Unique Monetary Equilibria with Interest Rate Rules*

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Abstract

In contrast to previous literature, we show that there are interest rate rules that implement unique global equilibria in standard monetary models. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. The interest rate

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rules we propose are price level targeting rules that respond to the forecasts of the future price level and future economic activity.

Key words: Monetary policy; interest rate rules; price level targeting; unique equilibrium.

JEL classification: E31; E40; E52; E58.

1. Introduction

Central banks in developed economies have been very successful in targeting low inflation in the last twenty years. The success is commonly attributed to central banks following some form of a Taylor (1993) rule where the short term nominal interest rate responds to inflation, to be raised when inflation is above target and lowered when it is below target, with a low target for inflation. Interest rate feedback rules of this type, have been extensively studied in the literature since Sargent and Wallace (1975) and McCallum (1981). Nevertheless, the interest rate rules proposed in the literature are unable to achieve in a monetary model what interest rate policy seems to achieve in reality. They are unable to implement a unique equilibrium with stable prices. The most they can ensure is local determinacy, meaning that there is a unique local equilibrium in some neighborhood of interest, but there are still multiple equilibria outside that neighborhood (see Cochrane (2007) for a criticism of the local determinacy approach).

In this paper we show, in contrast to previous literature, that there are interest rate rules that implement unique equilibria globally. The policy rules are price level targeting rules where the interest rate responds to the expected price level as well as other measures of economic activity. To the extent that the interest rate reacts to a forecast of future economic activity it resembles the rules that central banks appear to follow. In the response to the price level it is in the class of price level targeting rules that are further apart from the policy debate.

We show the results in a simple model, a cash-in-advance economy with flexible prices. The results are however robust to the consideration of nominal rigidities, as we also show in the paper. An important assumption, and one that is also standard in this literature, is that fiscal policy is endogenous, meaning that taxes can be adjusted residually to satisfy the budget constraint of the government.

As mentioned above, after Sargent and Wallace (1975), and McCallum (1981), there has been an extensive literature on multiplicity of equilibria when the government follows an interest rate rule. This includes the literature on local determinacy, with more recent contributions such as Woodford (2003), Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib, Schmitt-Grohe and Uribe (2001a), among many others. In this literature the analysis uses linear approximations of the models, in the neighborhood of a steady state, and identifies the conditions on preferences, technology, timing of markets, and policy rules, under which there is a unique local equilibrium.[1] There is a unique local equilibrium, in the neighborhood of a steady state, when there is also a continuum of divergent solution paths originating close to that steady state. In the linear approximation of the model, the divergent solutions are explosive, and are typically disregarded using arbitrary technical restrictions.[2] In the nonlinear model the alternative equilibria may converge to other steady states, or exhibit all kinds of cyclical behavior. It is on the basis of these results that the literature on local determinacy has been criticized by the recent work on global stability showing that the conditions for local determinacy may in fact be conditions for global indeterminacy (see Benhabib, Schmitt-Grohe and Uribe 2001b, 2002, 2003, Schmitt-Grohe and Uribe 2001). Christiano and Rostagno (2002) and Atkeson, Chari and Kehoe (2007) also criticize the local determinacy approach and show that there are alternative policies that implement unique equilibria.

Independent work by Loisel (2006) has shown in the linearized new keynesian model that there are feedback rules analogous to the ones we propose in this paper

[1] The conditions for local determinacy depend on considering investment as in Dupor (2001), the timing of transactions as in Carlstrom and Fuerst (2001), the role of money and the presence of nominal rigidities, as in Benhabib, Schmitt-Grohe and Uribe (2001a) and Carlstrom and Fuerst (2002).

[2] See Woodford (2003).

that rule out explosive solutions. Cochrane (2007) provides a critical evaluation of the local determinacy literature and compares the standard rules to Loisel's and ours.

This paper was motivated by previous work on optimal monetary policy in an economy under sticky prices. In Adao, Correia and Teles (2003), it is shown that after choosing the sequence of nominal interest rates there is still a large set of implementable allocations, each supported by a particular sequence of money supplies. Implicitly it is assumed that policy can set exogenous sequences for both interest rates and money supplies, subject to certain restrictions. Alternatively, as we show in this paper, there are interest rate feedback rules that implement the optimal allocation. Finally, the paper is also related to Adao, Correia and Teles (2004) that shows that it is possible to implement unique equilibria in environments with flexible prices and prices set in advance by pegging state contingent interest rates as well as the initial money supply.

