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2008/9

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An estimated DSGE model of the Hungarian economy

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December 2008

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MNB Working Papers 2008/9

An estimated DSGE model of the Hungarian economy

(A magyar gazdaság egy becsült, sztochasztikus, dinamikus általános egyensúlyi modellje)

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Published by the Magyar Nemzeti Bank

Szabadság tér 8–9, H–1850 Budapest

http://www.mnb.hu

ISSN 1585 5600 (online)

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Contents

Abstract

This paper presents and estimates a dynamic stochastic general equilibrium (DSGE) small-open-economy model for the Hungarian economy. The model features different types of frictions, real and nominal rigidities which are necessary to replicate the empirical persistence of Hungarian data.

Bayesian methods are applied, and the structural break due to changing monetary regime over the studied period is explicitly taken into account in the estimation procedure. A real-time adaptive learning mechanism describes agents' perception on underlying inflation. This creates an additional inertia in inflation.

We describe the properties of the estimated model by impulse-response analysis, variance decomposition and the analysis of identified structural shocks. Our results are compared with that of estimated euro-area DSGE models, and estimated non-DSGE models of the Hungarian economy. As a robustness check, a model without real time adaptive learning is also estimated and it's results are also compared to those of the original model.

JEL Classification: E40, E50.

Keywords: New Keynesian models, DSGE models, small open economy, Bayesian econometrics.

Összefoglalás

Tanulmányunkban egy a magyar, kis és nyitott gazdaságot leíró dinamikus, sztochasztikus általános egyensúlyi (DSGE) modellt mutatunk be. A modellbe olyan reál- és nominális ragadósságokat építettünk be, amelyek szükségesek a magyar adatokban meglévő perzisztenciák megfelelő leírásához.

A modell Bayes-i becslése során figyelembe vettük a monetáris rezsim 2001-ben történt megváltozása miatti strukturális törést. A gazdasági szereplők által ”érzékelt” alapinflációt egy adaptív tanulási folyamat írja le, ami az infláció inerciáját tovább növeli.

A modell dinamikus tulajdonságait az impulzus válasz függvények, a variancia dekompozíció és a strukturális sokkok elemzésével szemléltetjük. Eredményeinket az eurózónára becsült DSGE modellekkel illetve a magyar gazdaságot leíró nem-DSGE modellekkel is összevetjük. Ezen túlmenően, az eredmények robusztusságát egy tanulási folyamatot nem tartalmazó modell becslésének segítségével ellenőrizzük.

1 Introduction

This paper presents and estimates a two-sector dynamic stochastic general equilibrium (DSGE) small-open-economy model for the Hungarian economy. The two sectors produce domestic and exported final goods. Following Christiano *et al.* (2005) and Smets & Wouters (2003), our model features different types of frictions, real and nominal rigidities which are necessary to replicate the empirical persistence of Hungarian data. The model incorporates external habit formation in consumption, Calvo-type price and wage rigidity complemented with indexation to past prices and wages, adjustment costs of investments, adjustment cost of capital, labor and import utilization and fixed cost in production. The model also contains liquidity-constrained rule-of-thumb consumers introduced by Galí *et al.* (2007). We follow the approach of McCallum & Nelson (2001) which considers imports as production input.

In the past years a disinflation process occured in Hungary. In such an environment one cannot disregard how inflationary expectations were formed. Thus, as a special feature we incorporated an inertia in agents' perception on *underlying* inflation. It is assumed that price and wage setters index their prices to lagged cyclical component of inflation. Moreover, all agents always use an indexation to a measure of past inflation. This measure of past inflation is, in contrast to other models with inflation ineria, not observed aggregate inflation, but a measure of their inflationary perception. We describe the evolution of this perception of *underlying* inflation rate by a real-time adaptive-learning algorithm.

As now becomes standard, the model is estimated by Bayesian method described in An & Schorfheide (2005). The method based on maximization of the likelihood function, derived from the rational-expectations solution by the Kalman-filter, combined with prior distributions. To characterize the posterior density function of the estimted parameters the random-walk Metropolis-Hastings (MH) algorithm is applied.

There were two different monetary regimes in Hungary over the estimation sample: between 1995 and 2001 a crawling-peg regime, and since 2001 an inflation-targeting regime. This structural break is explicitly taken into account in the estimation procedure: we formulate two slightly different models for the two subperiods.

The main results are the following. The estimated values of the Calvo parameters for consumer price setting are close to the ones usually estimated for euro-zone. On the other hand, the Calvo coefficients for wage setting are generally estimated lower than euro-area estimates. Unlike Calvo coefficients, the monetary regime shift is mostly felt in the indexation of consumer prices as it is estimated to be significantly lower in the inflation targeting regime.In contrast, wage indexation parameters are estimated to be stable across the two regimes and generally lower than in the eurozone. Adjustment cost of investment is found to be high compared to other DSGE models.

The estimated value of the interest-rate smoothing parameter is significantly lower than various euro-area and US estimates. It is important to note that this result is not driven by the choice of the accompanying prior distribution. A relatively uninformative Uniform prior is imposed on this parameter.

Estimated impulse-response functions replicate qualitatively the behavior of other New Keynesian models quite well. The main difference is that investments react much less to most shocks than it is common in similar models. The model features a hump-shaped effect on both output and inflation to a monetary tightening. A positive productivity shock results in increasing output and production, but decreasing inflation and employment as documented in Galí (1999). The response of cyclical inflation and wages are less persistent than those adjusted for agents' perception on *underlying* inflation (the response of original price and wage inflation).

The crowding-out effect of a government-consumption shock is also observable, however, due to the presence of rule-of-thumb consumers the estimated model is able to replicate the co-movement of government and private consumption. It is important to note, that in general short term reactions are highly affected by the presence of non-optimizing consumers. The relatively high adjustment cost of investment implies a generally smoother reaction of investments to shocks.

We compared our estimated impulse responses to an interest-rate shock with that of other estimated non-DSGE models of Hungarian economy, presented in Jakab *et al.* (2006). The models are quite similar in replicating the short run adjustment of the real exchange rate. On the other hand, consumer prices are found to be less responsive in the longer run than in other models. On the other hand, the reaction of consumption, investments and net exports is different in our DSGE model than in the others. This results is due to the different structure of the models.

Variance decomposition reveals that both cyclical and permanent (*'perceived underlying'*) component of inflation can be explained by productivity, investment, consumer preference and markup shocks. That is, the endogenous learning process in this DSGE model was able to capture longer term inflation movements without introducing an additional exogenous shock.

To show this, an alternative model without endogenous real time adaptive learning of *'underlying inflation'* is also estimated. We conclude that the inclusion of adaptive learning is not really responsible for creating an *'intrinsic'* inertia in inflation. However, according to variance decomposition long-term movements of inflation are mostly captured by the shock to '*underlying'* inflation'. This reveals that the approach of the baseline model was necessary to explain long term disinflation in Hungary.

In the long run real variables are heavily influenced by both the external demand and the productivity shock. These shocks are the prime source of output fluctuation in a small, open economy like Hungary. Real effect of the financial premium and the monetary-policy shock is found negligible, which is in sharp contrast with the finding of Smets & Wouters (2003). This finding is, however, in accordance with Vonnák (2007) and Jakab *et al.* (2006) that monetary policy has a rather limited effect on output in Hungary.

This model may serve as a basis for policy simulations at the central bank of Hungary (Magyar Nemzeti Bank). Natural directions would be the refinement of labor market (by inserting search and marching frictions as in Jakab & Kónya (2007)) or the research on optimal monetary policy rules in a more detailed manner.

The paper is structured as follows. *Section 2* presents the estimated DSGE model. *Section 3* describes the applied estimation method, and presents estimation results. In *Section 4* the evolution of shocks is described. In *Section 5* and *6* we analyze impulse responses and forecast error variance decomposition of the model. In *Section 7* we present results from an alternative model where the real time adaptive learning mechanism on the *underlying* inflation was switched off. *Section 8* concludes.

2 The model

Our model is a simple open-economy extension of the DSGE model presented in Smets & Wouters (2003). To open their model it assumed that beyond labor and a capital an additional imported input is needed for domestic production. On the other hand, some part of domestic production is exported.

A further complication in our model, missing from that of Smets & Wouters (2003), is the presence of non-Ricardian rule-of-thumb consumers, as in Galí *et al.* (2007), in order to replicate the empirical co-movement of private and government consumption. There are two types of rule-of-thumb consumers in the model. The first type spends her entire labor income for consumption. However, since labor hours are relatively volatile it induce two much consumption fluctuation. To mitigate the volatility of aggregate consumption input adjustments cost is introduced. Furthermore, the income of the second type of rule-of-thumb consumer, representing pensioners, is independent of labor-hour movements, which decreases further consumption volatility. The assumption of input adjustment cost also helps to replicate the empirical behavior of imports. To simplify wage setting mechanism in the model it is assumed that both Ricardian and active rule-of-thumb consumers belong to unions, which set wages such a way that maximizes the weighted average of welfare functions of the two different types.

Domestic and exported final goods are produced in two different sectors. The structure of the sectors are identical, however the input requirement of production in the exports sector is higher. This assumption is necessary two reproduce the empirical co-movement of exports and imports. Price formation mechanisms are similar in both sectors, they are captured by the sticky-price model of Calvo, however the exporters set their prices in foreign currency.

2.1 PRODUCTION

Production has a hierarchical structure: at the first stage labor and imported inputs are transformed into an intermediate input in a perfectly competitive industry. At the second stage the intermediate input and capital are used to produce differentiated goods in a monopolistically competitive industry. Finally, a homogenous final good is produced by the differentiated goods in a perfectly competitive environment. There two sectors in the economy: a domestic production sector and exports sector, labeled by d and x, respectively.

Final good y_t^s in sector s $(s = d, x)$ is produced in a competitive market by a constant-returns-to-scale technology from a continuum of differentiated intermediate goods $y_t^s(i)$, $i \in [0,1]$. The technology is represented by the following CES production function:

$$y_t^s = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $\theta > 1$ measures the degree of the elasticity of substitution. As a consequence, the price index P_t^s is given by

$$P_t^s = \left(\int_0^1 P_t^s(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}, \tag{2}$$

where $P_t^s(i)$ denotes the prices of differentiated goods $y_t^s(i)$, and the demand for $y_t^s(i)$ is determined by

$$y_t^s(i) = \left(\frac{P_t^s}{P_t^s(i)} \right)^{\theta} y_t^s. \tag{3}$$

2.1.1 Cost minimization

The continuum of goods $y_t^s(i)$ is produced in a monopolistically competitive market. Each $y_t^s(i)$ is made by an individual firm, and they apply the same CES technology. Firm i uses technology

$$y_t^s(i) = A_t \left(\bar{\alpha}_s^{\frac{1}{\varrho}} \bar{k}_t^s(i)^{\frac{\varrho-1}{\varrho}} + (1-\bar{\alpha}_s)^{\frac{1}{\varrho}} z_t^s(i)^{\frac{\varrho-1}{\varrho}} \right)^{\frac{\varrho}{\varrho-1}} - \bar{f}_s, \tag{4}$$

where $\bar{k}_t^s(i)$ is the firm's effective utilization of physical capital, $\bar{k}_t^s(i) = u_t k_t^s(i)$, where u_t is the degree of capital utilization explained in detail in the next section, $k_t^s(i)$ the firm's utilization of the homogenous capital good. $z_t^s(i)$ is the firm's utilization of a composite intermediate input good z_t^s. Variable A_t is a uniform exogenous productivity factor and $\bar{f}_s$ is uniform real fixed cost of the industry. The parameter $0 < \varrho$ measures the elasticity of substitution between $\bar{k}_t^s$ and z_t^s and $0 < \bar{\alpha}_s < 1$. Good $z_t^s(i)$ is composed by composite labor and imported inputs,

Solution of firms' cost minimization problem implies that their marginal cost is

$$MC_t^s = A_t^{-1} \left[\bar{\alpha}_s \left(R_t^k \right)^{1-\varrho} + (1-\bar{\alpha}_s) \left(W_t^{zs} \right)^{1-\varrho} \right]^{\frac{1}{1-\varrho}}, \tag{5}$$

where R_t^k is the rental rate of capital and W_t^{zs} is the price of z_t^s. Solution of cost minimization also provides demand for inputs, represented by

$$\bar{k}_t^s(i) = \bar{\alpha}_s \left(\frac{mc_t^s}{r_t^k} \right)^{\varrho} \frac{y_t^s(i) + \bar{f}_s}{A_t^{1-\varrho}},$$

$$z_t^s(i) = (1-\bar{\alpha}_s) \left(\frac{mc_t^s}{w_t^z} \right)^{\varrho} \frac{y_t^s(i) + \bar{f}_s}{A_t^{1-\varrho}},$$

where $mc_t^s = MC_t^s / P_t$. Let us defined the following sectoral aggregate variables.

$$k_t^s = \int_0^1 k_t^s(i) di, \qquad z_t^s = \int_0^1 z_t^s(i) di.$$

Aggregating individual demand functions and using equation (3) result in

$$u_t k_t^s = \bar{\alpha}_s \left(\frac{mc_t^s}{r_t^k} \right)^{\varrho} \frac{y_t^s DP_t^s + \bar{f}_s}{A_t^{1-\varrho}}, \tag{6}$$

$$z_t^s = (1-\bar{\alpha}_s) \left(\frac{mc_t^s}{w_t^z} \right)^{\varrho} \frac{y_t^s DP_t^s + \bar{f}_s}{A_t^{1-\varrho}}, \tag{7}$$

where variable DP_t^s represents price dispersion,

$$DP_t^s = \int_0^1 \left(\frac{P_t}{P_t(i)} \right)^{\theta} di.$$

The composite intermediate input is produced in a competitive industry by the following CES technology,

$$z_t^s = \left(\bar{a}_s^{\frac{1}{\varrho_z}} \left(m_t^s \right)^{\frac{\varrho_z-1}{\varrho_z}} + (1-\bar{a}_s)^{\frac{1}{\varrho_z}} \left(l_t^s \right)^{\frac{\varrho_z-1}{\varrho_z}} \right)^{\frac{\varrho_z}{\varrho_z-1}} - z_t^s \Phi_z \left(z_t^s \right). \tag{8}$$

where l_t^s is labor and m_t^s is the imported input good m_t^s. Furthermore $0 < \varrho_z$ and $0 < \bar{a}_s < 1$, and the adjustment cost function

$$\Phi_{zs}\left(z_t^s\right) = \frac{\phi_z}{2z^s}\left(\frac{z_t^s}{z^s} - 1\right)^2, \quad \phi_z > 0.$$

Properties of this function are $\Phi'_{zs} > 0$, $\Phi_{zs}(z^s) = \Phi'_{zs}(z^s) = 0$. z^s is the steady state level of the composite input. The price of composite input W_t^{zs} is equal to the marginal cost of its production. In *Appendix A.1* it is shown that marginal cost is given by,

$$W_t^{zs} = \left[\bar{a}_s W_t^{1-\varrho_z} + (1-\bar{a}_s)\left(e_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}} \left[1 + \Phi_{zs}\left(z_t^s\right) + z_t^s \Phi'_{zs}\left(z_t^s\right)\right], \tag{9}$$

where W_t is the nominal wage, P_t^{m*} is the foreign-currency price of imported inputs and e_t is the nominal exchange rate. Furthermore, demand for production inputs are given by the following equations,

$$l_t^s = \bar{a}_s\left(\frac{\bar{w}_t^{zs}}{w_t}\right)^{\varrho_z} z_t^s\left[1 + \Phi_{zs}\left(z_t^s\right)\right], \tag{10}$$

$$m_t^s = (1-\bar{a}_s)\left(\frac{\bar{w}_t^{zs}}{q_t P_t^{m*}}\right)^{\varrho_z} z_t^s\left[1 + \Phi_{zs}\left(z_t^s\right)\right], \tag{11}$$

where

$$\bar{w}_t^{zs} = \left[\bar{a}_s w_t^{1-\varrho_z} + (1-\bar{a}_s)\left(q_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}},$$

and $w_t = W_t/P_t$ is the real wage, $q_t = e_t/P_t$ is the domestic component of the real exchange rate.

