint (see next table).

[10] Specifically, the share of imports in marginal cost is 0.6 in the export and 0.4 in the domestic sector for our baseline calibration. This implies a steady state share of imports to GDP amounting to 100% which is the approximate value for the Hungarian economy. In the less open economy, the corresponding figures are 0.2 for both sectors, implying imports having a 25% share in steady state GDP. We will refer to the latter parametrizaion as the "moderately open" economy.

Table 7: Optimised feedback coefficients in the baseline and the "moderately open" economy

	Baseline			Moderately open		
Optimised feedback parameter to	OSR1	OSR2	OSR3	OSR1	OSR2	OSR3
inflation (π)	4.77	5.08	2.8	2.66	2.66	0.85
exchange rate (de)	0	0.27	0	0	0.06	0
wage inflation (π^w)	0	0	9.12	0	0	3.41

From Table 7, we conclude that in a more open economy, monetary policy becomes more concerned for stabilizing inflation. That is, the weights attached to inflation is significantly increased in all specifications. Moreover, the relative importance of domestic inflation (as reflected by the relative magnitude of the feedback coefficient on inflaton to other variables) also increases with openness.

4 Conclusions an directions for further research

In this paper, we examined optimal monetary policy in a DSGE model for Hungary. We got the following results.

Compared to optimal policy, the empirical rule implies excess smoothing of real domestic variables, and excess volatility of nominal variables. The best monetary policy can achieve is to reduce inefficiencies generated by nominal rigidities. Stabilizing nominal variables has exactly this effect, as it indicates that monetary policy has successfully eliminated all incentives for adjusting prices, i.e. closed the inefficiency wedges. The excess smoothness in real variables, particularly in consumption, that characterizes the emprical rule is a natural consequence of the estimated policy rule being "dovish". In face of shocks hitting the economy, the empirical rule with its modest feedback coefficients, doesn't imply aggressive movements in the nominal rate, and hence generates only very subdued movements in the real rate. As a consequence, the path of consumption is smoothed out, too. Note that with flexible prices, the real interest rate would strongly adjust to shocks, and consumption would be more volatile, and that is exactly the allocation optimal policy seeks to replicate.

Simple optimal rules can approximate the fully optimal policy relatively well. Sizeable welfare gains can be achieved with the optimised feedback coefficients relative to the estimated rule. Simple policy rules with a higher coefficient on inflation relative to the estimated one imply more variable real interest rates and, consequently, more volatility in real variables. On the other hand, this also means that they reduce the variability of nominal variables.

Adding the nominal exchange rate to the policy rule doesn't improve welfare or the stabilization properties of the optimal simple rule. This results suggests that optimal monetary policy should not target the wedge related to price stickiness of the export sector separately. The reason behind is that fluctuations in the nominal exchange rate causing unintended variations in exporters' markup are already corrected for when the monetary authority aims to eliminate the inefficiency wedge of domestic producers, and thus are already controlled with the coefficient on domestic inflation. Moreover, responding to the exchange rate creates additional variance for domestic producers through the marginal cost channel, while responding to inflation has no such externality for the export sector.

Including wage inflation into the policy rule implies significant improvement in welfare. This result suggest that the welfare loss associated with sticky wage setting is more severe than those related to nominal rigidities in product markets. This is in line with the empirical estimates of wage stickiness. Furthermore, as imports are not consumed, the nominal exchange rate (a more conventional target variable for monetary policy) cannot close the inefficieny gap generated by sticky wage setting.

With increasing openness optimal monetary policy gets more concerned with stabilizing nominal, as opposed to real variables. Opening up the economy offers new channels of adjustment. In the two-sector open economy all shocks bring about changes in intratemporal relative prices. Movements in the terms-of-trade and the real exchange rate, then, provide incentives for reallocation of productive resources between sectors, and substitution of inputs for each other. The new channels of adjustment have consequences on the general conduct of optimal monetary policy as they provide more flexibility in terms of real adjustment. If real adjustment is less painful, the welfare loss associated with real (output) fluctuations becomes less important relative to fluctuations in inflation. Therefore, the inflation-output trade-off becomes tilted towards inflation stabilization.

Optimal policy should react to dometic inflation more strongly in a more open economy. For very open economies, the central bank can ignore exporters, because they basically re-export and hence are unaffected by foreign price stickiness. This allows the bank to respond fully to changes in inflation. But with moderate openness, the inefficiency created by sticky price setting in the export sector must be taken into account, resulting in a less agressive policy, because exporters are partially shielded from the full effect of depreciation due to selling in the foreign currency.

Our results suggests that a simple inflation targeting rule with a high enough feedback coefficient on domestic inflation can approximate the welfare-maximizing monetary policy relatively well. Surprisingly, after introducing a wide range of nominal frictions and a completely different way of modelling openness, we got back the classical result of the "science of monetary policy". However, this builds on some crucial assumptions (high share of imports in export sector marginal cost, perfect exchange rate pass-through for import prices, broadly similar movements in marginal costs for both sectors, no imported consumption) that all help in getting this result. A further research is needed to examine these assumptions and check the robustness of our conclusions to relaxing them.

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6 Appendix A: The log-linearized model

This section reviews the log-linearized model equations. The tilde denotes the log-deviation of a variable from its steady-state value. Variables without time indices represent their steady-state values.