The paper proceeds as follows: In Section 2 we illustrate the results in an endowment economy. We show that there are interest rate rules that implement unique equilibria, while the rules proposed in the literature only guarantee local determinacy. In Section 3 we describe the model, a simple cash-in-advance economy with flexible prices. In Section 4 we show that each equilibrium in a large set of equilibria can be implemented uniquely using interest rate rules analogous to the ones in the endowment economy. In Section 5 we interpret the results and in Section 6 we extend the results to environments with capital and sticky prices. Section 7 contains concluding remarks.

2. A simple endowment economy

To motivate and illustrate the results in this paper, we consider a simple endowment economy. The representative household can hold non-contingent bonds so that the Euler equation

$$\frac{u_c(Y_t)}{P_t} = R_t E_t \frac{\beta u_c(Y_{t+1})}{P_{t+1}} \tag{2.1}$$

must hold in equilibrium, where $\{Y_t\}$ is the endowment process, R_t is the gross nominal interest rate and P_t is the price level. The log linear approximation to the Euler equation, where the variables with a hat are in log deviations from a deterministic steady state with constant inflation π^*, can be written as

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t, \tag{2.2}$$

where $r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}$, or

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}. \tag{2.3}$$

Suppose now that policy was conducted by setting the nominal interest rate path, exogenously, equal to a sequence of numbers. This would allow to determine a unique path for the conditional expectation of inflation $E_t \widehat{\pi}_{t+1}$, but would not determine the initial price level, nor the distribution of realized inflation across states.

In this economy, if policy was conducted, instead, with an inflation targeting rule where the interest rate responded to contemporaneous, past or future inflation there still be multiple equilibria. Inflation targeting rules are able to determine locally a unique equilibrium in the neighborhood of a steady state, but do so at the expense of multiple other solutions of the linear system that diverge from that neighborhood.

Suppose policy was conducted with an interest rate rule where the nominal interest rate $\widehat{R}_t$ reacts to $\widehat{\pi}_t$, i.e. the deviations of inflation from the target π^*,[3]

$$\widehat{R}_t = \widehat{r}_t + \tau \widehat{\pi}_t.$$

[3] The real interest rate term $\widehat{r}_t$ is included in the rule only for the convenience of determining a particular path, the one where inflation is equal to the constant target. It is irrelevant for the issue of determinacy.

Then

$$\tau\widehat{\pi}_t - E_t\left(\widehat{\pi}_{t+1}\right) = 0.$$

With $\tau > 1$, there is a locally determinate solution $\widehat{\pi}_t = 0$. In deviations from the steady state levels, the solution for the nominal interest rate is equal to the real interest rate, $\widehat{R}_t = \widehat{r}_t$. This is the standard case discussed in the literature where the Taylor principle, with $\tau > 1$, is necessary to guarantee a locally determinate equilibrium.

With an active interest rate rule reacting to inflation, i.e. one with a coefficient $\tau > 1$, there is indeed in the linear model a single local equilibrium but multiple explosive solutions. If inflation in period zero was $\widehat{\pi}_t = \varepsilon > 0$, the solution would diverge. These divergent solutions may in the nonlinear model converge to another steady state or cycle around this steady state.[4]

Forward looking rules are not even able to guarantee local determinacy. If the rule is

$$\widehat{R}_t = \widehat{r}_t + \tau E_t\widehat{\pi}_{t+1}.$$

Then, for $\tau \neq 1$, only expected inflation is pinned down, not so the distribution of prices across states. With a backward rule

$$\widehat{R}_t = \widehat{r}_t + \tau\widehat{\pi}_{t-1},$$

the dynamic equation is

$$\tau\widehat{\pi}_{t-1} - E_t\left(\widehat{\pi}_{t+1}\right) = 0.$$

There are again multiple solutions and a locally determinate solution, $\widehat{\pi}_t = 0$, with $\tau > 1$.