2.1.2 Price setting

Let us consider how firms in the domestic production sector set their prices. To simplify notation we drop index d of the sectoral price index. It is assumed that prices are sticky: as in the model of Calvo (1983), each intermediate good producer at a given date changes its price in a rational, optimizing, forward-looking way with a constant probability of $1-\gamma_d$. Those firms which do not optimize at the given date follow a rule of thumb. Rule of thumb price setters increase their prices by the expected *underlying* rate of inflation, as in Yun (1996), and to some extent by the difference between the past actual and perceived *underlying* inflation rates, similarly to Christiano *et al.* (2005) and Smets & Wouters (2003). Formally, if firm i does not optimize at date t

$$P_t(i) = P_{t-1}(i)\Pi_{t-1}^I = P_{t-1}(i)\left(\frac{\Pi_{t-1}}{\bar{\Pi}_{t-1}}\right)^{\vartheta_d}\bar{\Pi}_t$$

where $\Pi_{t-1} = P_{t-1}/P_{t-2}$, $\bar{\Pi}_t$ is the perceived *underlying* inflation ϑ_d measures the degree of indexation according to past inflation. The above formula implies if a given firm does not optimize between $t+1$ and T its price at date T is given by

$$P_T(i) = P_t(i)\Pi_{T,t}^I = P_t(i)\Pi_T^I\Pi_{T-1}^I\cdots\Pi_t^I. \tag{12}$$

If $P_t(i)$ is the chosen price of a firm at date t, then its profit will be at T

$$\begin{aligned} V_T(P_t(i)) &= y_T^d(i)\left(\tau_t^d P_T(i) - MC_T^d\right) - \bar{f}_d. \\ &= y_T^d P_T^\theta\left[\left(P_t(i)\Pi_{T,t}^I\right)^{1-\theta} - \left(P_t(i)\Pi_{T,t}^I\right)^{-\theta} MC_T^d\right] - \bar{f}_d, \end{aligned}$$

where the second equation is a consequence of formulas (3) and (12). If firm i sets its price optimally at date t it solves the following maximization problem.

$$\max_{P_t(i)} = \mathrm{E}_t \left[\sum_{T=t}^{\infty} (\gamma_d)^{T-t} D_{T,t} V_T(P_t(i)) \right],$$

where $D_{T,t}$ is the stochastic discount factor,

$$D_{T,t} = \beta^{T-t} \frac{\Lambda_T^o / P_T}{\Lambda_t^o / P_t},$$

and Λ_t^o is the marginal utility of consumption of optimizing consumers, who owns the firms. It is explained in detail in the next section.

The first order condition is

$$\sum_{T=t}^{\infty} \gamma_d^{T-t} \mathrm{E}_t \left[D_{T,t} y_t^d \left(\frac{P_T}{P_t(i) \Pi_{T,t}^I} \right)^{\theta} \left(\tau_T^d \Pi_{T,t}^I - \frac{\theta}{\theta - 1} \frac{P_T mc_T^d}{P_t(i)} \right) \right] = 0,$$

where $mc_t^d = MC_T^d / P_T$ is the real marginal cost. Rearranging it yields

$$\frac{P_t(i)}{P_t} \sum_{T=t}^{\infty} (\gamma_d \beta)^{T-t} \mathrm{E}_t \left[\Lambda_T^o y_t^d \tau_t^d \left(\frac{P_T}{P_t \Pi_{T,t}^I} \right)^{\theta - 1} \right] =$$

$$\frac{\theta}{\theta - 1} \sum_{T=t}^{\infty} (\gamma_d \beta)^{T-t} \mathrm{E}_t \left[\Lambda_T^o y_t^d \left(\frac{P_T}{P_t \Pi_{T,t}^I} \right)^{\theta} mc_T^d \right]. \tag{13}$$

Equation (13) implies that all firms choose the same $P_t(i)$. Let us denote this uniform price by $P_t^\star$. Define $\mathscr{P}_t = P_t^\star / P_t$. In *Appendix A.1* it is shown that condition (13) can be expressed in a recursive form,

$$\mathscr{P}_t = \frac{\theta}{\theta - 1} \frac{\mathscr{Z}_t^1}{\mathscr{Z}_t^2}, \tag{14}$$

where

$$\mathscr{Z}_t^1 = \Lambda_t^o y_t^d mc_t^d + \beta \gamma_d \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^I} \right)^{\theta} \mathscr{Z}_{t+1}^1 \right], \tag{15}$$

$$\mathscr{Z}_t^2 = \tau_t^d \Lambda_t^o y_t^d + \beta \gamma_d \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^I} \right)^{\theta - 1} \mathscr{Z}_{t+1}^2 \right]. \tag{16}$$

Equation (1) and the price-setting assumptions imply that the evolution aggregate price index is given by

$$P_t^{1-\theta} = (1 - \gamma_d) \left(P_t^\star \right)^{1-\theta} + \gamma_d \left(P_{t-1} \Pi_{t-1}^I \right)^{1-\theta},$$

rearranging it yields

$$\mathscr{P}_t^{1-\theta} = \frac{1 - \gamma_d \left(\frac{\Pi_t}{\Pi_{t-1}^I} \right)^{\theta - 1}}{1 - \gamma_d}. \tag{17}$$

In export sector price setting is similar to that of domestic production sector, however prices are set in foreign currency That is, firms set $P_t^{x*} = P_t^x / e_t$, where e_t is the nominal exchange rate.

The price indexation scheme of the sector is

$$P_T^{x*}(i) = P_t^{x*}(i)\Pi_{T,t}^{I} = P_t(i)\Pi_T^{Ix}\Pi_{T-1}^{Ix}\cdots\Pi_t^{Ix},$$

where

$$\Pi_t^{Ix} = \left(\frac{\Pi_t^{x*}}{\bar{\Pi}_t^{x*}}\right)^{\vartheta_x}\bar{\Pi}_{t+1}^{x*}$$

and $\Pi_t^{x*} = P_t^{x*}/P_{t-1}^{x*}$, $\bar{\Pi}_t^{x*}$ is the perceived *underlying* inflation, ϑ_x represents the degree of indexation according to past inflation.

If $P_t^{x*}(i)$ is the chosen price of a firm at date t, then its profit will be at date T

$$\begin{aligned} V_T^x(P_t^{x*}(i)) &= y_T^d(i)\left(\tau_t^x e_T P_T^{x*}(i) - MC_T^x\right) - \bar{f}_x. \\ &= y_T^x P_T^{\theta}\left[\left(e_T P_t^{x*}(i)\Pi_{T,t}^{Ix}\right)^{1-\theta} - \left(e_T P_t^{x*}(i)\Pi_{T,t}^{Ix}\right)^{-\theta} MC_T^x\right] - \bar{f}_x. \end{aligned}$$

The maximization problem of firm i is

$$\max_{P_t^{x*}(i)} = \mathrm{E}_t\left[\sum_{T=t}^{\infty}(\gamma_x)^{T-t}D_{T,t}V_T^x(P_t^{x*}(i))\right].$$

The first order condition is

$$\sum_{T=t}^{\infty}\gamma_x^{T-t}\mathrm{E}_t\left[D_{T,t}y_t^x\left(\frac{P_T^{x*}}{P_t^{x*}(i)\Pi_{T,t}^{Ix}}\right)^{\theta}\left(\tau_T^x\Pi_{T,t}^{I*} - \frac{\theta}{\theta-1}\frac{e_T P_T^{x*} mc_T^{x*}}{P_t^{x*}(i)}\right)\right] = 0,$$

where

$$mc_t^{x*} = \frac{MC_T^x}{e_T P_T^{x*}} = \frac{P_T mc_T^x}{e_T P_T^{x*}}. \tag{18}$$

As in the previous case, all firms chooses the same $P_t^{x*}(i)/P_t^{x*}$. Its common value is denoted by $\mathscr{P}_t^x$. In *Appendix A.1* it is shown that the above first-order condition can be expressed recursively as,

$$\mathscr{P}_t^x = \frac{\theta}{\theta-1}\frac{\mathscr{Z}_t^{x1}}{\mathscr{Z}_t^{x2}}, \tag{19}$$

where

$$\mathscr{Z}_t^{x1} = \frac{P_t^{x*}}{q_t}\Lambda_t^o y_t^x mc_t^{x*} + \beta\gamma_x\mathrm{E}_t\left[\left(\frac{\Pi_{t+1}^{x*}}{\Pi_t^{Ix}}\right)^{\theta}\mathscr{Z}_{t+1}^{x1}\right], \tag{20}$$

$$\mathscr{Z}_t^{x2} = \tau_t^x\frac{P_t^{x*}}{q_t}\Lambda_t^o y_t^x + \beta\gamma_x\mathrm{E}_t\left[\left(\frac{\Pi_{t+1}^{x*}}{\Pi_t^{Ix}}\right)^{\theta-1}\mathscr{Z}_{t+1}^{x2}\right]. \tag{21}$$

As above, it is possible to show that

$$\left(\mathscr{P}_t^x\right)^{1-\theta} = \frac{1-\gamma_x\left(\frac{\Pi_t^{x*}}{\Pi_{t-1}^{Ix}}\right)^{\theta-1}}{1-\gamma_x}. \tag{22}$$

2.2 HOUSEHOLDS

2.2.1 Optimizing households

The domestic economy is populated by a continuum of infinitely-lived households. Fraction $\bar{\omega}^o$ of households choose their consumption stream in the standard rational optimizing manner. These optimizing households have labor and capital income and they own domestic firms. The expected utility function of optimizing household j is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\eta_t^c \left\{ u(H_t^o(j)) - \eta_t^l v(l_t(j)) \right\} \right], \tag{23}$$

for all $j \in [0,1]$. $H_t^o(j) = c_t^o(j) - h c_{t-1}^o$, where $c_t^o(j)$ denotes the consumption of household j at date t, c_{t-1}^o is aggregate consumption of optimizers at date $t-1$, parameter $h \in [0,1]$ measures the strength of external habit formation, $l_t(j)$ is the labor supply of household j, η_t^c and η_t^l are preference shocks. Furthermore, $u(H) = H^{1-\sigma}/(1-\sigma)$, and $v(l) = l^{1+\varphi}/(1+\varphi)$, $\sigma, \varphi > 0$.

The intertemporal budget constraint of a given household can be written in the form

$$\begin{aligned} & P_t c_t^o(j) + P_t I_t(j) + \frac{B_t(j)}{1+i_t} = \\ & B_{t-1}(j) + X_t^w(j) + W_t(j) l_t(j) + P_t r_t^k u_t(j) k_t(j) - \Psi(u_t(j)) P_t k_t(j) + Div_t - T_t^o, \end{aligned} \tag{24}$$

where P_t is the consumer price index, $B_t(j)$ is the household's holding of riskless nominal bonds at the beginning of time t, i_t is the corresponding one-period nominal interest rate, Div_t denotes dividends derived form firms. It is assumed that dividends are equally distributed among firms. $k_t(j)$ is the stock of physical capital supplied by the household, $u_t(j)$ is the utilization rate of capital ($\bar{k}_t = u_t k_t$). T_t^o denotes the lump-sum tax levied on optimizing households. Ψ is the cost of the capital utilization rate, it is assumed that

$$\Psi(u_t) = r^k \psi \left[\exp\left(\frac{u_t - 1}{\psi} \right) - 1 \right].$$

This implies that at the steady state ($u = 1$) $\Psi(1) = 0$, Ψ'^k, and $\psi = \Psi'(1)/\Psi''(1)$. $I_t(j)$ denotes investments in physical capital. $W_t(j)$ is the nominal wage paid to household j. Households supply differentiated labor, hence the wage paid to individual households can be different. On the other hand, X_t^w is a state-contingent security which eliminates the risk of heterogeneous labor supply and labor income. Physical capital accumulation is described by

$$k_{t+1}(j) = (1-\delta) k_t(j) + \left[1 - \Phi_I \left(\frac{\eta_t^I I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j), \tag{25}$$

where function Φ_I represents investments adjustment costs, and η_t^I is an exogenous shock. It is assumed that

$$\Phi_I \left(\frac{\eta_t^I I_t(j)}{I_{t-1}(j)} \right) = \frac{\phi_I}{2} \left(\frac{\eta_t^I I_t(j)}{I_{t-1}(j)} - 1 \right)^2, \qquad \phi_I > 0.$$

This implies that $\Phi_I' > 0$, and in the steady state $\Phi_I'(1) = \Phi_I'(1) = 0$.

An optimizing household chooses the trajectory of its consumption, bond-holding, investments, physical capital and capital utilization. It is assumed that a certain household supplying type j of labor belongs to a trade-union representing the interest of optimizing and non-optimizing households supplying type j of labor. The union determines the labor supply and the nominal wage of its members, all members accept its decision.

The formal optimization problem of the households is the following: they maximize the objective function (23) subject to the budget constraint (24), the investments equation (25), non-negativity constraints on consumption and investments, and no-Ponzi schemes. The characterization of the solution of the above optimization problem can be found in *Appendix A.2*.

Due to the existence of asset X_t^w the wage incomes of all households are the same. As a consequence, all households choose the same consumption allocation. We, therefore, drop index j from subsequent notations.

The path of consumption is determined by the following Euler equation.

$$\frac{\Lambda_t^o}{P_t} = \beta(1+i_t)\mathrm{E}_t\left[\frac{\Lambda_{t+1}^o}{P_{t+1}}\right], \tag{26}$$

where Λ_t^o is the marginal utility of consumption,

$$\Lambda_t^o = \eta_t^c(c_t^o - hc_{t-1}^o)^{-\sigma}. \tag{27}$$

The trajectory of investments is described by equation

$$\begin{aligned} Q_t\left[1-\Phi_I\left(\frac{\eta_t^I I_t}{I_{t-1}}\right)-\Phi_I'\left(\frac{\eta_t^I I_t}{I_{t-1}}\right)\frac{\eta_t^I I_t}{I_{t-1}}\right] &= \\ 1-\beta\mathrm{E}_t\left[D_{t+1,1}Q_{t+1}\Phi_I'\left(\frac{\eta_{t+1}^I I_{t+1}}{I_t}\right)\frac{\eta_{t+1}^I I_{t+1}^2}{I_t^2}\right]&, \end{aligned} \tag{28}$$

where Q_t is the shadow price of capital. The portfolio choice between bond and physical capital is given by

$$Q_t = \mathrm{E}_t\left[D_{t+1,1}\left(Q_{t+1}(1-\delta)+u_{t+1}r_{t+1}^k-\Psi(u_{t+1})\right)\right]. \tag{29}$$

Finally, the following condition describes the choice of capital utilization.

$$r_t^k = \Psi'(u_t(j)). \tag{30}$$

2.2.2 Non-optimizing households

Fraction $\bar{\omega}^{no}$ of households are liquidity constrained. Their consumption follows a simple rule of thumb.

$$P_t c_t^{no}(j) = X_t^w(j) + W_t(j)l_t(j).$$

Due to the existence of asset X_t^w their wage income and consumption are uniform. As a consequence,

$$P_t c_t^{no} = W_t l_t. \tag{31}$$

Fraction $\bar{\omega}^p$ of households are pensioners. We assume that they also consume their total income adjusted by the "Swiss indexation formula". That is, it is assumed that their consumption proportional to the average of price and wage level,

$$P_t c_t^p = c^p w^{-\frac{1}{2}}\left(W_t P_t\right)^{\frac{1}{2}}, \tag{32}$$

where c^p and w are the steady-state values of pensioners' consumption and real wages, respectively.