6.1 Aggregate demand

The Euler equation of households'

$$\tilde{c}_t = \frac{h}{1+h}\tilde{c}_{t-1} + \frac{1}{1+h}\mathrm{E}_t\left[\tilde{c}_{t+1}\right] - \frac{1-h}{(1+h)\sigma}\mathrm{E}_t\left[\hat{\imath}_t - \hat{\pi}_{t+1}\right] + \tilde{\varepsilon}_t^c,$$

where the shock to the Euler equation can be expressed as function of current and future preferences shocks:

$$\tilde{\varepsilon}_t^c = \frac{(1-h)}{(1+h)\sigma}\left(\tilde{\eta}_t^c - \mathrm{E}_t\left[\tilde{\eta}_{t+1}^c\right]\right).$$

The arbitrage condition equating expected yields on physical capital and bonds, is described by

$$\begin{aligned}\mathrm{E}_t\left[\hat{\imath}_t - \hat{\pi}_{t+1}\right] &= \frac{1-\delta}{1-\delta+r^k}\mathrm{E}_t\left[\widetilde{Q}_{t+1}\right] - \widetilde{Q}_t \\ &+ \frac{r^k}{1-\delta+r^k}\mathrm{E}_t\left[\tilde{r}_{t+1}^k\right] + \tilde{\varepsilon}_t^Q.\end{aligned}$$

The trajectory of investments is given by

$$\tilde{I}_t = \frac{1}{1+\beta}\tilde{I}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\left[\tilde{I}_{t+1}\right] + \frac{1}{(1+\beta)\phi_I}\widetilde{Q}_t + \tilde{\varepsilon}_t^I,$$

where

$$\tilde{\varepsilon}_t^I = \frac{\beta\mathrm{E}_t\left[\tilde{\eta}_{t+1}^I\right] - \tilde{\eta}_t^I}{1+\beta}.$$

Capital accumulation equation is standard.

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{I}_t + \tilde{\varepsilon}_t^k.$$

The log-linear version of the export-demand equation is

$$\tilde{x}_t = h_x\tilde{x}_{t-1} - \theta_x\tilde{P}_t^{x*} + \tilde{x}_t^*.$$

Equilibrium conditions on domestic and export markets, respectively,

$$\begin{aligned}y^d\tilde{y}_t^d &= c\tilde{c}_t + I\tilde{I}_t + g\tilde{g}_t + r^k k^d\psi\tilde{r}_t^k, \\ y^x\tilde{y}_t^x &= x\tilde{x}_t + r^k k^x\psi\tilde{r}_t^k,\end{aligned}$$

where the last term stands for the $r^k k\psi\tilde{r}_t^k$ the loss associated with changes in capacity utilization.

6.2 Aggregate supply

Demand for production inputs (capital and composite production input) is represented by the following log-linear equations,

$$\begin{aligned}
\tilde{k}_t^s &= \varrho(1-\alpha_s)\left(\widetilde{w}_t^{zs}-\tilde{r}_t^k\right)-\psi\tilde{r}_t^k+\frac{\tilde{y^s}_t}{1+f_s}-\widetilde{A}_t, \qquad s=d,\,x,\\
\tilde{z}_t^s &= \varrho\alpha_s\left(\tilde{r}_t^k-\widetilde{w}_t^{zs}\right)+\frac{\tilde{y^s}_t}{1+f_s}-\widetilde{A}_t, \qquad s=d,\,x,
\end{aligned}$$

where $f^s=\bar{f}^s/y$ represents the share of fix costs in steady production, while ϱ is the elasticity of substitution between k and z (the composite input). The price of z, denoted by w^z is the weighted average cost of labor ($\widetilde{w}_t$) and imports ($\tilde{q}_t+\widetilde{P}_t^{m*}$), net of adjustment costs (ϕ_z):

$$\begin{aligned}
\widetilde{w}_t^{zs} &= a_s\widetilde{w}_t+(1-a_s)\left(\tilde{q}_t+\widetilde{P}_t^{m*}\right)+\phi_z\tilde{z}_t^s, \qquad s=d,\,x.\\
\tilde{q}_t &= \tilde{e}_t-\widetilde{P}_t,
\end{aligned}$$

Furthermore, labor and import demand is given by

$$\begin{aligned}
\tilde{l}_t^s &= \varrho_z(1-a_s)\left(\tilde{q}_t+\widetilde{P}_t^{m*}-\widetilde{w}_t\right)+\tilde{z}_t^s, \qquad s=d,\,x,\\
\widetilde{m}_t^s &= \varrho_z a_s\left(\widetilde{w}_t-\tilde{q}_t-\widetilde{P}_t^{m*}\right)+\tilde{z}_t^s, \qquad s=d,x,
\end{aligned}$$

and the aggregate quantities of the individual inputs are given by

$$\begin{aligned}
l\tilde{l}_t &= l^d\tilde{l}_t^d+l^x\tilde{l}_t^x,\\
m\widetilde{m}_t &= m^d\widetilde{m}_t^d+m^x\widetilde{m}_t^x,\\
k\tilde{k}_t &= k^d\tilde{k}_t^d+k^x\tilde{k}_t^x.
\end{aligned}$$

The Calvo price-setting rule with indexation to lagged inflation implies the following log-linear *hybrid Phillips curve.*

$$\begin{aligned}
\hat{\pi}_t &= \frac{\beta}{1+\beta\vartheta_d}\mathrm{E}_t\left[\hat{\pi}_{t+1}\right]+\frac{\vartheta_d}{1+\beta\vartheta_d}\hat{\pi}_{t-1}\\
&+\frac{\xi_d}{1+\beta\vartheta_d}\left[\alpha_d\tilde{r}_t^k+(1-\alpha_d)\widetilde{w}_t^{zd}-\widetilde{A}_t\right]+\tilde{\upsilon}_t^d,
\end{aligned}$$

where

$$\xi_d=\frac{(1-\gamma_d)(1-\beta\gamma_d)}{\gamma_d}, \qquad \tilde{\upsilon}_t=-\frac{\xi_d}{1+\beta\vartheta_d}\tilde{\tau}_t.$$