Wicksellian interest rate rules as in Woodford (2003) have the interest rate respond to the price level rather than inflation. In deviations from the deterministic

[4] See Benhabib, Schmitt-Grohe and Uribe (2001b).

steady state, with a constant inflation target,[5] that rule is

$$\widehat{R}_t = \widehat{r}_t + \phi \widehat{P}_t,$$

where $\phi > 0$.[6] Substituting the rule in the equilibrium condition (2.3) with $\widehat{\pi}_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t$, we get

$$(1+\phi)\,\widehat{P}_t - E_t \widehat{P}_{t+1} = 0 \tag{2.4}$$

With $\phi > 0$, there is a determinate equilibrium, locally, in the neighborhood of the steady state. The price level will be growing at the constant inflation π^*. There are however, also in this case, other solutions of the linear model, that diverge from the neighborhood of the steady state. Also with these rules it is not possible to exclude those other, divergent, solutions as possible candidates to equilibria that cannot be analyzed in the linear model.

The rule we propose in this paper is also a price targeting rule in the sense that the interest rate reacts to the price level rather than inflation. In the linear endowment economy model the rule would be

$$\widehat{R}_t = \widehat{r}_t + \phi E_t \widehat{P}_{t+1} - \widehat{\xi}_t \tag{2.5}$$

where $\widehat{\xi}_t$ is an exogenous random variable and $\phi = 1$. It is a forward rule with the coefficient on the price level equal to 1. The policy rule together with the Fisher equation, (2.3), implies

$$\widehat{P}_t = \widehat{\xi}_t$$

which determines a unique solution for the deviations from a steady-state.

For any $\phi < 1$, there is one local solution and a continuum of explosive solutions

[5] $\widehat{P}_t$ are log deviations of the price level from the stationary path $P_t = P_{-1}(\pi^*)^{t+1}$, where π^* is the constant (gross) inflation target.

[6] Again, here, the term $\widehat{r}_t$ is irrelevant for the issue of determinacy.

of the dynamic equation

$$(1-\phi)\, E_t \widehat{P}_{t+1} = \widehat{P}_t - \widehat{\xi}_t \tag{2.6}$$

This is still the case as $\phi \to 1$. For $\phi = 1$, there is a unique solution. This is the argument in Cochrane (2007) to relate our rule to standard Wicksellian, or Taylor rules, that generate an infinite forward-looking eigenvalue.

We have shown, in the linear model, that the policy rule (2.5), with $\phi = 1$, is able to generate a unique global solution for the deviations from steady state, eliminating the other solutions that are present when the alternative rules considered above are followed, whether inflation or price level targeting rules.

The policy rule in the linearized model pins down the deviations from the steady-state path, but not the steady-state path itself. The linearization also rules out by construction other solutions that could be present in the non linear model. For these reasons, we analyze now the non linear model.

3. A model with flexible prices

We first consider a simple cash-in-advance economy with flexible prices. The economy consists of a representative household, a representative firm behaving competitively, and a government. The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$ and the history of its realizations up to period t (state or node at t), $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. The initial realization s_0 is given. We assume that the history of shocks has a discrete distribution.

Production uses labor according to a linear technology. We impose a cash-in-advance constraint on the households' transactions with the timing structure described in Lucas and Stokey (1983). Each period is divided into two subperiods, with the assets market operational in the first subperiod and the goods market in the second.

3.1. Competitive equilibria

Households The households have preferences over consumption C_t, and leisure L_t, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(C_t, L_t \right) \right\} \tag{3.1}$$

where β is a discount factor. The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t riskless nominal bonds that pay $R_t B_t$ one period later. R_t is the gross nominal interest rate at date t. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t \leq \mathbb{W}_t. \tag{3.2}$$

Consumption must be purchased with money according to the cash-in-advance constraint

$$P_t C_t \leq M_t, \tag{3.3}$$

where P_t is the price of the consumption good in units of money.

At the end of the period, the households receive the labor income $W_t N_t$, where $N_t = 1 - L_t$ is labor and W_t is the nominal wage rate and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t - P_t C_t + W_t N_t - T_t \tag{3.4}$$

The households' problem is to maximize expected utility (3.1) subject to the restrictions (3.2), (3.3), (3.4), together with a no-Ponzi games condition on the holdings of assets.

The following are first order conditions of the households problem:

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{R_t P_t} \tag{3.5}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right] \tag{3.6}$$

Condition (3.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money, R_t. Condition (3.6) is an intertemporal marginal condition necessary for the optimal choice of riskless nominal bonds.