2.2.3 Wage setting

There is monopolistic competition in the labor market, different types of labor are supplied by households. Wages is set by which representing the interest of households active on the labor market, that is, that of optimizers and non-optimizers. Union j sets $W_t(j)$, the nominal wage level belonging to type j of labor. The composite labor good of the economy is a CES aggregate of different types of labor,

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\theta_w - 1}{\theta_w}} \, dj \right)^{\frac{\theta^w}{\theta^w - 1}},$$

where $\theta^w > 1$ is elasticity of substitution between different types of labor. This implies that the demand for labor supplied by household j is given by

$$l_t(j) = \left(\frac{W_t}{W_t(j)} \right)^{\theta_w} l_t, \tag{33}$$

where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta_w} \, dj \right)^{\frac{1}{1-\theta_w}}.$$

It is assumed that there is sticky wage setting in the model, as in the paper of Erceg *et al.* (2000). Similarly to Calvo (1983), every union at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1 - \gamma_w$. All those unions, which do not optimize at the given date follow a rule of thumb similar to that of producers. Using the notation introduced in the previous subsection, the price setting scheme of the non-optimizers is described by formula

$$W_T(i) = W_t(i) \Pi^{Iw}_{T,t} = P_t(i) \Pi^{Iw}_T \Pi^{Iw}_{T-1} \cdots \Pi^{Iw}_t,$$

where

$$\Pi^{Iw}_t = \left(\frac{\Pi^w_t}{\bar{\Pi}_t} \right)^{\vartheta_w} \bar{\Pi}_{t+1}$$

and $\Pi^w_t = W_t / W_{t-1}$, ϑ_w represents the degree of indexation according to past inflation.

If a union chooses it wage optimally at date t it has to take into account that it will follow the rule of thumb at $t+1$ with a probability of γ_w, at $t+2$ with γ_w^2, and so on. Hence it has to weight the objective function with the above sequence of probabilities. That is, it maximizes formula

$$\sum_{t=T}^{\infty} (\gamma_w \beta)^{T-t} \mathrm{E}_t \left[\eta^c_t \left\{ \frac{\bar{\omega}^o U(H^o_T) + \bar{\omega}^{no} U(H^{no}_T)}{\bar{\omega}^o + \bar{\omega}^{no}} - \eta^l_t V(l_T(j)) \right\} \right],$$

subject to the budget constraints (24) and (31), the labor demand equation (33) and the above indexation formula, where $H^{no}_t = c^{no}_t - h c^{no}_{t-1}$.

The first-order conditions of this problem with respect to the two types of consumption and the nominal wages are the following.

$$\lambda_T^o = (\gamma_w \beta)^{T-t} \frac{\Lambda_t^o}{P_T}, \qquad \lambda_T^{no} = (\gamma_w \beta)^{T-t} \frac{\Lambda_t^{no}}{P_T} \tag{34}$$

$$\sum_{T=t}^{\infty} (\gamma_w \beta)^{T-t} \mathrm{E}_t \left[l_T \left(\frac{W_T}{W_t(j) \Pi_{T,t}^{Iw}} \right)^{\theta_w} \lambda_T^l \tau_T^w \Pi_{T,t}^{Iw} \right] = \tag{35}$$

$$\frac{\theta_w}{\theta_w - 1} \sum_{T=t}^{\infty} (\gamma_w \beta)^{T-t} \mathrm{E}_t \left[l_T \left(\frac{W_T}{W_t(j) \Pi_{T,t}^{Iw}} \right)^{\theta_w} \eta_T^c \eta_T^l \frac{l_T(j)^{\varphi}}{W_t(j)} \right],$$

where λ_T^o and λ_T^{no} are state dependent Lagrange multipliers, $\Lambda_T^{no} = \eta_T^c \left(H_t^{no} \right)^{-\sigma}$ and $\lambda_T^l = \lambda_T^o + \lambda_T^{no}$. It is shown in *Appendix A.3* that the above first-order conditions imply that aggregate wage setting can be describes by the following recursive form,

$$\mathscr{W}_t = \left(\frac{\theta^w}{\theta^w - 1} \frac{\mathscr{Z}_t^{w1}}{\mathscr{Z}_t^{w2}} \right)^{1+\theta_w \varphi}, \tag{36}$$

where

$$\mathscr{Z}_t^{w1} = \eta_t^c \eta_t^l l_t^{\varphi+1} + \beta \gamma_w \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}^w}{\Pi_t^{Iw}} \right)^{\theta_t^w (1+\varphi)-1} \mathscr{Z}_{t+1}^{w1} \right], \tag{37}$$

$$\mathscr{Z}_t^{w2} = \tau_t^w \Lambda_t^a w_t l_t + \beta \gamma_w \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}^w}{\Pi_t^{Iw}} \right)^{\theta_t^w - 1} \mathscr{Z}_{t+1}^{w2} \right]. \tag{38}$$

Finally,

$$\mathscr{W}_t^{1-\theta_w} = \frac{1 - \gamma_w \left(\frac{\Pi_t^w}{\Pi_{t-1}^{Iw}} \right)^{\theta_w - 1}}{1 - \gamma_w}. \tag{39}$$

2.3 EXPORTS DEMAND

Export is determined by an ad-hoc demand equation

$$\frac{x_t}{x_{t-1}^{h_x}} = \left(P_t^{x*} \right)_t^{-\theta_x} x_t^*, \tag{40}$$

where x_t denotes exports, P_t^{x*} is the price index of exported goods denominated in foreign currency, variable x_t^* is an exogenous shock $0 \le h_x \le 1$, $0 < \theta_x$.

2.4 GOVERNMENT

$$P_t g_t + \bar{\omega}^p P_t c_t^p = \bar{\omega}^o T_t^o. \tag{41}$$

$$P_t g_t = T_t^{no}. \tag{42}$$

2.5 CURRENT ACCOUNT

The evolution of net foreign assets (measured in foreign currency) is given by

$$b_t = P_t^{x*} x_t - P_t^{m*} m_t + (1 + i_{t-1}^*) b_{t-1}. \tag{43}$$

2.6 THE INTEREST RATE AND THE EXCHANGE RATE

As mentioned before, there were two different monetary regimes in Hungary. Untill 2001, monetary policy followed an exchange rate targeting regime (with some small tolerance band). The central bank preannonced a rate of depreciation and allowed some (±2.25 per cent) deviation from this preannonced exchange rate path. In 2001, the interventioned band was widened to ±15 per cent and the preannonced rate of depreciation was fixed to zero. In this relatively wide band, the nominal exchange rate fluctuated significantly, and the central bank started to operate in an inflation targeting regime. In 2008. the intervention band was abandoned.

In order to model monetary policy, we describe it by two reaction functions: one which focuses on stabilizing the nominal exchange rate around the preannonced (exogenous) exchange rate path and one in which it follows a simple Taylor-type rule.

We assume that in the crawling-peg regime the main focus of monetary policy is determination of the rate of crawl. Hence the behavior of cyclical part of the nominal interest rate is captured by the following simple equation.

$$i_t = \zeta_e^{cr}(e_t - d\bar{e}_t) + d\bar{e}_t + \tilde{\varepsilon}_t^r, \tag{44}$$

where e_t denotes nominal exchange rate, $d\bar{e}_t$ is the exogenously given deterministic part of depreciation, $\tilde{\varepsilon}_t^r$ is an exogenous stochastic shock, and $\zeta_e^{cr} > 0$ ensures that deviations of exchange rate from its deterministic component (the one determined by the rate of crawl) is stationary. Since the presence of ζ_e^{cr} is due to this technical requirement its magnitude is set to be negligible.

In the inflation-targeting regime the behavior of the monetary authority is captured by the following *interest-rate rule*:

$$i_t - (r^k - \delta) = \varsigma^i(i_{t-1} - (r^k - \delta)) + (1 - \varsigma^i)(\varsigma^\pi(\hat{\pi}_t - d\bar{q}_t) + \zeta_e^{it}(e_t - 1)) + \tilde{\varepsilon}_t^r.$$

Recall that $-d\bar{q}_t = \bar{\pi}_t$ in the inflation targeting regime. Again, the only role of $\zeta_e^{it} > 0$ is to ensure the stationarity of e_t. Note that $\hat{\pi}_t - d\bar{q}_t$ equals to the demeaned inflation and the steady state level of nominal exchange rate is 1.

Nominal exchange rate is determined by the uncovered interest rate parity with financial premium shock ($\tilde{\varepsilon}_t^{pr}$):

$$\frac{1 + i_t}{1 + i_t^*} = \frac{E_t e_{t+1}}{e_t}, \tag{45}$$

Where stationary of b_t is achived by (following Schmitt-Grohe & Uribe (2002)) the assumption that the relevant foreign interest rate (i_t^*) depends on a net-foreign asset determined premium, on an exogenous foreign interest rate and on financial premium shocks.

$$\frac{1 + i_t^*}{1 + r} = \exp(-\nu(b_t - b))(1 + \tilde{\varepsilon}_t^{pr}) \tag{46}$$

The deviation of real exchange rate from its steady state is determined by the following identity.

$$\tilde{q}_t - \tilde{q}_{t-1} = d\hat{e}_t - \hat{\pi}_t + d\bar{q}_t. \tag{47}$$

Finally, equation (72) of the Appendix implies the following law of motion for $d\bar{q}_t = d\bar{e}_t - \bar{\pi}_t$,

$$d\bar{q}_t = \frac{\rho_\pi - g}{1 - g} d\bar{q}_{t-1} - \frac{g}{1 - g}\hat{\pi}_t + \tilde{\chi}_t, \tag{48}$$

where

$$\tilde{\chi}_t = d\bar{e}_t - \frac{\rho_\pi - g}{1-g} d\bar{e}_{t-1}$$

is an exogenous shock.

2.7 EQUILIBRIUM CONDITIONS

This section discusses the equilibrium conditions and aggregation issues.

The goods market equilibrium conditions are

$$y_t^d = c_t + I_t + g_t + \Psi(u_t) k_t^d, \tag{49}$$

$$y_t^x = x_t + \Psi(u_t) k_t^x. \tag{50}$$

where g_t is real government consumption determined by an exogenous shock.

Equilibrium conditions of the input markets are

$$l_t = l_t^d + l_t^x, \quad m_t = m_t^d + m_t^x, \quad k_t = k_t^d + k_t^x. \tag{51}$$

2.8 PERCEIVED *UNDERLYING* INFLATION RATE

Agents apply a real-time adaptive algorithm to determine the *underlying* inflation rate,

$$\bar{\pi}_t = \rho_\pi \bar{\pi}_{t-1} + g\left(\pi_t - \bar{\pi}_{t-1}\right),$$

where $\pi_t = \widetilde{P}_t - \widetilde{P}_{t-1}$ is the observed actual, $\bar{\pi}_t = \widetilde{\bar{\Pi}}_t$ is the perceived *underlying* inflation rate and $0 < \rho_\pi < 1$. The gain parameter $0 < g < 1$ influences the speed of learning. If one defines the cyclical component of inflation as $\hat{\pi}_t = \pi_t - \bar{\pi}_t$, then the previous formula can be expressed in the following way,

$$\bar{\pi}_t = \frac{\rho_\pi - g}{1-g} \bar{\pi}_{t-1} + \frac{g}{1-g} \hat{\pi}_t. \tag{52}$$

2.9 LOG-LINEARIZED MODEL

We first log-linearize the model around the steady state. The log-linearized model is described in Appendix A.2. The behavior of the crawling-peg regime is captured by a 23-equation system of formulas (73) – (92) and (94) – (96). It determines the trajectories of 23 endogenous variables, namely, $\hat{\pi}_t$, $\hat{\pi}_t^{x*}$, $\hat{\pi}_t^w$, $\tilde{y}_t^d$, $\tilde{y}_t^x$, $\tilde{c}_t$, $\tilde{c}_t^o$, $\tilde{c}_t^{no}$, $\tilde{c}_t^p$, $\widetilde{w}_t$, $\tilde{x}_t$, $\widetilde{m}_t$, $\tilde{l}_t$, $\tilde{n}_t$, $\tilde{k}_t$, $\tilde{I}_t$, $\tilde{q}_t$, $d\bar{q}_t$, $\hat{e}_t$, $\hat{\imath}_t$, $\tilde{b}_t$, $\widetilde{Q}_t$, and $\tilde{r}_t^k$. The system is driven by 15 exogenous shocks, $\tilde{g}_t$, $\widetilde{P}_t^{m*}$, $\tilde{\chi}_t$, $\tilde{x}_t^*$, $\widetilde{A}_t$, $\tilde{\upsilon}_t^d$, $\tilde{\upsilon}_t^w$, $\tilde{\upsilon}_t^x$, $\tilde{\varepsilon}_t^c$, $\tilde{\varepsilon}_t^k$, $\tilde{\varepsilon}_t^r$, $\tilde{\varepsilon}_t^{pr}$, $\tilde{\varepsilon}_t^n$, $\tilde{\varepsilon}_t^I$ and $\tilde{\varepsilon}_t^Q$.

The inflation-targeting regime is described by equations (73) – (91) and (93) – (96). This set of equations determines the same 23 endogenous variables. The system is driven by the same 15 shocks as previously.

3 Bayesian Estimation

In order to estimate the parameters of the DSGE model presented in *Section 2* we used quarterly Hungarian data of thirteen macroeconomic variables: real consumption, real investments, real exports, real imports, real government consumption, real wages, employment, capital stock, CPI inflation rate, nominal interest rate, import and export prices denominated in foreign currency and the preannounced rate of the nominal-exchange-rate crawl. Estimation is based on the database of the Quarterly Projection Model of the Magyar Nemzeti Bank (data set presented in Benk *et al.* (2006)). This covers the period of 1995:2-2007:2. Detailed description of the data and the applied data transformations can be found in *Appendix B*.

To estimate the model, we apply a likelihood-based Bayesian method described in An & Schorfheide (2005). The first step is to construct the likelihood function. This needs the reduced form rational-expectations solution. Then one has to write the model in its state-space form, and formulate the Kalman filter for calculating the likelihood function. The construction of the Kalman filter is described in detail in *Appendix A.12*. In the next step, the likelihood function is combined with prior distributions in order to derive the posterior density function of parameters. Then one has to find numerically the mode of the posterior density function. Finally, the random-walk Metropolis-Hastings (MH) algorithm is used to generate the posterior distribution. The applied MH algorithm is based on $500,000$ draws (2 parallel chains of $250,000$ draws discarding the initial burn-in period of $50,000$ iterations). To monitor the convergence of the MH algorithm we applied the method of Brooks & Gelman (1998).[1] In order to compare different model versions, the marginal likelihoods of models are calculated by the modified harmonic mean algorithm of Geweke (1998).

In the studied time period there is one obvious structural break: in 2001 the crawling-peg regime was abandoned and inflation targeting was introduced. To capture this change in monetary policy practice we estimated different policy rules in the two subperiods, as was discussed in the previous section. Our estimation procedure also allowed some other parameters to change between the two regimes. Namely, price setting parameters and parameters of the financial premium shock and the labor market shock are time varying.[2]

3.1 FIXED PARAMETERS

Some parameters are not estimated but kept fixed from the start of the procedure (see *Table 1*). This can be viewed as a very strict prior.

First, we estimate the standard-deviation and autoregressive parameters of exogenous shock with observable time series, (namely, the government-spending $\tilde{g}_t$, the measurement error of capital accumulation $\tilde{\varepsilon}_t^k$ and the import-price $\widetilde{P}_t^{m*}$ shocks) by single-equation OLS. Then we use these results to fix those parameters in the estimation procedure of the full system.

Second, the time series of the deterministic part of depreciation $d\bar{e}_t$ can be constructed. Using the constructed time series we also estimate the standard-deviation and autoregressive parameters of this shock by OLS.

Third, some other parameters can be directly related to the steady-state values of endogenous variables. These are the production function parameters, the subjective discount rate of households, depreciation rate and the elasticity-of-substitution between varieties of differentiated goods and that of differentiated labor.

Fourth, there were parameters we were not able to identify. Concretely, our algorithm of searching the mode of the posterior density function failed if these parameters were not fixed. To identify these parameters we

[1] For the numerical implementation of the estimation procedure we developed our own MATLAB code. Reduced form RE solutions were calculated by the MATLAB routine of Uhlig (1999). For finding the mode of a posterior distribution, we used the algorithm and code of Kuntsevich & Kappel (1997).