Similarly, the *Phillips curve* of the exports sector is given by

$$
\begin{aligned}
\hat{\pi}_t^{x*} &= \frac{\beta}{1+\beta\vartheta_d}\mathrm{E}_t\left[\hat{\pi}_{t+1}^{x*}\right] + \frac{\vartheta_x}{1+\beta\vartheta_x}\hat{\pi}_{t-1}^{x*} \\
&+ \frac{\xi_x}{1+\beta\vartheta_x}\left[\alpha_x\tilde{r}_t^k + (1-\alpha_x)a_x\widetilde{w}_t - \left[\alpha_x + (1-\alpha_x)a_x\right]\tilde{q}_t + (1-\alpha_x)(1-a_x)\widetilde{P}_t^{m*}\right] \\
&+ \frac{\xi_x}{1+\beta\vartheta_x}\left[(1-\alpha_x)\phi_z\tilde{z}_t^x - \widetilde{P}_t^{x*} - \widetilde{A}_t\right] + \tilde{\upsilon}_t^x,
\end{aligned}
$$

where $\hat{\pi}_t^{x*} = \pi_t^{x*} - \bar{\pi}_t^{x*}$, $\pi_t^{x*} = \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}$ and

$$
\xi_x = \frac{(1-\gamma_x)(1-\beta\gamma_x)}{\gamma_x}, \qquad \tilde{\upsilon}_t^x = -\frac{\xi_x}{1+\beta\vartheta_x}\tilde{\tau}_t^x.
$$

Wage setting in the model is based on similar assumptions as price formation. In can be shown that the log-linear *wage Phillips curve* is given by

$$
\begin{aligned}
\hat{\pi}_t^w &= \frac{\beta}{1+\beta\vartheta_w}\mathrm{E}_t\left[\hat{\pi}_{t+1}^w\right] + \frac{\vartheta_w}{1+\beta\vartheta_w}\hat{\pi}_{t-1}^w \\
&+ \frac{\xi_w}{1+\beta\vartheta_w}\left[\frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right) + \varphi\tilde{l}_t - \widetilde{w}_t\right] + \tilde{\upsilon}_t^w,
\end{aligned}
$$

where

$$
\xi_w = \frac{(1-\gamma_w)(1-\beta\gamma_w)}{\gamma_w(1+\theta_w\varphi)}, \qquad \tilde{\upsilon}_t = \frac{\xi_w}{1+\beta\vartheta_w}\left(\eta_t^l - \tau_t^w\right)
$$

and

$$
\hat{\pi}_t^w = \widetilde{w}_t - \widetilde{w}_{t-1} + \hat{\pi}_t.
$$

6.3 Current account

The evolution of net foreign assets is given by:

$$
\tilde{b}_t = (1+i^*)\tilde{b}_{t-1} + \frac{P^{x*}x}{GDP^*}\left(\widetilde{P}_t^{x*} + \tilde{x}_t\right) - \frac{P^{m*}m}{GDP^*}\left(\widetilde{P}_t^{m*} + \widetilde{m}_t\right),
$$

since it is assumed that $b = 0$, $\tilde{b}_t = b_t/GDP^*$, where $GDP^* = y^d + qP^{x*}x - qP^{m*}m$.

6.4 The interest rate and the exchange rate

Uncovered interest rate parity with financial premium shock can be expressed as

$$
\hat{\imath}_t = \mathrm{E}_t\left[d\hat{e}_{t+1}\right] + \tilde{\imath}_t^* + \tilde{\varepsilon}_t^{pr},
$$

following Schmitt-Grohe and Uribe (2002), it is assumed that $\tilde{\imath}_t^* = -\nu \tilde{b}_t$, this assumption ensures stationary of $\tilde{b}_t$.

In the inflation-targeting regime the behavior of the monetary authority is captured by the following *interest-rate rule*.

$$\hat{\imath}_t = \zeta_i \hat{\imath}_{t_1} + (1 - \zeta_i) \left[\zeta_\pi \hat{\pi}_t + \zeta_e^{it} \hat{e}_t \right] + \tilde{\varepsilon}_t^r.$$

Note, that the only role of $\zeta_e^{it} > 0$ is to ensure the stationarity of $\hat{e}_t$.

The evolution of the real exchange rate is determined by the following identity.

$$\tilde{q}_t - \tilde{q}_{t-1} = d\hat{e}_t - \hat{\pi}_t.$$

7 Appendix B: Parameters

Table 1 Fixed parameter values

Parameter		
Name		Value
standard error of gov't. consumption	σ_g	4.72
standard error of import prices	$\sigma_{p^{m*}}$	2.19
standard error of capital measurement error	σ_k	0.15
autoreg. coeff. of gov't. consumption	ρ_g	0.46
autoreg. coeff. of import prices	$\rho_{p^{m*}}$	0.74
autoreg. coeff of capital measurement error	ρ_k	0.60
autoreg. coeff of perceived average inflation	$\rho_{\bar{\pi}}$	0.99
discount factor	β	0.99
steady-state share of capital in real marginal costs, domestic	α_d	0.17
steady-state share of capital in real marginal costs, export	α_d	0.14
steady-state share of labor in w_t^z,domestic	a_d	0.50
steady-state share of labor in w_t^z, export	a_x	0.30
depreciation rate	δ	0.025
elasticity of subt. of goods	θ	6.00
elasticity of subt. of labor	θ_w	3.00
disutility parameter of labour	φ	8.00
Calvo parameter of employment	γ_n	0.70
elasticity of subt. between capital and z	ϱ	0.80
elasticity of subt. between labor and import	ϱ_z	0.50
ratio of fixed cost relative to total output	$\bar{f}_d, \bar{f}_x$	0.20
capacity utilization adj. cost	ψ	0.20
investments adjustment cost	Φ''	13.00
labour utilization adjustment cost	Φ_l''	3.00
import utilization adjustment cost	Φ_m''	3.00
exchange rate elasticity of the policy rule	ζ_e^{it}	0.025
debt elasticity of financial premium	ν	0.001