Firms The firms are competitive and prices are flexible. The production function of the representative firm is

$$Y_t \leq A_t N_t.$$

The firms maximize profits $P_t Y_t - W_t N_t$. The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t. \tag{3.7}$$

Government The policy variables are lump sum taxes, T_t, interest rates, R_t, money supply, M_t, supply of state-noncontingent public debt, B_t. We can define a policy as a mapping for the sequence of policy variables $\{T_t, R_t, M_t, B_t,\ t \geq 0,\ \text{all } s^t\}$, that maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables that satisfy the government budget constraints (see Kocherlakota and Phelan, 1999).

The period by period government budget constraints are

$$M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}G_{t-1} - P_{t-1}T_{t-1},\ t \geq 0.$$

Let

$$Q_{t,s} = \beta^{s-t} \frac{u_C(s)}{u_C(t)} \frac{P_t}{P_s},\ t \geq 0,\ s \geq t. \tag{3.8}$$

With $\lim_{T\to\infty} E_t Q_{t,T+1} \mathbb{W}_{T+1} = 0$, the sequence of budget constraints are

$$\sum_{s=t}^{\infty} E_t Q_{t,s+1} M_s \left(R_s - 1\right) = \mathbb{W}_t + \sum_{s=t}^{\infty} E_t Q_{t,s+1} P_s \left[G_s - T_s\right], \tag{3.9}$$

where $\mathbb{W}_t$ is given by (3.4).

Market clearing Market clearing in the goods and labor market requires

$$C_t + G_t = A_t N_t \tag{3.10}$$

and

$$N_t = 1 - L_t. \tag{3.11}$$

We have already imposed market clearing in the money and debt markets.

Equilibrium An equilibrium is a sequence of policy variables, quantities and prices such that the private agents solve their problems given the sequences of policy variables and prices, the sequence of policy variables is in the set defined by the policy and markets clear.

An equilibrium sequence $\{C_t, L_t, P_t, R_t, M_t, \mathbb{W}_{t+1}, T_t,\ t \geq 0, \text{ all } s^t\}$ satisfies the following equilibrium conditions: The resources constraints

$$C_t + G_t = A_t(1 - L_t),\ t \geq 0, \tag{3.12}$$

the intratemporal condition that is obtained from the households intratemporal condition (3.5) and the firms optimal condition (3.7)

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},\ t \geq 0, \tag{3.13}$$

the cash in advance constraints (3.3), the intertemporal conditions (3.6), and the budget constraints (3.9), as well as the government policy rules.

We now consider a particular government policy, in the class defined above, where policy is the set of all sequences for the policy variables that satisfy the government budget constraint. This allows us to define a set of implementable allocations, prices and policy variables $\{C_t, L_t, P_t, R_t \geq 1, M_t, \mathbb{W}_{t+1}, T_t\}$ as follows.

Definition 3.1. *The set of implementable allocations, prices and policy variables* $\{C_t, L_t, P_t, R_t \geq 1, M_t, B_t, T_t\}$ *is the set of sequences that satisfies conditions (3.3), (3.6), (3.9), (3.12) and (3.13).*

The question we ask in this paper is whether there is a policy rule that is able to implement uniquely each and every sequence in the implementable set defined above. In particular, suppose the policy is a rule for the nominal interest rate where the rate depends on prices and quantities. Is an interest rate rule of that type able to implement a unique equilibrium? Is it able to implement the best equilibrium in the implementable set?

4. Interest rate rules that implement unique equilibria.

We now assume that policy is conducted with an interest rate feedback rule. The remaining policy variables are such that the government budget constraints are satisfied. We show the main result of the paper, that there are interest rate rules that implement unique equilibria, globally, for the allocations and prices. The proposition follows:

Proposition 4.1. *Every equilibrium in Definition 1, can be implemented (uniquely) with the interest rate rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}, \tag{4.1}$$

where ξ_t *is an exogenous variable.*

Proof: When policy is conducted with the rule (4.1), the intertemporal condition (3.6) can be written as

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0, \tag{4.2}$$

so that

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \tag{4.3}$$

It follows that the intratemporal conditions (3.13) can be written as

$$\frac{u_C(t)}{u_L(t)} = \frac{\frac{\xi_t}{\beta E_t \xi_{t+1}}}{A_t},\ t \geq 0 \tag{4.4}$$

These conditions together with the resource constraints, (3.12), determine uniquely the variables C_t, L_t, P_t, R_t, for every date and state. The money balances, M_t, are restricted by the cash-in-advance conditions (3.3). They are determined uniquely if the constraint holds with equality.[7]

The budget constraints (3.9) are satisfied for multiple paths of the taxes and state noncontingent debt levels.