[2] There is a vast literature that the rigidity of prices and wages depends on monetary-policy regimes, see, e.g., Taylor (2000)).

Table 1

Fixed parameter values

Parameter		
Name		Value
standard error of gov't. consumption	σ_g	4.72
standard error of import prices	$\sigma_{p^{m*}}$	2.19
standard error of χ_t	σ_χ	0.12
standard error of capital measurement error	σ_k	0.15
autoreg. coeff. of gov't. consumption	ρ_g	0.46
autoreg. coeff. of import prices	$\rho_{p^{m*}}$	0.74
autoreg. coeff. of χ_t	ρ_χ	0.53
autoreg. coeff of capital measurement error	ρ_k	0.60
autoreg. coeff of perceived *underlying* inflation	$\rho_{\bar{\pi}}$	0.99
discount factor	β	0.99
steady-state share of capital in real marginal costs, domestic	α_d	0.17
steady-state share of capital in real marginal costs, export	α_d	0.14
steady-state share of labor in w^z_t,domestic	a_d	0.50
steady-state share of labor in w^z_t, export	a_x	0.36
depreciation rate	δ	0.025
elasticity of subt. of goods	θ	6.00
elasticity of subt. of labor	θ_w	3.00
fraction of optimizing households's consumption	$1-\omega^{no}$	0.75
fraction of pensioners' consumption to total non-optimizer consumption	ω^p/ω^{no}	0.35
disutility parameter of labour	φ	8.00
Calvo parameter of employment	γ_n	0.70
elasticity of subt. between capital and z	ϱ	0.80
elasticity of subt. between labor and import	ϱ_z	0.50
ratio of fixed cost relative to total output	$\bar{f}_d, \bar{f}_x$	0.20
capacity utilization adj. cost	ψ	0.20
investments adjustment cost	Φ''	13.00
labour-import utilization adjustment cost	Φ''_z	3.00
exchange rate elasticity of the policy rule	ζ^{cr}_e	0.001
exchange rate elasticity of the policy rule	ζ^{it}_e	0.025
debt elasticity of financial premium	ν	0.001

chose such values common in the business cycle literature. The exception is the adjustment-cost parameters of investments Φ'' and that of the import-labour boundle.Φ''_z. We picked several values of them, and compared the accompanying marginal likelihoods of the estimated model version. We chose the parameter value with the highest marginal likelihoods (although we have not found large differences between the different versions). Finally, we arrived at a relatively costly adjustment of capital ($\Phi''=13$) and more moderate adjustment cost for the labor-import boundle ($\Phi''_z=3$) [3]

Finally, we fixed ζ^{cr}_e, ζ^{it}_e and ν. These are technical parameters. There only role is to assure stationarity of the model.

3.2 SPECIFYING PRIOR DISTRIBUTIONS

Prior distributions for parameters of non-observed exogenous shocks are displayed in *Table 2*. All the standard deviations of the shocks are assumed to be distributed as an inverted Gamma distribution with a degree of freedom equal to 2. This distribution guarantees a positive standard deviation with a rather large domain.

[3] The share of non-optimizing households ($1-\bar{\omega}^{no}$) was set to be 0.25, based on some survey evidence stating that 25 per cent of Hungarian households do not have connections with the banking sector. The share of pensioners among non-optimizers (ω^p) was determined by a regression such that income of pensioners equals to half of real wage according to the 'Swiss-index-formula' determining real pensions in Hungary. The production function parameters ϱ and ϱ_z were calibrated, in such a way that imports and labor are complements and capital and the import-labor bundle has an elasticity of transformation of 0.8.

Prior distributions of autoregressive parameters are assumed to follow Beta distributions with mean of 0.8 and standard error of 0.1.

Prior distributions for the rest of estimated parameters are shown in *Table 3*. Calvo parameters of consumer and export price setting and that of nominal wages were set to be equal for both regimes with a relatively uninformative prior, a Beta distribution with mean of 0.5 and standard error of 0.2. Similarily, indexation parameters (ϑ_p, ϑ_x and ϑ_w) also received a not very tight prior of Beta with standard deviation of 0.2 and mean of 0.6.

The choice of prior for the parameter of interest rate smoothing ζ_i is different to the literature. We imposed a relatively uninformative Uniform prior distribution on it. Our prior for the learning gain (g) parameter was relatively tight, a Beta distribution with mean of one-sixth and standard error of 0.03. We chose a mean value for parameter σ higher than that of Smets & Wouters (2003) due to some stylized evidence on low real-interest-rate elasticity of consumption in Hungary.

Prior distributions for the rest of the parameters were chosen similarly to Smets & Wouters (2003).

3.3 ESTIMATION RESULTS

As was mentioned, we estimated different values for some parameters in the crawling-peg and the inflation-targeting periods, namely for the standard-deviation and autoregressive parameters of shocks $\tilde{\upsilon}_t^w$ and $\tilde{\varepsilon}_t^{pr}$, and the Calvo and indexation parameters, (γ_p, γ_x, γ_w, ϑ_p, ϑ_x and ϑ_w). Their different values are denoted by superscripts cr and it, respectively. Recall, that σ_{de} belongs to the nominal-exchange rate shock of the crawling-peg regime, and σ_r, ζ_i, and ζ_π belong to the the policy-rule equation of the inflation-targeting period. Estimation results are summarized in *Tables 2* and *3*.

None of the estimated values of the Calvo parameters are very different in the two monetary policy regimes. This seems surprising at the first glance as one would assume a change in these key parameters of Phillips curves. As shown later, the regime change rather had an effect on the indexation behavior. The Calvo-parameters of domestic prices are close to that of euro-area estimates, see, e.g., Smets & Wouters (2003) and the *new area wide model* of the ECB, described in Christoffel *et al.* (2007). Export prices are estimated to be less sticky than consumer prices. This conforms to our intuition that exporters in Hungary mostly produce intermediate goods with probably less relevant price stickiness. There is a significant difference with respect to the Calvo parameters of wages: in Hungary nominal wages are estimated to be less sticky than in the eurozone. In addition, we estimate wages as the more flexible than either consumer or export prices.

Unlike Calvo coefficients, the monetary regime shift is mostly felt in the indexation properties in pricing (indexation of consumer prices dropped in the second regime). This migh indicate that the crawling-peg regime served as a natural way for indexation-mechanisms. Indexation parameter of consumer prices in the inflation targeting regime is lower than that of Christoffel *et al.* (2007), but comparable to that in Smets & Wouters (2003). That is, their no consensus on the issue of price indexation in the literature. As far as nominal wage indexation is concerned, it is much lower than in NAWM and in SW in both monetary regimes.

However, it is important to note that one should be cautious to interpret our results of price and wage indexation. Indexation formulas reveal that in this both prices and wages are fully indexed to the perceived long-run component of inflation. Besides that, the parameters ϑ_p and ϑ_w represent the degree of additional indexation to the cyclical components of past price and wage inflation rates.

The mean speed of learning the *underlying* inflation (g) is estimated to be higher than our prior mean. In estimated US and euro-area models the value of the interest-rate-smoothing parameter ζ_i is quite high.[4] On the other hand, we found a relatively low value for this parameter, it is around 0.75. It is important to note that our result also contrasts with previous Hungarian estimates. For example, Hidi (2006) in his estimated

[4] See CEE, SW, NAWM,. Flat prior Rabanal & Rubio-Ramirez (2005), Világi (2007)

Table 2

Estimated parameters of exogenous shocks

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Standard errors							
productivity	σ_A	I.Gam.	0.5	2*	2.080	2.152	[1.82, 2.55]
export demand	σ_x	I.Gam.	0.5	2*	2.352	2.464	[2.06, 2.95]
cons. pref.	σ_c	I.Gam.	0.5	2*	0.146	0.203	[0.11, 0.33]
cons. price markup	σ_p	I.Gam.	0.5	2*	0.397	0.420	[0.34, 0.52]
export price markup	σ_{px}	I.Gam.	0.5	2*	1.873	2.182	[1.64, 2.87]
labor market	σ_w^{cr}	I.Gam.	0.5	2*	0.824	0.922	[0.65, 1.26]
labor market	σ_w^{it}	I.Gam.	0.5	2*	0.609	0.932	[0.54, 1.42]
investments	σ_I	I.Gam.	0.5	2*	0.969	1.003	[0.76, 1.27]
Equity premium	σ_Q	I.Gam.	0.5	2*	0.167	0.393	[0.11, 0.99]
policy rule	σ_r^{cr}	I.Gam.	0.5	2*	0.574	0.602	[0.48, 0.76]
policy rule	σ_r^{it}	I.Gam.	0.5	2*	0.229	0.247	[0.19, 0.32]
fin. premium	σ_{pr}^{cr}	I.Gam.	0.5	2*	0.221	0.372	[0.17, 0.67]
fin. premium	σ_{pr}^{it}	I.Gam.	0.5	2*	0.486	0.666	[0.36, 1.06]
employment	σ_n	I.Gam.	0.5	2*	0.369	0.383	[0.30, 0.49]
Autoregressive coefficients							
productivity	ρ_A	Beta	0.8	0.1	0.577	0.552	[0.43, 0.68]
export demand	ρ_x	Beta	0.8	0.1	0.616	0.625	[0.51, 0.73]
cons. pref.	ρ_c	Beta	0.8	0.1	0.833	0.767	[0.60, 0.88]
labor market	ρ_w^{cr}	Beta	0.8	0.1	0.486	0.480	[0.32, 0.64]
labor market	ρ_w^{it}	Beta	0.8	0.1	0.797	0.661	[0.43, 0.87]
export markup	ρ_x	Beta	0.5	0.15	0.317	0.318	[0.15, 0.50]
investments	ρ_I	Beta	0.8	0.1	0.497	0.488	[0.33, 0.65]
fin. premium	ρ_{pr}^{cr}	Beta	0.8	0.1	0.899	0.833	[0.69, 0.93]
fin. premium	ρ_{pr}^{it}	Beta	0.8	0.1	0.872	0.820	[0.70, 0.92]
employment	ρ_n	Beta	0.8	0.1	0.790	0.770	[0.64, 0.89]

*For the Inverted Gamma function the degrees of freedom are indicated.

single-equation policy rule found a much higher interest-rate smoothing parameter comparable with the values in the international literature. A possible explanation for this can be found in Goodhart (2004), he argues that non-structural single-equation methods overestimates the value of interest-rate smoothing parameter, since they are not able to identify some persistent structural shocks influencing the behavior of the policy rate. As mentioned earlier, the adjustment cost of investment was chosen at a value higher than usually estimated in other DSGE models. In addition, the presence of cost of adjustment for labor-import boundle is not usually assumed in the literature.

Comparing posterior and prior density graphs, data were informative, prior and posterior density graphs differ, the only exceptions are the export price elasticity and the export smoothing parameters (θ_x and h_x) where prior and posterior distributions are close to each other.[5]

[5] One should also note that posterior density graphs of γ_p are not fully satisfactory, as the modes of posteriors are not close to the maximum likelihood estimates.

Table 3

Estimated parameters

		Prior distribution			Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.
Utility function parameters							
consumption	σ	Norm.	2.00	0.40	1.680	1.814	[1.18, 2.46]
habit	h	Beta	0.75	0.15	0.597	0.646	[0.45, 0.83]
Price and wage setting parameters							
ind. cons. prices	ϑ_p^{cr}	Beta	0.60	0.20	0.873	0.821	[0.63, 0.96]
ind. cons. prices	ϑ_p^{it}	Beta	0.60	0.20	0.416	0.431	[0.22, 0.66]
ind. exp. prices	ϑ_x^{cr}	Beta	0.60	0.20	0.215	0.290	[0.10, 0.55]
ind. exp. prices	ϑ_x^{it}	Beta	0.60	0.20	0.383	0.494	[0.18, 0.83]
ind. wages	ϑ_w^{cr}	Beta	0.60	0.20	0.112	0.169	[0.05, 0.34]
ind. wages	ϑ_w^{it}	Beta	0.60	0.20	0.107	0.185	[0.05,0.40]
Calvo cons. prices	γ_p^{cr}	Beta	0.50	0.20	0.939	0.938	[0.92, 0.96]
Calvo cons. prices	γ_p^{it}	Beta	0.50	0.20	0.929	0.921	[0.88, 0.95]
Calvo exp. prices	γ_x^{cr}	Beta	0.50	0.20	0.850	0.824	[0.75, 0.89]
Calvo exp. prices	γ_x^{it}	Beta	0.50	0.20	0.827	0.810	[0.73, 0.89]
Calvo wages	γ_w^{cr}	Beta	0.50	0.20	0.714	0.698	[0.58, 0.81]
Calvo wages	γ_w^{it}	Beta	0.50	0.20	0.711	0.657	[0.46, 0.88]
Other parameters							
exp. elasticity	θ_x	Beta	0.50	0.10	0.510	0.534	[0.40, 0.67]
exp. smooth.	h_x	Beta	0.75	0.15	0.503	0.507	[0.35, 0.66]
ir. smooth.	ζ_i	U(0,1)	0.50	0.29	0.766	0.761	[0.67, 0.84]
policy rule	ζ_π	Norm.	1.50	0.16	1.375	1.379	[1.12, 1.65]
gain	g	Beta	0.167	0.03	0.229	0.234	[0.17, 0.30]

4 Analysis of structural shocks and perceived *underlying* inflation

There are fifteen structural shocks determining the economy. Two of them, $\tilde{g}_t$ and $\widetilde{P}_t^{m*}$, are observable, and one, the measurement error for capital accumulation ($\tilde{\varepsilon}_t^k$) was estimated from an OLS estimate *Figure 1* displays the three exogenous series. The deterministic part of depreciation, $d\bar{e}_t$, is also treated as an exogenous shock in our estimation exercise, it equals to the rate of crawl in the crawling-peg regime, and captures a one-off trend appreciation after the introduction of the new exchange rate regime accompanied by a widening of the intervention band. (see *Figure 2*)

The rest of the shock are unobservable, and they are as latent variables in the estimation procedure, and calculated by the two-sided Kalman-smoother. The analysis of this section based on shock trajectories belonging to a model version parameterized by the estimated mean values of the inflation-targeting period.

Figure 2 shows the estimated trajectories of shock directly influencing the nominal interest rate and the nominal exchange rate. Namely, the nominal depreciation shock in the crawling-peg regime, χ_t , the monetary-policy shock of the inflation-targeting period, $\tilde{\varepsilon}_t^r$, and the financial-premium shock in the uncovered-interest-rate-parity equation, $\tilde{\varepsilon}^{pr}$. Since in the estimation procedure we did not use a foreign-interest-rate series our estimated financial-premium shock incorporates foreign-interest-rate movements as well.

The evolution of the above shocks fits some well-documented events of the Hungarian economy of the past decade. Credibility in the exchange rate regime was somewhat weak at the outset of the crawling peg regime (in 1996) and this is reflected in the financial premium shock. In addition the change in monetary regime in 2001, accompanied by a significant appreciation of the Hungarian forint, can also be clearly observed as a series of negative shocks. To interpret this, one can also think of this shock mirroring the substantial change in portfolios (i.e. an increase in forint denominated government debt among the assets of international investors). A period of increasing risk of Hungarian assets is demonstrated also in 2003, when the central parity was devalued and the forint depreciated markedly as financial markets became vulnerable. The appreciating speculation in early 2003 is also shown as a negative premium shock. The shock also describes the gradual tightening of the ECB at the end of the sample. Moreover, in the summer of 2006 exchange rate depreciated after the announcement of the fiscal stabilization and this shows up in a temporary financial premium shock as the reaction of monetary policy was relatively smooth and exchange rate only strengthened back to the pre-stabilization levels later.

The estimated trajectories of the rest of the shocks can be seen in *Figure 3*. The export-demand shock, $\hat{x}_t^*$, also matches to common perception of the economy: the slowdown in Europe because of Russian crisis and financial market evolutions in US in 1998 and 1999 and the sluggish demand for exports between 2002-2003. It also shows a gradual recovery at the end of the sample.