Table 2 Estimated parameters of exogenous shocks

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Standard errors							
productivity	σ_A	I.Gam.	0.5	2*	2.080	2.152	$[1.82, 2.55]$
export demand	σ_x	I.Gam.	0.5	2*	2.352	2.464	$[2.06, 2.95]$
cons. pref.	σ_c	I.Gam.	0.5	2*	0.146	0.203	$[0.11, 0.33]$
cons. price markup	σ_p	I.Gam.	0.5	2*	0.397	0.420	$[0.34, 0.52]$
export price markup	σ_{px}	I.Gam.	0.5	2*	1.873	2.182	$[1.64, 2.87]$
labor market	σ_w^{it}	I.Gam.	0.5	2*	0.609	0.932	$[0.54, 1.42]$
investments	σ_I	I.Gam.	0.5	2*	0.969	1.003	$[0.76, 1.27]$
Equity premium	σ_Q	I.Gam.	0.5	2*	0.167	0.393	$[0.11, 0.99]$
policy rule	σ_r^{cr}	I.Gam.	0.5	2*	0.574	0.602	$[0.48, 0.76]$
policy rule	σ_r^{it}	I.Gam.	0.5	2*	0.229	0.247	$[0.19, 0.32]$
fin. premium	σ_{pr}^{it}	I.Gam.	0.5	2*	0.486	0.666	$[0.36, 1.06]$
employment	σ_n	I.Gam.	0.5	2*	0.369	0.383	$[0.30, 0.49]$
Autoregressive coefficients							
productivity	ρ_A	Beta	0.8	0.1	0.577	0.552	$[0.43, 0.68]$
export demand	ρ_x	Beta	0.8	0.1	0.616	0.625	$[0.51, 0.73]$
cons. pref.	ρ_c	Beta	0.8	0.1	0.833	0.767	$[0.60, 0.88]$
labor market	ρ_w^{it}	Beta	0.8	0.1	0.797	0.661	$[0.43, 0.87]$
export markup	ρ_x	Beta	0.5	0.15	0.317	0.318	$[0.15, 0.50]$
investments	ρ_I	Beta	0.8	0.1	0.497	0.488	$[0.33, 0.65]$
fin. premium	ρ_{pr}^{it}	Beta	0.8	0.1	0.872	0.820	$[0.70, 0.92]$
employment	ρ_n	Beta	0.8	0.1	0.790	0.770	$[0.64, 0.89]$

* For the Inverted Gamma function the degrees of freedom are indicated.

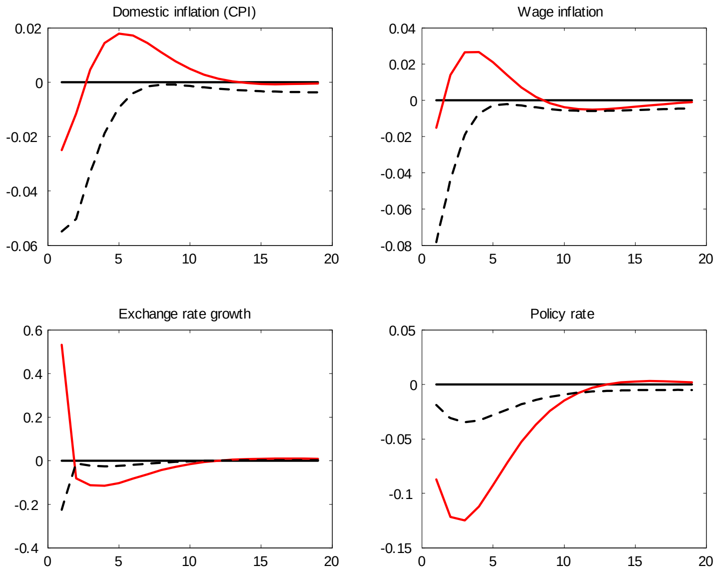
Table 3 Structural parameters

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Utility function parameters							
consumption	σ	Norm.	2.00	0.40	1.680	1.814	$[1.18, 2.46]$
habit	h	Beta	0.75	0.15	0.597	0.646	$[0.45, 0.83]$
Price and wage setting param.							
ind. cons. prices	ϑ_p^{it}	Beta	0.60	0.20	0.416	0.431	$[0.22, 0.66]$
ind. exp. prices	ϑ_x^{it}	Beta	0.60	0.20	0.383	0.494	$[0.18, 0.83]$
ind. wages	ϑ_w^{it}	Beta	0.60	0.20	0.107	0.185	$[0.05, 0.40]$
Calvo cons. prices	γ_p^{it}	Beta	0.50	0.20	0.929	0.921	$[0.88, 0.95]$
Calvo exp. prices	γ_x^{it}	Beta	0.50	0.20	0.827	0.810	$[0.73, 0.89]$
Calvo wages	γ_w^{it}	Beta	0.50	0.20	0.711	0.657	$[0.46, 0.88]$
Other parameters							
exp. elasticity	θ_x	Beta	0.50	0.10	0.510	0.534	$[0.40, 0.67]$
exp. smooth.	h_x	Beta	0.75	0.15	0.503	0.507	$[0.35, 0.66]$
ir. smooth.	ζ_i	U(0,1)	0.50	0.29	0.766	0.761	$[0.67, 0.84]$
policy rule	ζ_π	Norm.	1.50	0.16	1.375	1.379	$[1.12, 1.65]$