To see that the rule is able to implement each equilibrium in Definition 1, notice that the set of implementable nominal interest rates and prices $\{R_t, P_t\}$ in the definition is given by

$$\frac{u_C\left(C\left(R_t\right), L\left(R_t\right)\right)}{P_t} = R_t E_t\left[\frac{\beta u_C\left(C\left(R_{t+1}\right), L\left(R_{t+1}\right)\right)}{P_{t+1}}\right] \tag{4.5}$$

$$R_t \geq 1$$

where the functions $C_t = C(R_t)$ and $L_t = L(R_t)$ are obtained using (3.12) and (3.13). For each sequence of $\{R_t \geq 1\}$, there are multiple sequences of $\{P_t\}$. The

[7] Notice that when the nominal interest rate is zero the cash-in-advance constraint does not have to hold with equality. This multiplicity of the money stock has no implications for the uniqueness of the price level or allocation.

initial price P_0 can be anything and there are $\Phi_{t+1} - \Phi_t$ degrees of freedom, where Φ_t is the number of states at t, to determine P_{t+1}, $t \geq 0$.

In every equilibrium using the rule we have

$$\frac{u_C\left(C\left(R_t\right), L\left(R_t\right)\right)}{P_t} = \xi_t,\ t \geq 0,$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

Consider a sequence of $\{R_t \geq 1\}$, associated with a particular allocation. ξ_t determines P_t, $E_t\xi_{t+1}$ is restricted by R_t. There are enough degrees of freedom to choose ξ_{t+1} to determine P_{t+1} in $\Phi_{t+1} - \Phi_t$ states.$\blacksquare$

Depending on the exogenous process for ξ_t, with the interest rate rule we consider, it is possible to implement every equilibrium in the implementable set defined in Definition 1. In particular, the first best allocation, at the Friedman rule of a zero nominal interest rate, can be implemented.

At the Friedman rule there is only one (first best) allocation but there are many possible equilibrium processes for the price level associated with that allocation. Varying the process for ξ_t, it is possible to implement uniquely each of those equilibria.

With

$$\xi_t = \frac{1}{k\beta^t}, t \geq 0,$$

where k is a positive constant, from (4.3), $R_t = 1$. Condition (4.4) becomes

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t},\ t \geq 0$$

which, together with the resource constraint (3.12) gives the first best allocation described by the functions $C_t = C^*(A_t, G_t)$, $L_t = L^*(A_t, G_t)$. The price level

$P_t = P(A_t, G_t; .)$ can be obtained as the solution for P_t of (4.2), i.e.

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{1}{k\beta^t},\ t \geq 0.$$

For each k, which is a policy parameter, there is a unique equilibrium process for the price level.[8] The equilibrium money stock is obtained using the cash-in-advance constraint, $M_t = P(A_t, G_t; .)C^*(A_t, G_t)$, if it holds with equality. If it did not hold exactly, there would be multiple equilibrium paths for the money stock that would have no implications for the determination of the prices and allocations.

The forward looking interest rate feedback rules that implement unique global equilibria resemble to some extent the rules that appear to be followed by central banks. The nominal interest rate reacts positively to the forecast of future consumption. It also reacts positively to the forecast of the future price level. While the reaction to future economic activity is standard in the policy debate, the reaction to the price level is not. Central banks appear to respond to forecasts of future inflation, rather than the price level, when deciding on nominal interest rates.