The consumer preference shock, $\tilde{\varepsilon}_t^c$, shows the effects of the fiscal stimulus during 2002 and 2003. In contrast to the observed government spending shock, this shock mostly captures indirect effects of fiscal policy, namely, rise in transfers and the easing of household mortgage subsidies. One could also explain the rise in consumer prefence as a result of wealth effects generated by the fiscal policy, as well. In addition, the deepening of financial markets can also account for this rise.[6] The model detects a strong negative preference shock after the fiscal consolidation package introduced in 2006.

The price markup shock $\tilde{\upsilon}_t$ is relatively volatile, but some part of the Hungarian inflation history can be realized. For example, the drop in price markup in early 1999 might be the result of a decrease of unprocessed food, oil and import prices due to the Russian crisis. On the other hand, a rise in food prices also captured

[6] Since 2001, credits to households started to accelerate and part of this might be explained by widening access to financial instruments. Liquidity constraints continuously eased.

by the estimated shock just before the introduction of the inflation targeting regime in 2001. A VAT-hike in 2004 is also detected by the model as a markup shock. Price markup started to decline after 2004, which might be the consequence of growing competition in retail sector due to Hungary's accession to the EU. The effects of fiscal consolidation accompanied by VAT and regulated price hikes in 2006 are also estimated as a price markup shock.

The labor-market shock $\tilde{\upsilon}_t^w$ is a combination of two structural shocks, a labor-supply shock and a wage-markup shock. *Figure 3* reveals that this shock is heteroskedastic in the sample, the variance of the shock increased in the inflation-targeting regime. Nominal wages were severely perturbed by government measures in this period. The effects of minimum wage hikes in 2001, 2002 and 2006 was detected by the model. Adjustment of nominal wages to the new low inflation environment after the introduction of the more disinflation oriented inflation-targeting system might have also created large nominal wage fluctuations. In 2004 and 2006, when the VAT was hiked, a negative labor-market shock can be observed, this might point to the fact that the increase in tax-wedge was not translated into higher wages that time. On the other hand, the coincidence of wage and price markup shocks might point to some specification problems, as well. This might call for a more precise modelling of labor markets e.g. as tried by Jakab & Kónya (2007).

The evolution of the productivity shock $\widetilde{A}_t$, predicts a slowdown in productivity during 1997 and 2001 and a higher productivity era since 2002. This is in contrast with other micro-level data based estimates (e.g. MNB (2006)). They argue that at the end of our sample a slowdown in productivity occurred. Moreover, other studies measure labor as employment, while in our model hours enter into production function. Hence, the difference between our productivity measure and the one estimated by e.g. Benk *et al.* (2005) might contain the possibly different evolution of hours and employment.

In summary: in most cases the estimated shocks conform to the documented special events of the Hungarian and world economy. However, the productivity and the labor-market shocks might indicate the presence of events not captured by our model. The treatment of hours and capital as latent variables and the heteroskedasticity of labour market shock are worth analyzing more deeply in the future.

An interesting feature of the model is that it contains an adaptive learning of agents about *underlying* inflation. Percception of *underlying* inflation is measured by $\bar{\pi}$. It is worth looking at the estimated evolution of this latent variable. *Figure 4* shows that our estimated perceived *underlying* inflation matches the long term disinflation in Hungary. This is not very surprising, by construction of the model perceived *underlying* inflation is a 'filtered' inflation. It is still worth comparing the two series to check it plausibility. The model prediccts that in the first three-four years of the crawling-ped regime (until around late 1998-early 1999), inflation and its perception closely moved together. There was a significant drop in quarterly inflation from around 13 percent to around 7. However, this was only gradually reflected in the estimated perceived *underlying* inflation. Our model suggests that agents only *'beleived'* in the lower-inflation era with a considerable lag. During 2000 and 2001, perceived inflation again started to stagnate. Thenafter, actual inflation was fuelled and perceived inflation also followed it with some lagged reaction. After the change in monetary policy regime to an inflation targeting regime, the relatively sudden drop in inflation, the model estimates that it was not fully perceived as permanent disinflation. The new regime needed a two-to-three years period to gain some credibility. The VAT increase in 2004 had only a temporary effect on inflation and on perceived inflation. In contrast, the VAT and regulated price hikes in 2006 had some unpleasant consequences: perceived inflation accelerated heavily.

Figure 1

Government spending, import-price and capital measurement error shocks

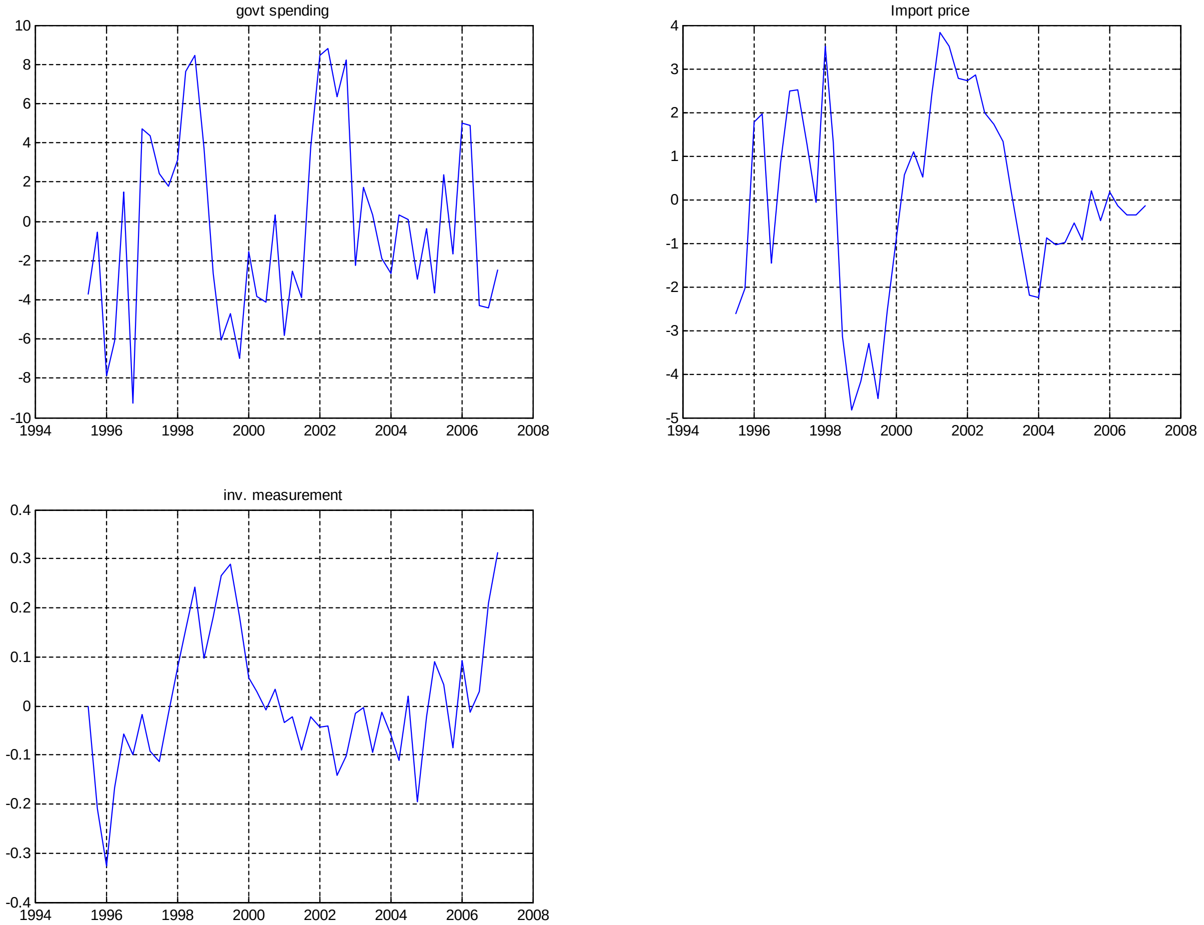
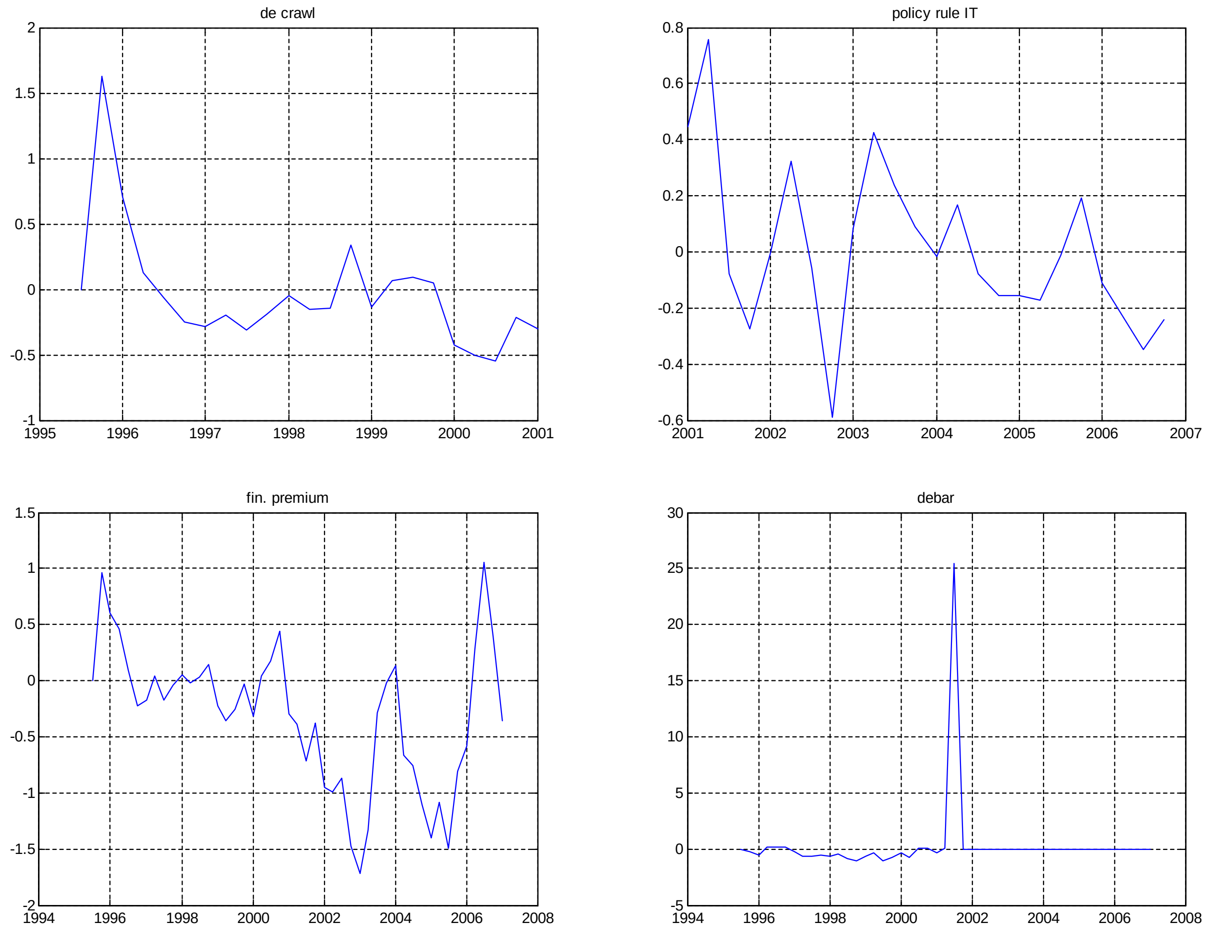


Figure 2

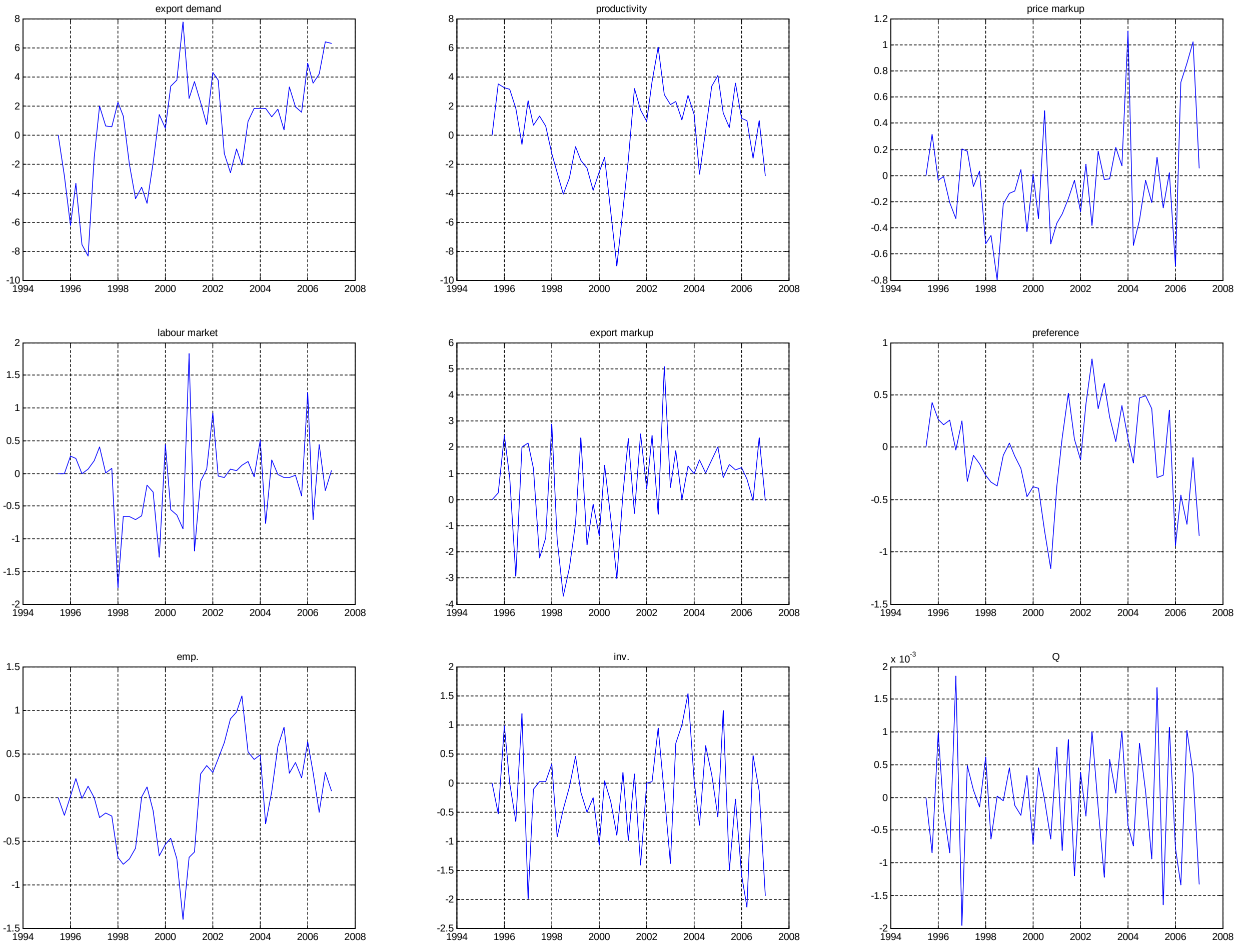
Deterministic part of nominal depreciation ($d\bar{e}_t$) rate of crawl, monetary-policy and financial-premium shocks*