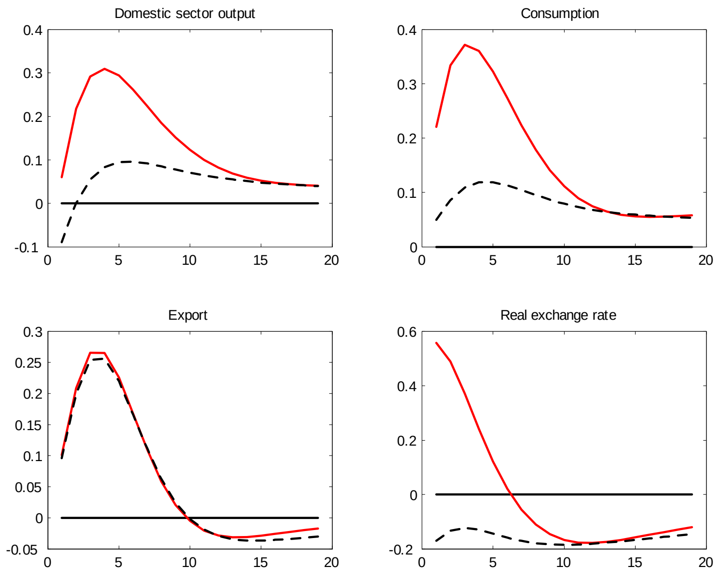
Appendix C: Impulse responses

7.1 Comparing the fully optimal rule and the empirical one

7.1.1 Productivity shock



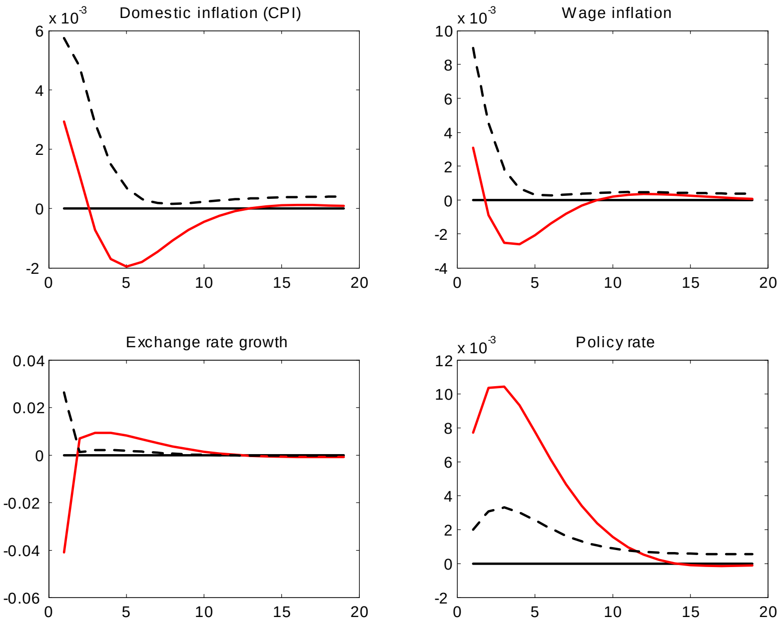
LQ solid red, EMP dashed black



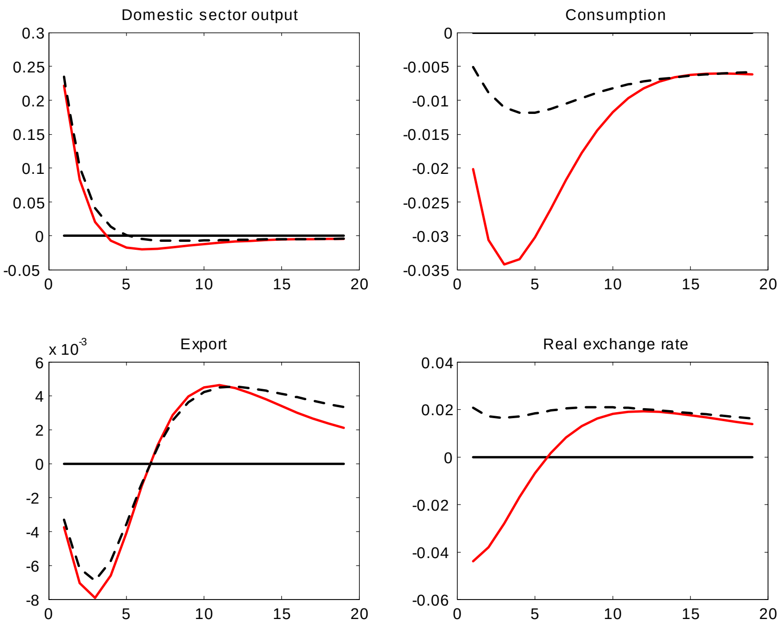
LQ solid red, EMP dashed black

7.1.2 Government spending shock

Figure here

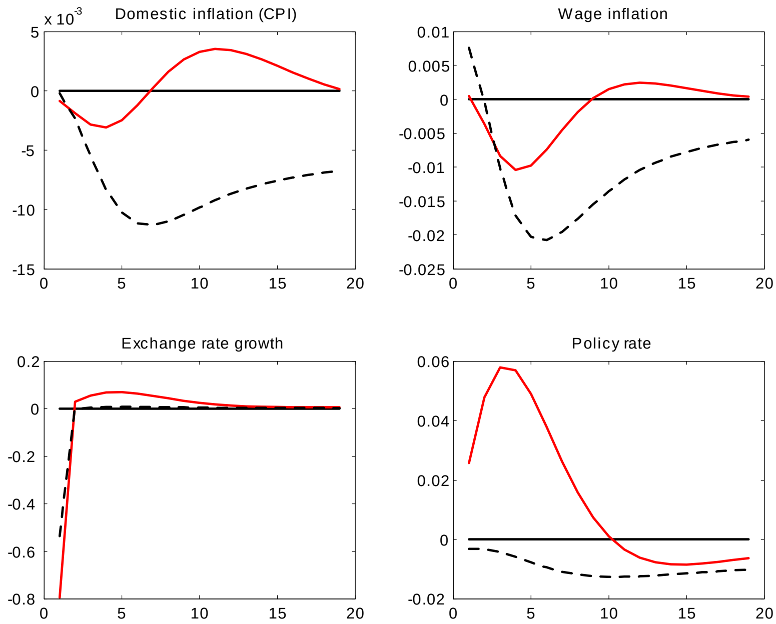


LQ solid red, EMP dashed black