4.1. Money supply rules

An analogous result to the one for the interest rate above is obtained when policy is conducted with a particular money supply feedback rule, provided that the cash-in-advance constraint holds exactly. Every equilibrium in Definition 1 can be implemented (uniquely) with the money supply feedback rule,

$$M_t = \frac{C_t u_C(t)}{\xi_t}, \tag{4.6}$$

[8] There are other possible equilibrium processes for the path of the price level associated with the Friedman rule. The rule with $\xi_t = \frac{\mu_t}{k(\rho\beta)^t}$,where $\mu_t = \rho\mu_{t-1} + \varepsilon_t$ and ε_t is a white noise, also implies $R_t = 1$ and achieves the first best allocation with different processes for the price level depending on the choice of k, ρ and ε_t.

where ξ_t is an exogenous variable. To see this notice that if policy is conducted according to (4.6), then, using the cash in advance conditions (3.3) with equality, we obtain

$$\frac{u_C(t)}{P_t} = \xi_t \tag{4.7}$$

Using the intertemporal conditions (3.6), we have

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \tag{4.8}$$

The two conditions above, (4.7) and (4.8), together with the intratemporal conditions (3.13) and the resource constraints, (3.12) determine uniquely the four variables, C_t, L_t, P_t, R_t in each period $t \geq 0$ and state s^t. The taxes and debt levels satisfy the budget constraint (3.9).

Also for this money supply rule, for a particular choice of the process of ξ_t, it is possible to implement a particularly desirable equilibrium. The same process ξ_t implements the same equilibrium whether the rule is the interest rate rule (4.1) or the money supply rule (4.6), with one qualification. The implementation of a unique equilibrium with a money supply rule relies on the cash in advance constraint holding exactly. That is not necessarily the case when the interest rate is zero. Instead, with an interest rate rule there is always a unique equilibrium for the allocations and price level. The money stock is not unique when the cash in advance constraint does not hold with equality.

5. Interpreting the results

In general neither an interest rate rule nor a money supply rule is able to pin down a unique equilibrium. In particular, when policy is conducted with sequences of numbers for either the nominal interest rate or the money supply there are in general multiple equilibria. We now illustrate this and give a counterexample to the general result. The counterexample is useful because the mechanism is similar to the one that guarantees uniqueness when the interest rate rules are as in Proposition 3.1.

As before, from the resource constraints, (3.12), and the intratemporal conditions, (3.13), we obtain the functions $C_t = C(R_t)$ and $L_t = L(R_t)$. Assuming the cash-in-advance constraints, (3.3), hold with equality, the system of equilibrium conditions can be summarized by the following dynamic equations:

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], \, t \geq 0 \tag{5.1}$$

together with the budget constraints, which are satisfied by the choice of lump-sum taxes.

Suppose the path of interest rates is set exogenously in every date and state. It is straightforward to see that in general there are multiple solutions for the path of the money supply and therefore also for the path of the price level. Similarly, there would also be multiple equilibria if the money supply was set exogenously in every date and state. In that case there would be multiple solutions for the interest rates and also for the allocations.

We now describe a well known counterexample to the general result of multiplicity with money supply policy. For preferences that are additively separable and logarithmic in consumption, a money supply policy guarantees a unique equilibrium, again, provided that the cash-in-advance constraint holds with equality. The difference equation (5.1) would become

$$\frac{1}{M_t} = \beta R_t E_t \left[\frac{1}{M_{t+1}} \right], \, t \geq 0. \tag{5.2}$$

When the money supply is set exogenously in every state, there is a unique equilibrium for the path of the nominal interest rates $\{R_t\}$. The allocations are therefore uniquely determined from (3.12) and (3.13). With the allocations uniquely determined and the money supply set exogenously, the price level is also determined uniquely from the cash-in-advance constraint (3.3) with equality. In this case there is a unique equilibrium with policy conducted with only one instrument, the money supply.

The mechanism that allows to pin down a unique equilibrium for these preferences with money supply policy is similar to the one that allows the rules in Proposition 3.1 to guarantee unique global equilibria. Notice that the right hand side of the difference equation (5.1) is exogenous, as in the case of the rules in that proposition. The dynamic system of equations becomes a static system of equations for the endogenous variables.

6. Robustness

We have shown the results in the simplest possible model with flexible prices. The results are robust to more complex structures. It is straightforward to show that, in general, more complex real structures, such as capital, do not make a difference, and the results also hold under sticky prices.