*Calculated at mean parameter values

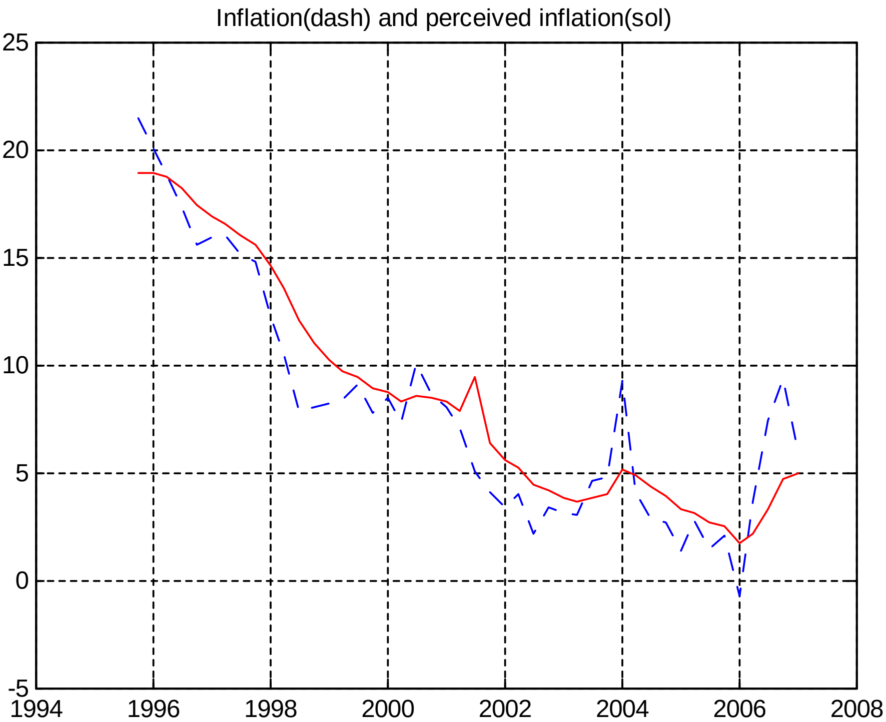
Figure 3

Other structural shocks*



*Calculated at mean parameter values

Figure 4
Perceived and actual inflation*



*Annualised quarter-on-quarter growth rates, calculated at mean parameter values. One should note that in this graph perceived inflation is defined as the one transformed back to be comparable to actual figures: perceived inflation $= \bar{\pi}_t + \chi_t + \text{mean}(\pi_t) + 0.4$

5 Impulse response analysis

Impulse responses of our model to different structural shocks are displayed in *Figures 5–16*. Price and wage inflation, user cost, nominal and real interest rates are defined as annualized quarter-on-quarter growth rates. Impulse response functions are calculated at mean parameters. Impulse responses are calculated as reactions of endogenous variables for a 1 percentage increase of innovation in the initial period. The exceptions are the two price markup shocks, labour market, policy rule and financial premium shocks where the initial increase is 0.25 percent.

To understand impulse responses, let us briefly describe some distinctive features of our estimated model. First, agents in this model continuously learn about *underlying* inflation. Consumer prices and wages are fully indexed to the perceived *underlying* inflation. Hence, due to gradual learning impulse responses of nominal wages and consumer prices are more persistent than that of the cyclical ones. Adaptive learning has a consequence for both consumption and investment response, as well. As there are non-optimizer consumers, the sluggish response of price and nominal wages creates relatively long lasting real wage and consumption responses, as well. Agents also use their perception of *underlying* inflation when real interest rate and rental rate of capital are determined. Thus investments are also sluggish due to the adaptive learning feature of the model.

Second, the estimated adjustment cost of investments is higher than usually found in the literature (e.g. SW). This implies that the response of investments is slower and less volatile than in other DSGE models, and the reaction of investments has usually the same magnitude as that of output or consumption.

Third, in most of the cases consumer prices are generally less responsive on impact and more persistent than nominal wages. This can be partly explained by the higher Calvo parameter of consumer prices than that of nominal wages.

As is usual in New Keynesian models, a positive productivity shock decreases labor (hours). Consumption is higher in the long run, but in the short run this translates into lower consumption of non-optimizers.[7]

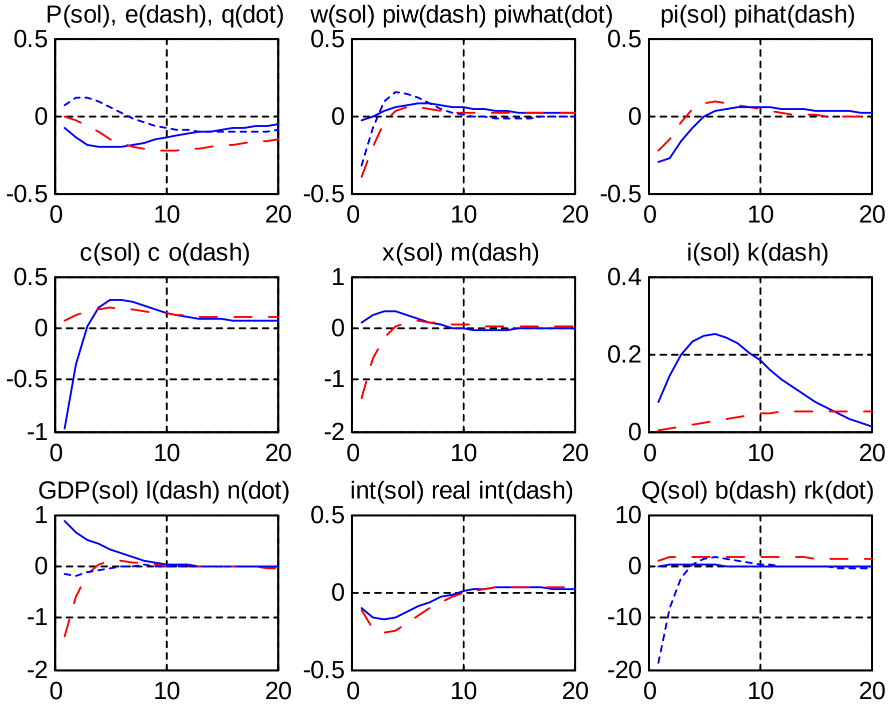
Monetary-policy shock has a negative effect on price and wage inflation. As mentioned above, part of the drop in inflation is devoted to the change in perceived *underlying* inflation, which induces agents to index to lower inflation. Indexation mechanisms amplify monetary policy shocks. In the case of financial premium shocks, GDP increases. This is mostly the result of growing consumption due to the presence of non-optimizers. Investments drop and export hardly change . The latter is the consequence of the relatively low price elasticity of exports.

Cost push shocks (consumer price markup and labour market shocks) have large impact on price and wage inflation. The two shocks result in different nominal and real interest rate paths. In the former case monetary policy tightens immediately, while in the in the latter case policy response is only gradual. Cost push shocks also accompany by significant real responses. In the case of labor market shock, the increase in non-optimizers income offset the reaction of optimizers and thus, total consumption is somewhat higher in the short run. Under both two shocks GDP, employment and investments drop. An interesting feature is that a foreign-import-price shock increases GDP which is a consequence of the large drop in imports due to relative price changes.

If a positive government spending shock occurs, one can observe that in the short run the increase in consumption of non-optimizers offset the decrease in optimizers consumption. Hence, the model replicates a (weakly) Keynesian multiplier effect in the short run. However, in the long run due to a crowding-out effect,

[7] Nominal exchange rate appreciates even though there is a drop in interest rates in the short run. At a first glance this is difficult to explain. However, the short-run response of the nominal exchange rate depends on the sum of all future nominal-interest-rate changes. In other words, the nominal exchange rate is determined by forward-looking factors. However, the reaction of the nominal exchange rate might not be very important in this case, since according to variance decomposition the behavior of the nominal exchange rate is largely explained by non-productivity factors.

Figure 5
Productivity shock



investment activity has a negative effect on GDP which also feeds back into income of nonoptimizers and thus in the medium run the response of total consumption becomes negative.

5.1 COMPARISON WITH OTHER ESTIMATED MODELS OF THE HUNGARIAN ECONOMY

Jakab *et al.* (2006) compare the reactions of different estimated Hungarian non-DSGE models to a certainly specified monetary-policy shock. They studied the behavior of the quarterly projection model of the MNB described in Benk *et al.* (2006), the small-size structural model of Várpalotai (2003), and the SVAR model of Vonnák (2005)

The benchmark monetary-policy shock is identified in SVAR. The identified shock implied a certain paths for the nominal interest rate and the nominal exchange rate.[8] Then the above defined nominal paths were treated as exogenous, and studied the impulse responses of the above models to these trajectories

In this section we perform the same exercise with our DSGE models. As previously, our impulse responses calculated by a model version parameterized with the estimated mean values of the inflation-targeting period (see *Figure 17*)

In the short run, price and real exchange rate reactions of our model to the above shock are rather similar to that of other models. In our model consumer prices are less responsive than in other models in the longer run. However, the behavior of consumption and real wages are quite different in our DSGE model and in NEM and SVAR. In the DSGE model the exogenous monetary contraction is followed by a slight decrease of real wages, while in the other models real wage first increases. This difference is due to the different structures of equations related to labor market. In the DSGE model the labor-supply equation implies that a decline of

[8] The monetary shock is described as an approximately 0.4 percentage point increase in (annual) nominal interest rate and an accompanying 0.7 percent appreciation of the nominal exchange rate.