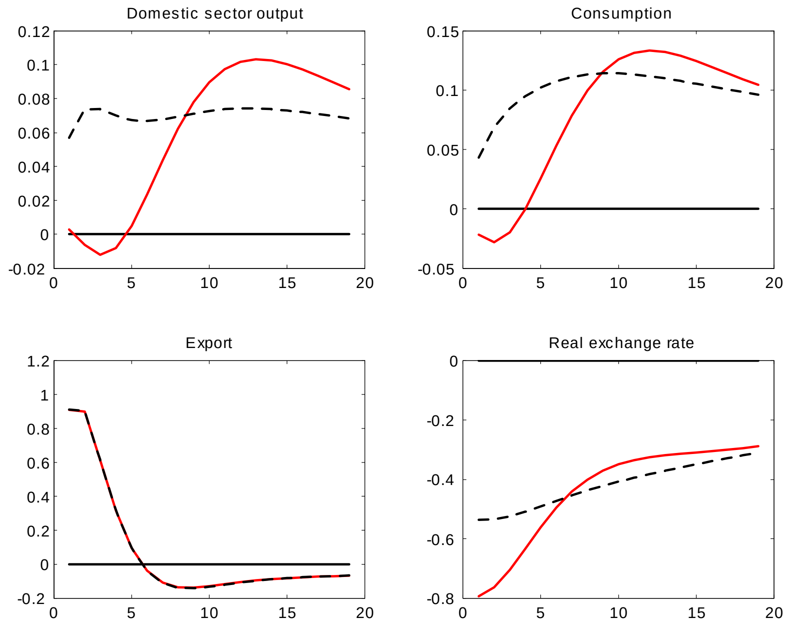


LQ solid red, EMP dashed black

7.1.3 Export demand shock

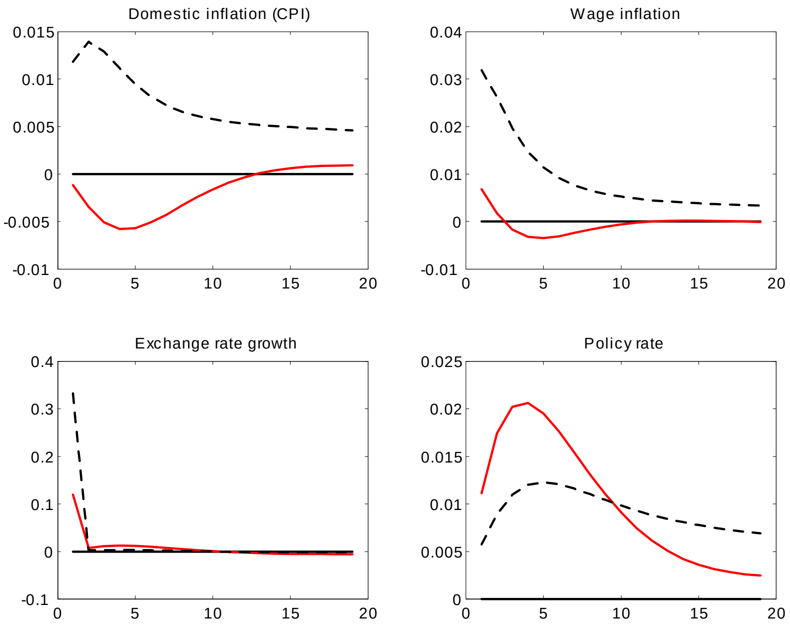


LQ solid red, EMP dashed black

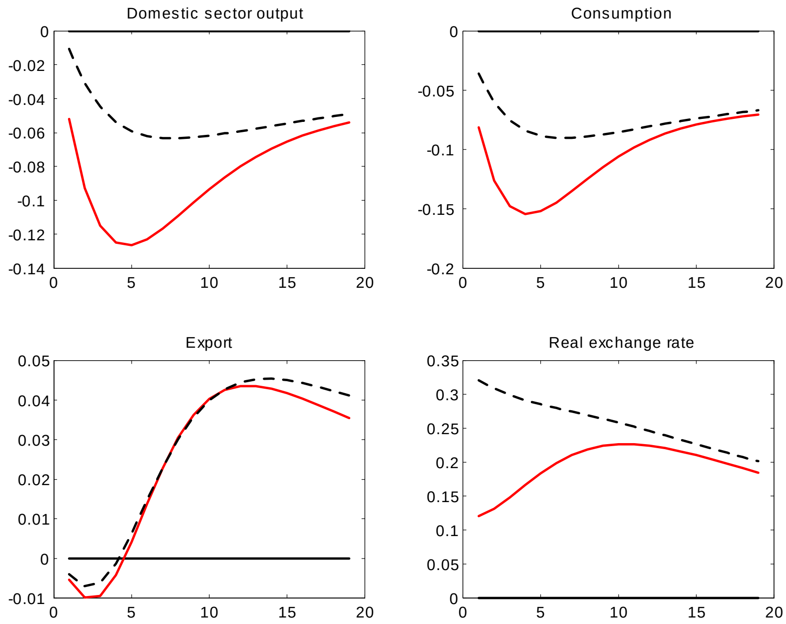


LQ solid red, EMP dashed black

7.1.4 Import price shock



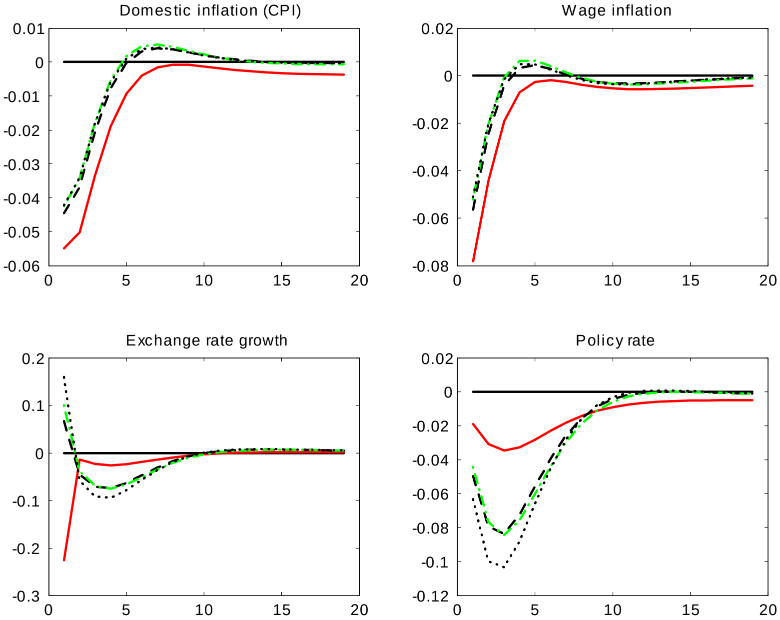
LQ solid red, EMP dashed black



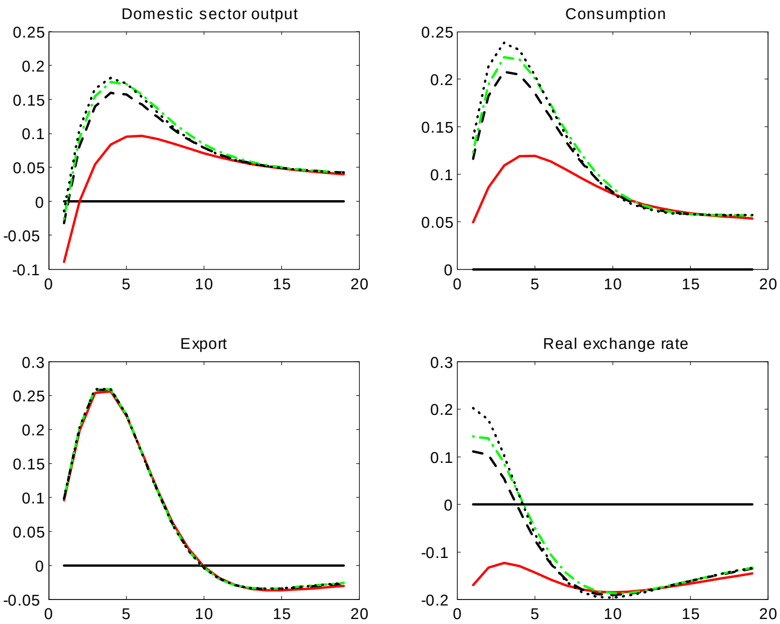
LQ solid red, EMP dashed black