6.1. Capital

In an economy with capital the technology will be

$$C_t + K_{t+1} - (1-\delta) K_t + G_t \leq A_t F(K_t, N_t).$$

The households own the capital stock and rent it out to the firms so that the budget constraints of the households are now

$$M_t + B_t \leq \mathbb{W}_t$$

and

$$\mathbb{W}_{t+1} = M_t + R_t B_t - P_t C_t + W_t N_t + P_t r_t K_t + P_t (1-\delta) K_t - P_t K_{t+1} - T_t$$

The intertemporal conditions of the households for nominal riskless bonds

(3.6), are still

$$\frac{u_C\left(t\right)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}}\right]$$

When policy is conducted with the interest rate rule above (4.1)

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium. Indeed, as before, the intertemporal condition (3.6) can be written as (4.2),

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0,$$

so that the nominal interest rate is given by (4.3),

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

Once the sequence of nominal interest rates $\{R_t\}$ is determined, the allocations in the model with capital are also uniquely determined. The real allocations in the model with flexible prices are only a function of the shocks and the process for the nominal interest rate. Once the allocations are determined, the price level is also determined uniquely from (4.2).

6.2. Sticky prices

Under flexible prices, an interest rate target, in the sense of a policy that sets the path of nominal interest rates equal to a sequence of numbers, is able to pin down a unique equilibrium for the real allocations, but not for the price level. Instead if prices are sticky, the same policy will generate multiplicity of real allocations. For this reason the interest of policy rules that may guarantee uniqueness is higher when nominal rigidities are considered.

In this section we show that the results derived above extend to an environment with prices set in advance. We modify the environment to consider price setting restrictions. There is a continuum of firms, indexed by $i \in [0,1]$, each producing a differentiated good also indexed by i. The firms are monopolistic competitive and set prices in advance with different lags.

The households have preferences described by (3.1) where C_t is now the composite consumption

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{6.1}$$

and $c_t(i)$ is consumption of good i. Households minimize expenditure $\int_0^1 p_t(i) c_t(i) di$, where $p_t(i)$ is the price of good i in units of money, to obtain a given level of the composite good C_t, (6.1). The resulting demand function for each good i is given by

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t, \tag{6.2}$$

where P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{6.3}$$

The households' intertemporal and intratemporal conditions are, as before, (3.5) and (3.6). (3.8) must also hold.

The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0. \tag{6.4}$$

Given the prices on each good i, $p_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta}. \tag{6.5}$$

Market clearing for each good implies

$$c_t(i) + g_t(i) = A_t n_t(i), \tag{6.6}$$

while in the labor market it must be that, in equilibrium,

$$\int_0^1 n_t(i)di = N_t. \tag{6.7}$$

Using (6.2), (6.5), (6.6) and (6.7), we can write the resource constraints as

$$(C_t + G_t) \int_0^1 \left(\frac{p_t\,(i)}{P_t} \right)^{-\theta} di = A_t N_t. \tag{6.8}$$

We consider now that firms set prices in advance. A fraction α_j firms set prices j periods in advance with $j = 0, ...J - 1$. Firms decide the price for period t with the information up to period $t - j$ to maximize profits[9]:

$$E_{t-j} \left[Q_{t-j,t+1} \left(p_t(i) y_t(i) - W_t n_t(i) \right) \right],$$

subject to the production function

$$y_t(i) \leq A_t n_t(i)$$

and the demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{6.9}$$

where $y_t(i) = c_t(i) + g_t(i)$ and $Y_t = C_t + G_t$.

The optimal price for a firm that is setting the price for period t, j periods in advance, is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j} \left[\eta_{t,j} \frac{W_t}{A_t} \right], \tag{6.10}$$

[9]Profits at t are priced by $Q_{t-j,t+1}$ because of the timing of transactions where profits are received at the end of the period to be used for consumption the period after.

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1} P_t^{\theta} Y_t}{E_{t-j}\left[Q_{t-j,t+1} P_t^{\theta} Y_t\right]}.$$

From (6.3), the price level at date t can be written as

$$P_t = \left[\sum_{j=0}^{J-1} \alpha_j \left(p_{t,j}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{6.11}$$

When we compare the two sets of equilibrium conditions, under flexible and prices set in advance, here we are adding more variables, the prices of the differently restricted firms, but we also add the same number of equations. To show that the same arguments in the previous section also work here, it is useful to rewrite the equilibrium conditions.

Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions (6.10), and using the intertemporal condition (3.6) as well as the households' intratemporal condition (3.5), we obtain the intratemporal conditions

$$E_{t-j}\left[\frac{u_C(t)}{R_t} P_t^{\theta-1} A_t \left(1 - L_t\right) - \frac{\theta}{(\theta-1)} u_L(t) P_t^{\theta-1} \left(1 - L_t\right) \frac{P_t}{p_{t,j}}\right] = 0,\ j = 0, ...J-1. \tag{6.12}$$

Notice that, if $J = 1$, meaning that there are only flexible price firms, $p_{t,0} = P_t$ and we would get the intratemporal condition obtained under flexible prices,

$$\frac{u_C(t)}{u_L(t)} = \frac{\theta R_t}{(\theta-1) A_t}, \tag{6.13}$$

corresponding to (3.13), for the case where $\theta \to \infty$.

The resource constraints can be written as

$$\left(C_t + G_t\right) \sum_{j=0}^{J-1} \alpha_j \left(\frac{p_{t,j}}{P_t}\right)^{-\theta} = A_t N_t. \tag{6.14}$$

The proposition follows:

Proposition 6.1. *When prices are set J periods in advance, if policy is conducted with the interest rate feedback rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium.

Proof: When policy is conducted with the interest rate feedback rule $R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$, then the intertemporal condition (3.6) implies

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0 \tag{6.15}$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0 \tag{6.16}$$

These conditions together with the resource constraints (6.14), the intratemporal conditions (6.12), the conditions on the price level, (6.11), and the cash in advance constraints, (3.3), with equality, determine uniquely all the variables C_t, L_t, P_t, $p_{t,j}$, $j = 0, ...J-1$, and M_t. $p_{s,j}$, $j = 1, ...J-1$, $s < j$, are exogenous.

The budget constraints (3.9) are satisfied for multiple paths of the taxes and state noncontingent debt levels.■

We have shown that the results extend to environments with sticky prices, in particular when prices are set in advance in a staggered fashion.

7. Concluding Remarks

The problem of multiplicity of equilibria under interest rate policy has been addressed, after Sargent and Wallace (1975) and McCallum (1981), by an extensive literature on local determinacy under interest rate rules. Interest rate feedback rules on endogenous variables such as the inflation rate can, with appropriately chosen coefficients, deliver locally determinate equilibria, i.e. unique local equilibria in the neighborhood of a steady state. There are still multiple solutions to the system of linear difference equations that approximates the model. Those additional solutions suggest other equilibria that can be analyzed in the nonlinear model. Indeed, it is a well known result that there are multiple equilibria when policy is conducted with an interest rate rule.

In this paper we show that there are interest rate feedback rules that implement unique equilibria. This result does not depend on preferences or other similar characteristics of the environment. It is also robust to the consideration of nominal rigidities. The way this rule works in pinning down unique equilibria is by eliminating expectations of future variables from the dynamic equations.

The feedback rules that we propose can be used to pin down the welfare maximizing equilibria, but the policy maker can also implement other, less desirable, equilibria.

An important assumption is that fiscal policy is Ricardian, in the sense that taxes can be used as a residual variable to satisfy the budget constraint of the government. As is standard in the literature we also assume that the nominal interest rate must be nonnegative in equilibrium but is unrestricted out of equilibrium, as in, for example, Bassetto (2004) and Schmitt-Grohe and Uribe (2001). Benhabib, Schmitt-Grohe and Uribe (2001b) assume that the zero bound restriction applies not only in equilibrium but also to the government actions out of equilibrium. Under this alternative approach there would also be multiple equilibria in our set up. The alternative assumptions cannot be assessed empirically. In a more deeply founded model, Bassetto (2004) shows that the zero bound restriction should only be satisfied in equilibrium[10].

[10] There is a resemblance between this issue and the controversy on Ricardian versus non-Ricardian policies in the fiscal theory of the price level. Ricardian policies are policies such that the budget constraint of the government holds also for prices, that are not necessarily equilibrium prices, while non-Ricardian policies satisfy the budget constraint only for the equilibrium prices. In Bassetto (2004) while the budget constraint must hold also out of equilibrium, the zero bound restriction only holds in equilibrium.

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