Figure 6

Gov't spending shock

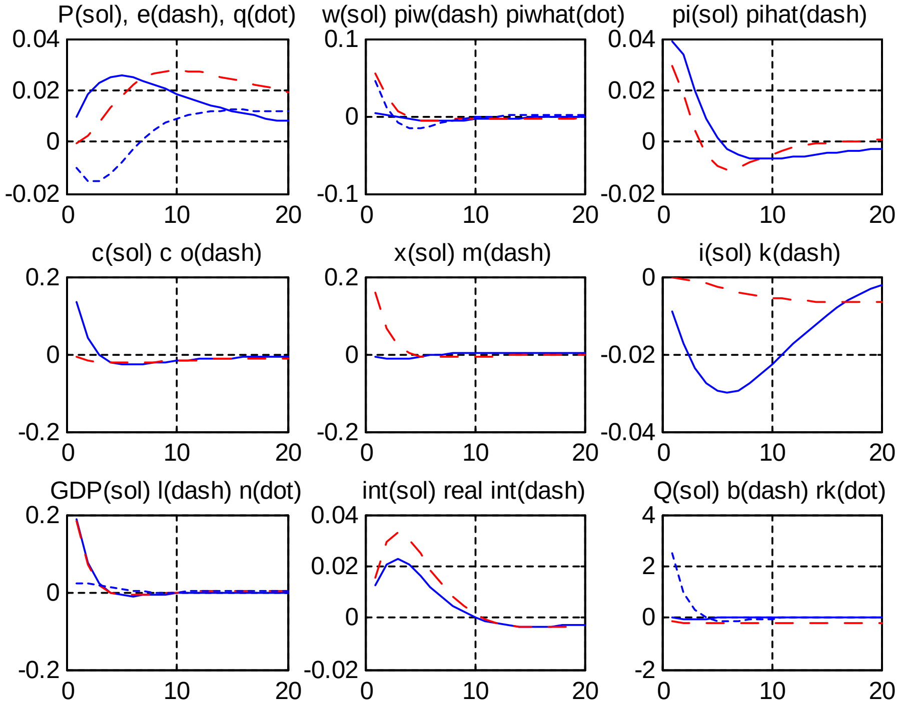


Figure 7

Monetary policy rule shock

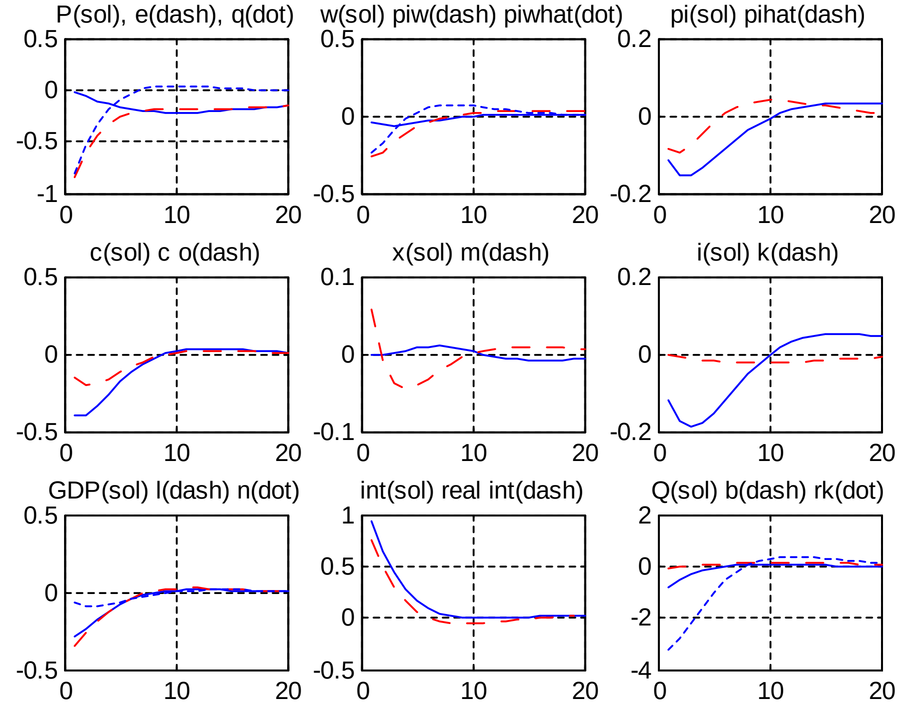


Figure 8
Financial premium shock

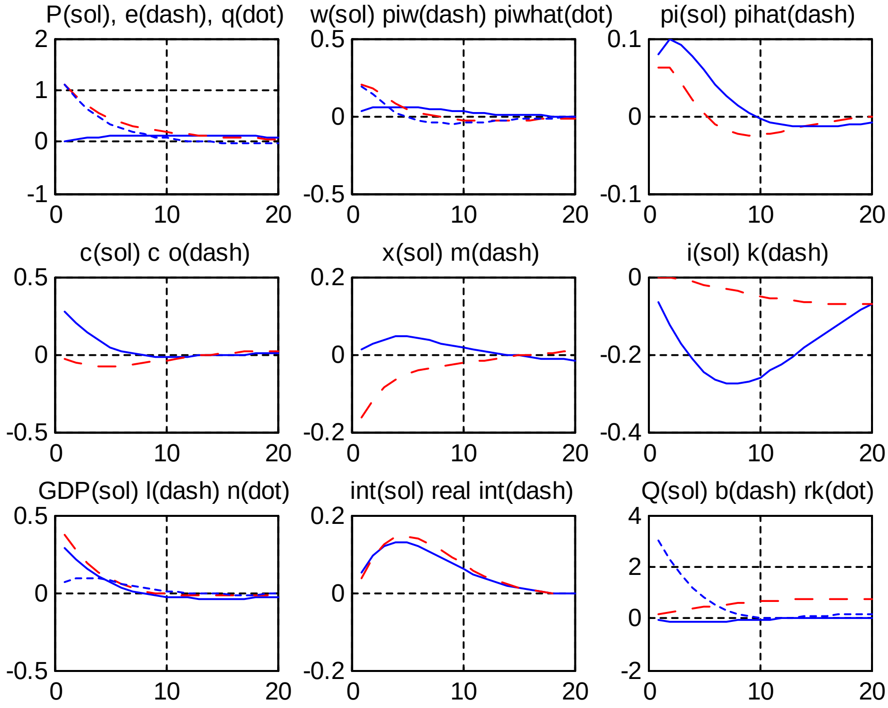


Figure 9
Consumer price markup shock

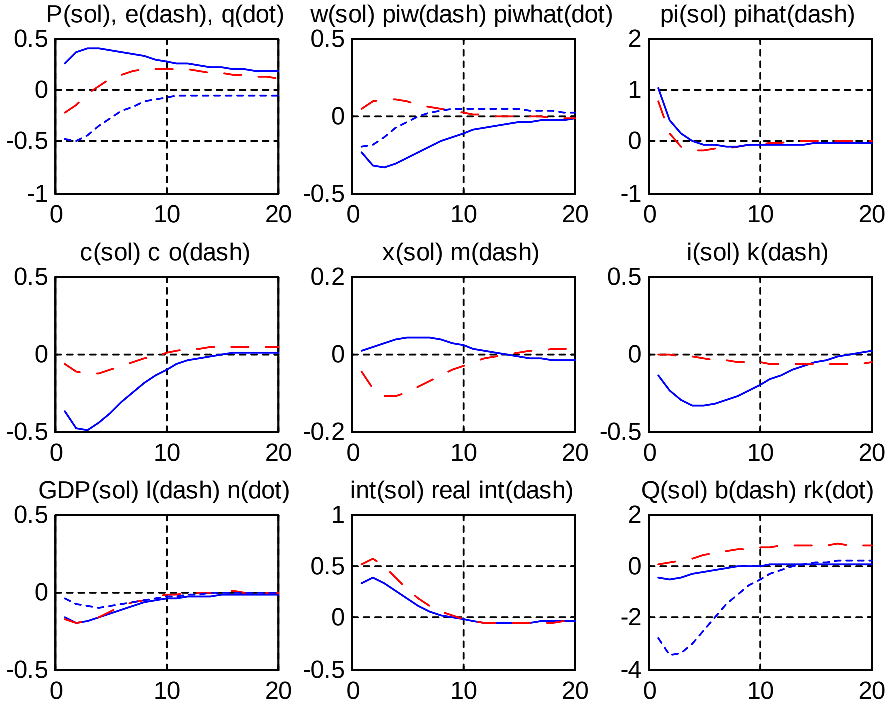


Figure 10

Export price markup shock

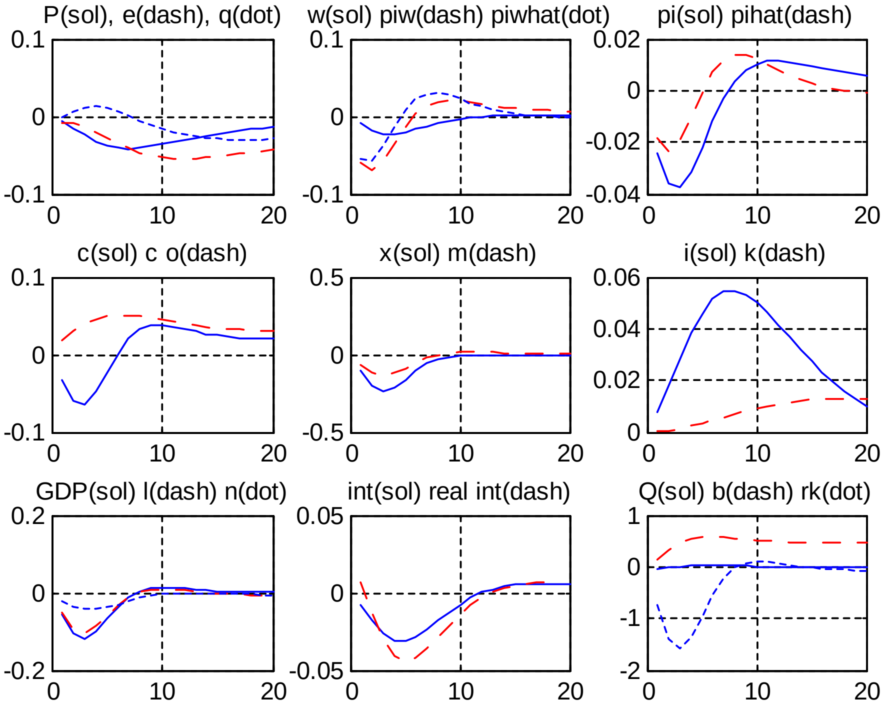


Figure 11

Labor market shock

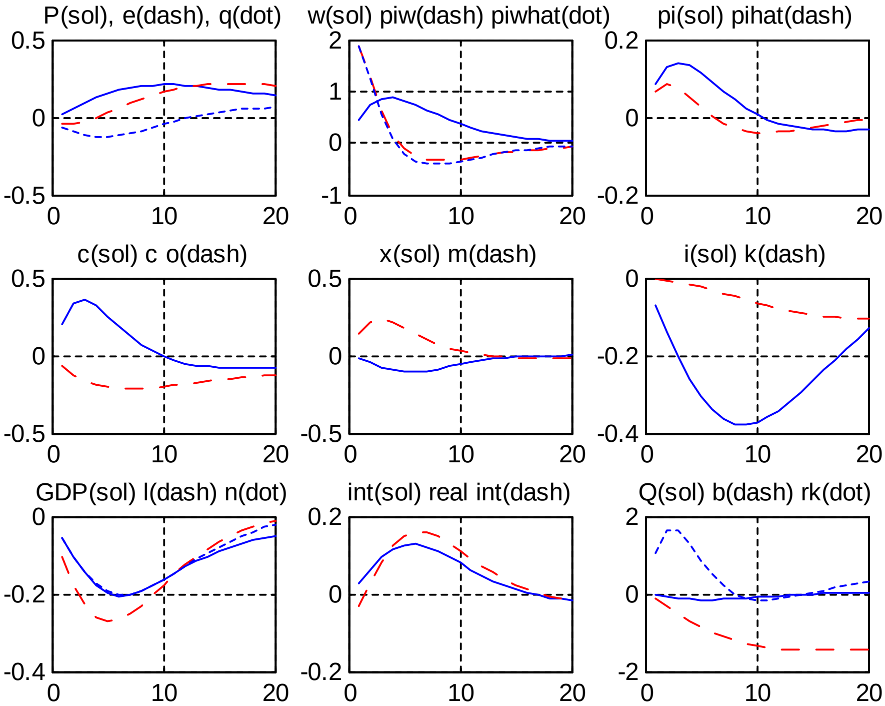


Figure 12

Preannounced nominal depreciation (crawl) shock

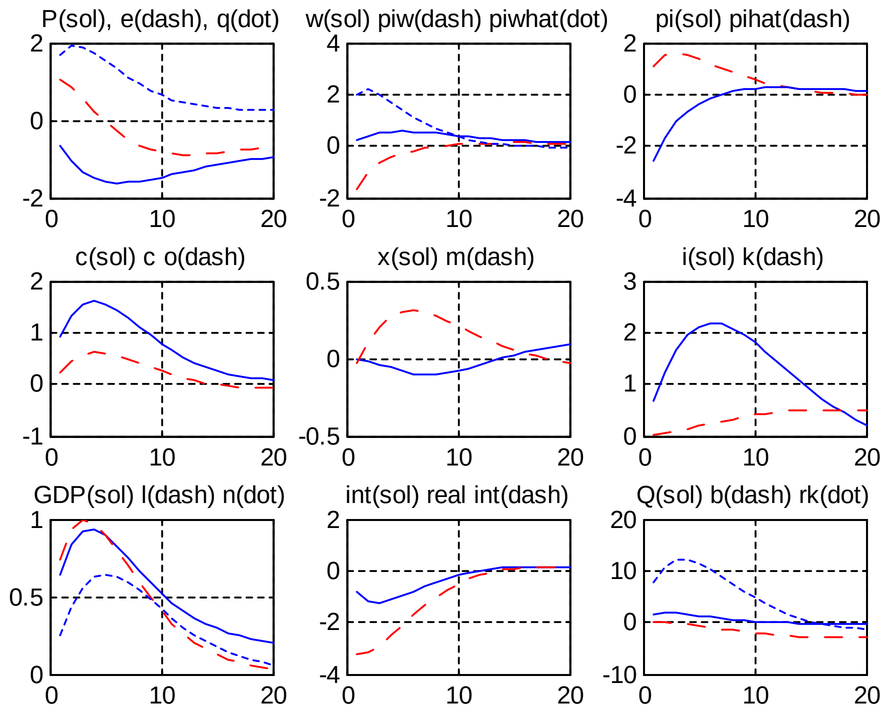


Figure 13

Import price shock

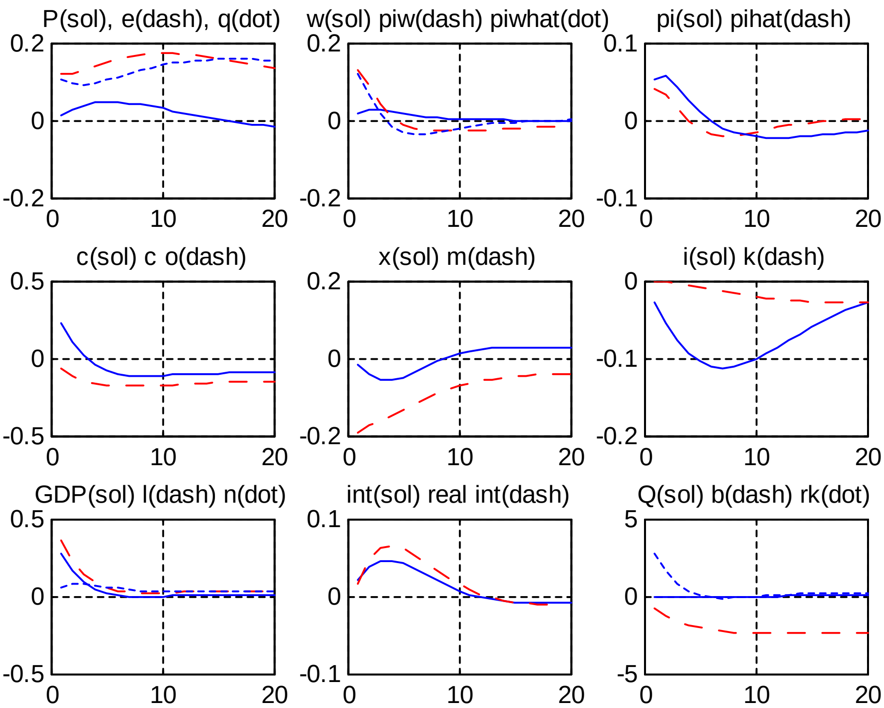


Figure 14
Export demand shock

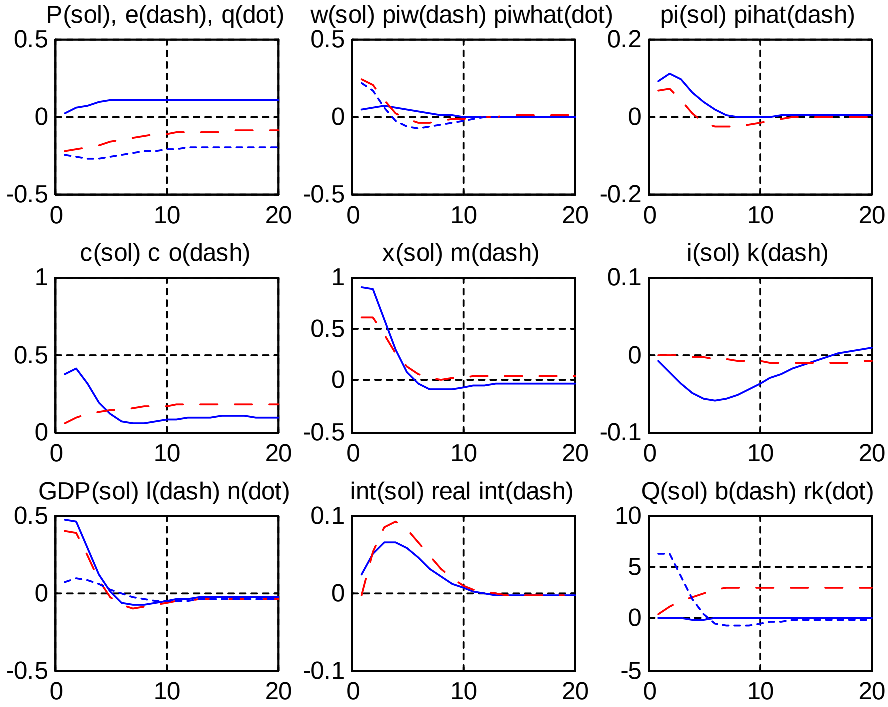


Figure 15
Investment shock

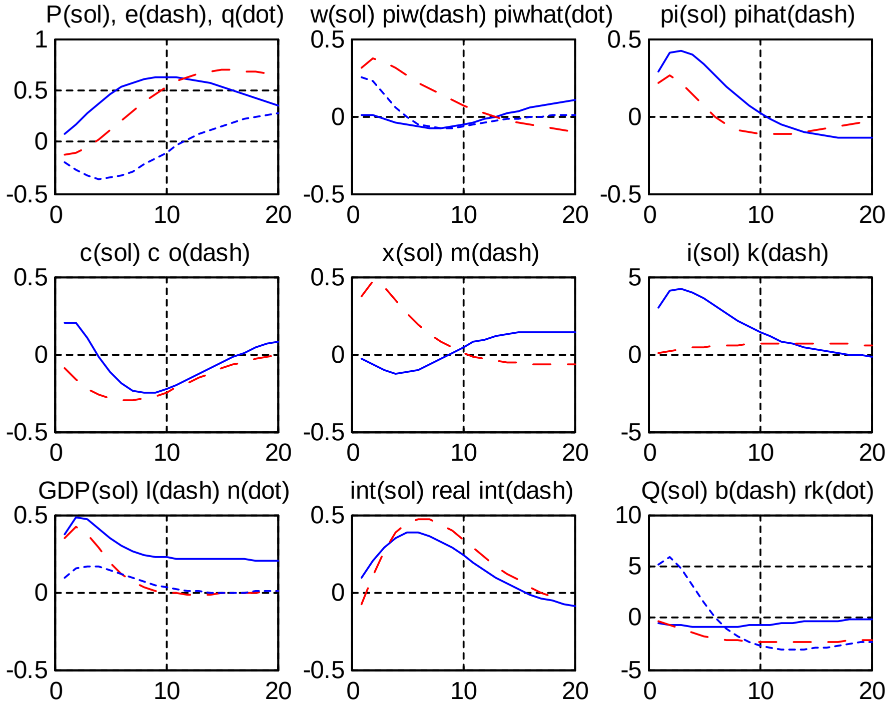
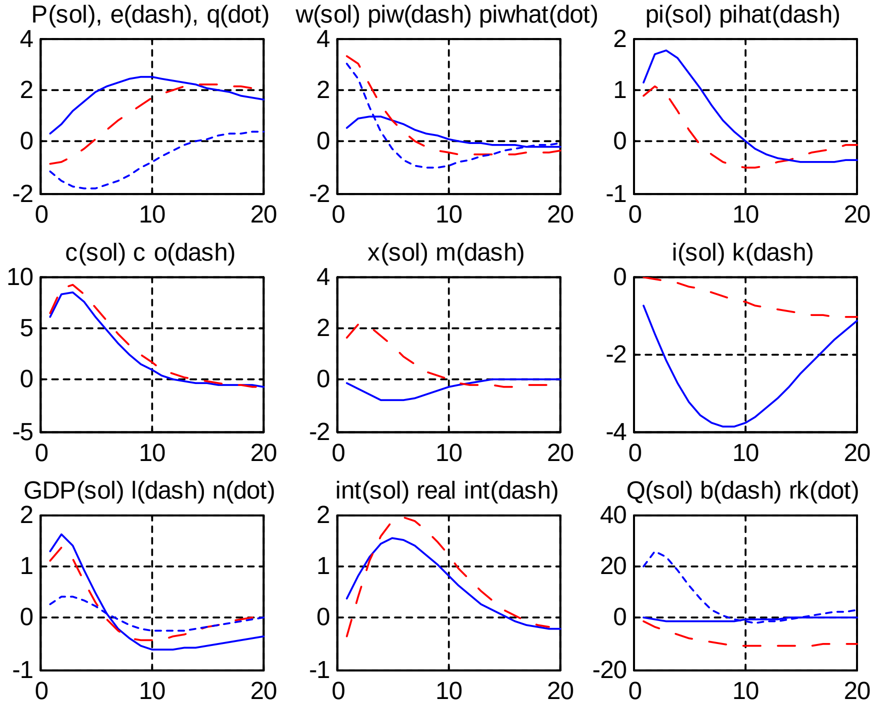