7.2 Comparing the empirical rule to the OSRs

7.2.1 Productivity shock

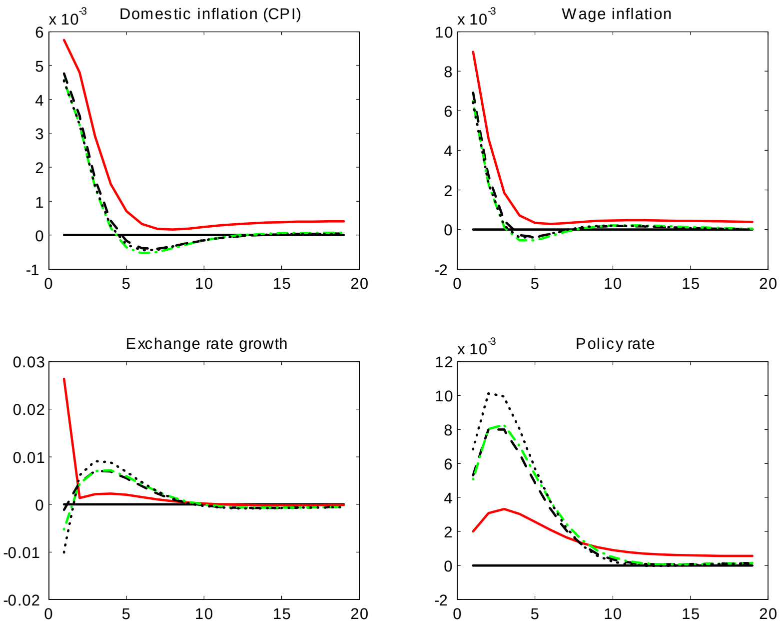


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

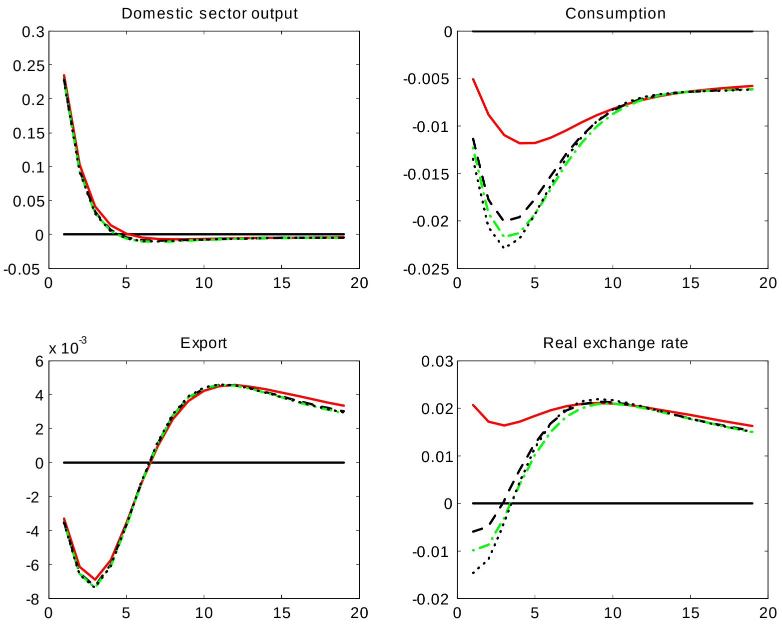


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

7.2.2 Government spending shock

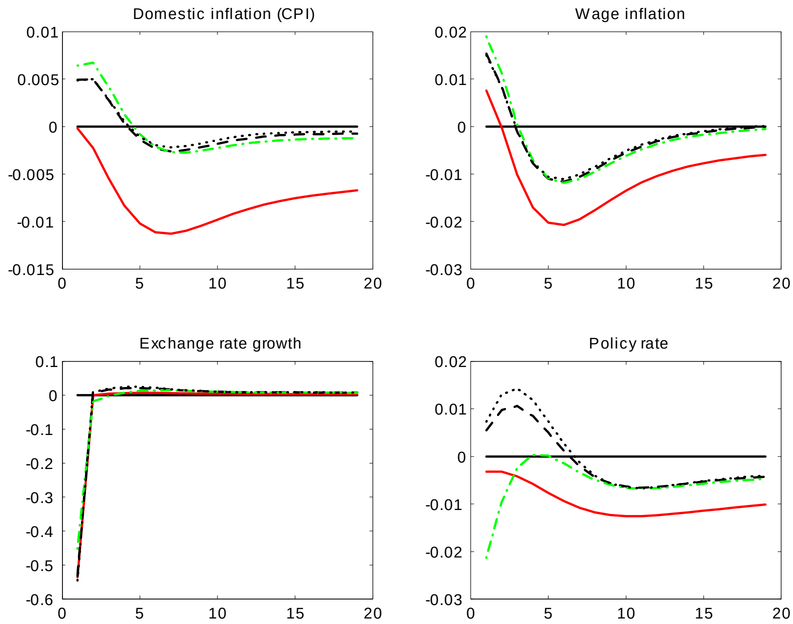


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

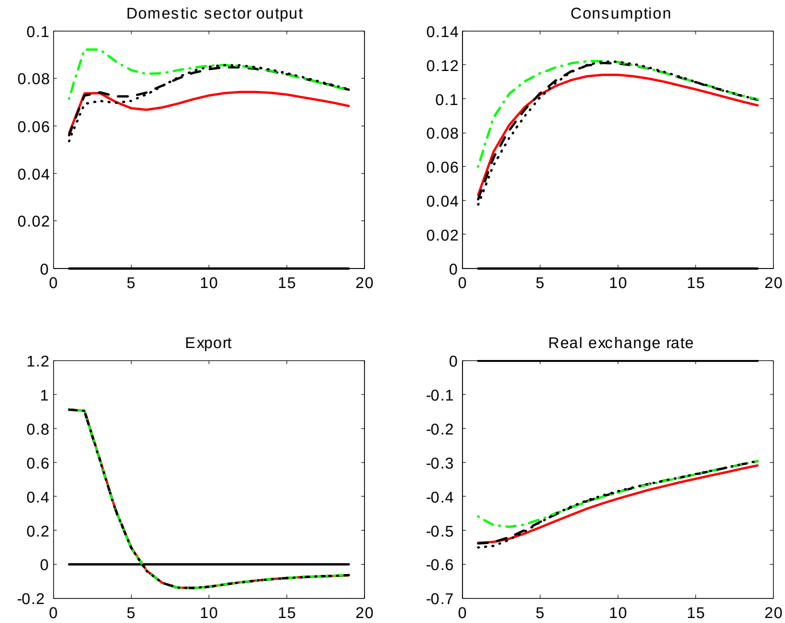


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

7.2.3 Export demand shock

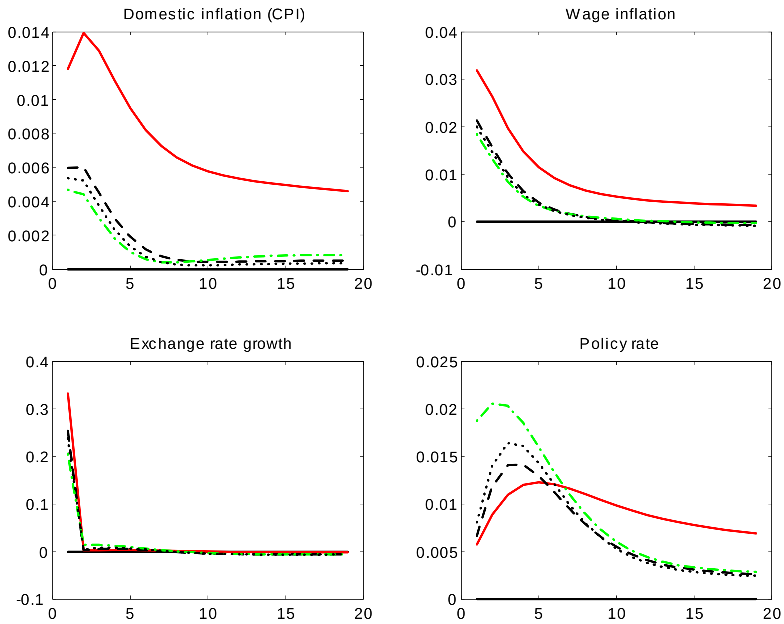


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

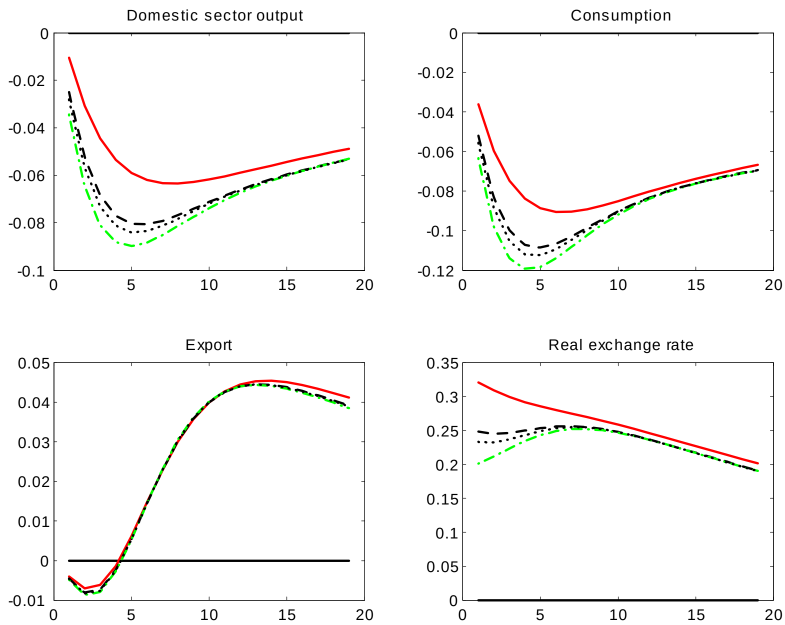


EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted

7.2.4 Import price shock



EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted



EMP solid, OSR1 dashed, OSR2 dotted, OSR3 dashed-dotted