Figure 16
Consumption preference shock



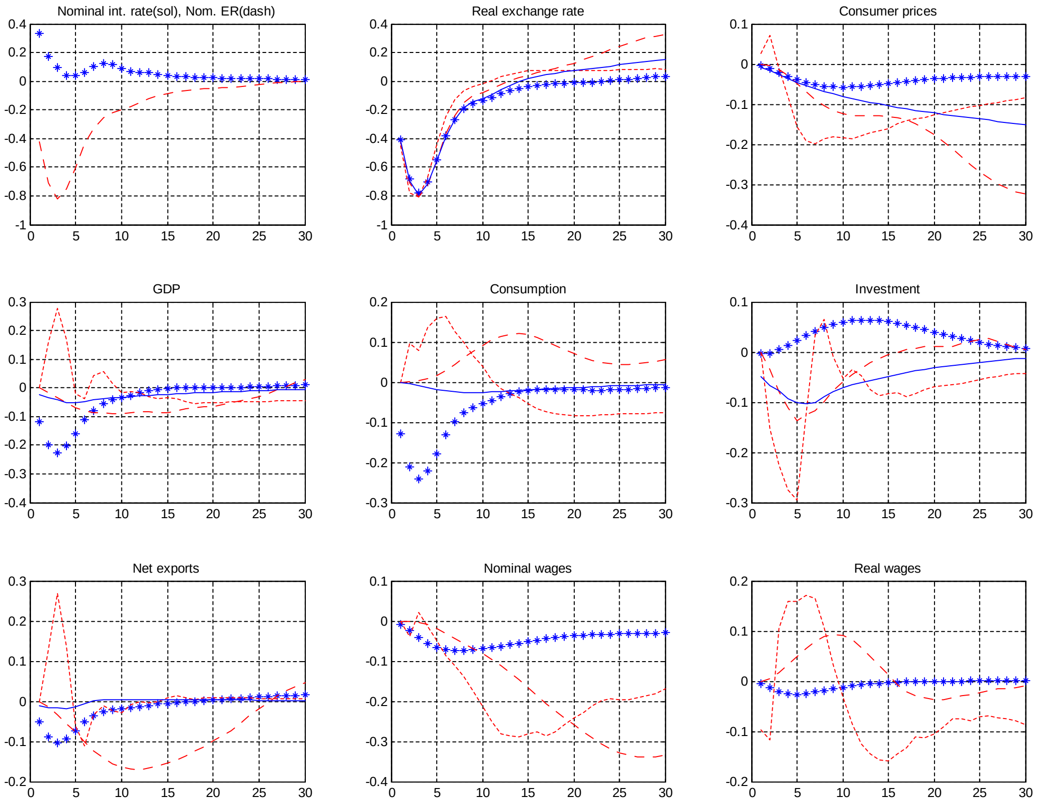
hours always accompanies with a decline of real wages. On the other hand, in the NEM model the decline of nominal wages is smaller than that of prices, hence real wages increase. This phenomenon partly explains the divergent reaction of consumption, since in both models consumption is directly influenced by real wages.

Regarding the behavior of investments one can detect larger qualitative differences. In SVAR, NEM and 5GAP models a monetary tightening is followed by an investment fall. In our model there is only a slight drop in investments in the first few quarters and then they start to increase. The reason for the increase in investment lies on the special shape of the interest rate and nominal exchange rate evolution. As shown in *Figure 7* monetary policy shock leads to lower investments. In this exercise, however, an additional (positive) risk premium shock should enter to capture the exogenously given nominal exchange rate dynamics. Moreover, nominal interest rate is less persistent here. Hence, forward looking agents expect monetary tightening to cease out and thus, the immediate drop in investment is subdued and the positive financial premium shock after the second quarter also pushes investments higher. Furthermore, as there is an adaptive learning on *underlying* inflation, agents perceive monetary tighteting as a temporary decrease in perceived *underlying* inflation, which also reduces rental rate of capital, at least in the medium run.

Finally, there is some difference in the response of net exports: in contrast with NEM net expors only show a transitory decline in the DSGE model.

Figure 17

The effects of a monetary shock in different models for Hungary*



*Our model's results are calculated at mean parameter values, our model: blue stars, SVAR: dotted, NEM: dashed, 5GAP: solid

6 Variance decomposition

Variance decomposition are calculated with parameters describing the inflation targeting regime. Therefore, it is not surprising that the trend depreciation shock (χ_t) does not influence forecast errors, as in this regime this observed shock is constantly kept zero. The results of the unconditional (long run) variance decomposition are summarized in *Table 7*, while forecast error variance decompositions are shown in *Tables 4–6*.

Consumer price inflation is affected in two channels: one through the perceived *underlying* inflation ($\bar{\pi}_t = -d\bar{q}_t$), and one through changes in cyclical inflation ($\hat{\pi}_t$). An interesting feature that in the long run perceived inflation is mostly explained by consumer price markup and labor market shocks. Productivity, investment and preference shocks are also important. As shown before, consumer preference shocks are related to financial deepening or fiscal stimulus (through transfers). That is, inflationary perception is also affected by the changing pattern in consumption demand. The second factor of inflation, the cyclical one is also driven by the above shocks, though here, consumer price markup shocks plays more significant role and the labor market shock is less relevant. That is, consumer price markup shocks have more explanatory power for cyclical inflation, while labor market shocks have more influence on the long term inflationary perception of agents. Cyclical nominal wages ($\hat{\pi}^w_t$) are governed by their own shocks both in the long and in the short run. This might indicate the model has rather limited ability to explain nominal wage fluctuations.[9]

In the long run, the cyclical behaviour of real exchange rate is explained by financial premium, foreign import price, labor market, export demand and export price markup shocks. In the shorter run, however,real exchange rate movements are almost entirely driven by financial premium shocks. though consumer price markup and monetary policy rule shock gain some importance.

In the shorter run the nominal interest rate is explained by consumer price markup and monetary rule shocks. In addition, productivity, preference, financial premium and the labor market shock gain importance in determining monetary policy in the long run. Interestingly, financial premium shocks has only a limited effect on interest rates in the short run. That is, monetary policy tried to react to foreign interest rate fluctuations and changes in risk premium mostly in a longer time span.

Real wages are governed by foreign shocks (foreign demand and export price markup shocks) showing that in a small, open economy, there is a close link between real wages in the export and in the domestic sector. Productivity shocks effects almost all real variables except for investments (consumption, imports, demand for labor and the rental rate of capital ($\tilde{r}^k_t$)). The export-demand shock ($\hat{x}^*_t$) is important in explaining the behavior of exports, import and labor demand of the export sector, real wages, but prices are isolated from this shock. Generally real variables are driven by productivity and export demand shocks. Not surprisingly, this result shows that in a small open economy, like Hungary, these are the prime determinants of output fluctuations. The overwhelming role of investment shock in investments and capital determination might show that the model is not very efficient in explaining investment behavior in Hungary.

In all horizons, financial premium and monetary shocks have only a negligible effects on real variables except for the real exchange rate. They mostly influence the cyclical components of real and nominal exchange rate and the nominal interest rate. Real effects of financial premium and monetary-policy shock are only minor, which is in contrast with eurozone estimates of Smets & Wouters (2003). This conforms to Vonnák (2007) and Jakab *et al.* (2006) that monetary policy's effects in Hungary are rather limited on output.

The government consumption shock plays only a minor role in determining real variables in the short run, the only exception is the import demand of the domestic goods producing sector. In the longer run, imports and labor demand is only influenced in a limited extent by this shock. This might indicate that fiscal policy mostly affected the economy in indirect ways, through transfers, tax and regulated price changes etc., and not by direct purchases of goods and services.

[9] This serves partly as a motivation to extend this model by a more detailed labor market setup. Jakab & Kónya (2007) insert search and matching frictions into a simplified version of this model.

Table 4

Forecast error variance decomposition $t = 1$ (one quarter)*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}$	0.7	0.2	0.0	0.7	8.2	85.0	0.9	0.6	1.1	0.6	0.2	0.0	1.8	0.0	0.0
$\hat{\pi}^x_t$	0.0	0.1	0.0	5.4	5.8	0.0	0.2	88.1	0.1	0.1	0.0	0.0	0.1	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.1	0.0	0.4	0.8	0.2	96.7	0.4	0.6	0.4	0.1	0.0	0.1	0.0	0.0
$\bar{\pi}^{**}_t$	0.7	0.2	0.0	0.7	8.2	85.0	0.9	0.6	1.1	0.6	0.2	0.0	1.8	0.0	0.0
π^{***}_t	0.7	0.2	0.0	0.7	8.2	85.0	0.9	0.6	1.1	0.6	0.2	0.0	1.8	0.0	0.0
$\tilde{q}_t$	0.0	0.6	0.0	3.5	0.2	7.1	0.1	0.0	0.5	81.4	6.2	0.0	0.3	0.0	0.1
$\tilde{e}_t$	0.0	0.7	0.0	3.2	0.0	1.8	0.0	0.1	0.3	86.9	6.8	0.0	0.1	0.0	0.1
$d\tilde{e}_t$	0.0	0.7	0.0	3.2	0.0	1.8	0.0	0.1	0.3	86.9	6.8	0.0	0.1	0.0	0.1
$\hat{i}_t$	0.2	0.1	0.0	0.1	2.6	25.8	0.3	0.2	0.3	1.1	68.7	0.0	0.5	0.0	0.0
$\tilde{c}^{no}_t$	4.8	4.4	0.0	7.3	55.1	3.7	12.9	2.0	0.7	7.4	0.9	0.0	0.9	0.0	0.0
$\tilde{c}^o_t$	0.1	1.0	0.0	1.3	1.2	1.1	2.1	0.9	90.3	0.2	1.3	0.0	0.3	0.0	0.2
$\widetilde{w}_t$	0.0	0.1	0.0	0.2	0.1	5.6	93.1	0.3	0.3	0.2	0.0	0.0	0.0	0.0	0.0
$\tilde{x}_t$	0.0	0.0	0.0	83.4	1.0	0.0	0.0	15.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\widetilde{m}^d_t$	11.6	3.6	0.0	4.3	68.3	0.0	2.0	0.2	2.2	4.6	0.1	0.0	3.2	0.0	0.0
$\widetilde{m}^x_t$	0.6	2.3	0.0	34.3	53.6	0.0	0.9	5.4	0.2	2.5	0.1	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	12.0	3.9	0.0	1.1	69.5	0.7	2.2	0.2	1.1	6.0	0.9	0.0	2.4	0.0	0.0
$\tilde{l}^x_t$	0.4	5.3	0.0	23.9	51.4	0.3	3.7	5.4	0.0	8.7	0.7	0.0	0.0	0.0	0.0
$\tilde{n}_t$	1.3	2.0	0.0	3.5	15.0	0.6	11.1	3.7	0.3	3.8	0.5	57.1	1.1	0.0	0.0
$\tilde{k}_t$	0.0	0.0	0.0	0.0	0.1	0.2	0.1	0.0	0.0	0.0	0.0	0.0	20.9	0.0	78.7
$\tilde{I}_t$	0.0	0.0	0.0	0.0	0.3	0.7	0.3	0.0	0.2	0.2	0.2	0.0	97.9	0.0	0.0
$\tilde{b}_t$	2.9	21.7	0.0	14.7	41.7	0.0	0.9	14.2	0.6	2.5	0.1	0.0	0.8	0.0	0.0
$\widetilde{Q}_t$	0.1	0.3	0.0	2.1	1.9	34.8	0.6	2.6	0.1	2.3	34.3	0.0	12.0	7.9	1.2
$\tilde{r}^k_t$	6.3	1.1	0.0	11.0	74.0	0.8	0.3	2.1	0.7	2.1	0.4	0.0	1.3	0.0	0.0
P^x_t	0.0	0.1	0.0	5.4	5.8	0.0	0.2	88.1	0.1	0.1	0.0	0.0	0.1	0.0	0.0

* Calculated at mean parameter values.

** $d\bar{q}_t = -\bar{\pi}_t$ in IT.

*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

Table 5

Forecast error variance decomposition $t = 4$ (one year)*

	$\tilde{g}_t$	$\widetilde{P}_t^{m*}$	$\tilde{\chi}_t$	$\tilde{x}_t^*$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}_t^w$	$\tilde{\upsilon}_t^x$	$\tilde{\varepsilon}_t^c$	$\tilde{\varepsilon}_t^{pr}$	$\tilde{\varepsilon}_t^r$	$\tilde{\varepsilon}_t^n$	$\tilde{\varepsilon}_t^I$	$\tilde{\varepsilon}_t^Q$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}$	0.8	0.2	0.0	1.3	9.9	71.5	3.2	1.8	3.5	1.3	0.6	0.0	5.8	0.0	0.1
$\hat{\pi}_t^x$	0.0	0.1	0.0	9.0	6.2	0.0	0.5	83.6	0.1	0.2	0.0	0.0	0.1	0.0	0.0
$\hat{\pi}_t^w$	0.1	0.1	0.0	0.4	0.7	0.3	96.2	0.7	0.7	0.5	0.1	0.0	0.2	0.0	0.0
$\bar{\pi}_t^{**}$	1.0	0.3	0.0	2.1	13.5	57.7	4.8	2.9	5.5	2.1	0.9	0.0	9.0	0.0	0.1
π_t^{***}	0.9	0.3	0.0	1.8	11.6	62.0	4.6	2.6	5.1	1.8	0.9	0.0	8.4	0.0	0.1
$\tilde{q}_t$	0.1	0.9	0.0	6.7	0.8	11.0	0.5	0.0	1.4	72.8	4.4	0.0	1.2	0.0	0.3
$\tilde{e}_t$	0.0	1.4	0.0	5.0	0.2	1.2	0.0	0.3	0.3	85.4	5.7	0.0	0.1	0.0	0.2
$d\tilde{e}_t$	0.0	0.7	0.0	2.9	0.1	2.3	0.1	0.1	0.3	86.2	7.0	0.0	0.2	0.0	0.1
$\hat{\imath}_t$	0.6	0.4	0.0	0.8	8.5	34.1	2.9	1.9	3.1	4.9	37.4	0.0	5.3	0.0	0.0
$\tilde{c}_t^{no}$	1.8	2.7	0.0	6.2	21.3	6.8	44.4	8.0	1.2	5.7	0.7	0.0	1.1	0.0	0.0
$\tilde{c}_t^o$	0.2	2.0	0.0	2.1	2.5	1.8	4.4	1.9	82.4	0.5	1.0	0.0	0.8	0.0	0.4
$\widetilde{w}_t$	0.0	0.1	0.0	0.2	0.1	3.3	95.5	0.4	0.2	0.3	0.0	0.0	0.0	0.0	0.0
$\tilde{x}_t$	0.0	0.1	0.0	47.9	4.6	0.0	0.5	46.3	0.1	0.2	0.0	0.0	0.1	0.0	0.0
$\widetilde{m}_t^d$	7.4	5.0	0.0	7.5	46.0	0.5	10.1	1.8	7.0	5.4	0.1	0.0	9.3	0.0	0.0
$\widetilde{m}_t^x$	0.3	2.5	0.0	36.7	25.9	0.0	2.9	29.5	0.3	1.7	0.1	0.0	0.2	0.0	0.0
$\tilde{l}_t^d$	8.1	3.5	0.0	1.4	49.2	2.1	17.1	1.0	3.3	6.1	1.2	0.0	7.1	0.0	0.0
$\tilde{l}_t^x$	0.2	3.5	0.0	21.3	22.6	0.3	19.0	25.1	0.2	7.3	0.5	0.0	0.0	0.0	0.0
$\tilde{n}_t$	0.4	1.4	0.0	2.0	5.0	0.8	25.3	5.9	0.3	3.1	0.4	54.0	1.3	0.0	0.0
$\tilde{k}_t$	0.0	0.0	0.0	0.0	0.3	0.7	0.4	0.0	0.3	0.3	0.1	0.0	43.6	0.0	54.3
$\tilde{I}_t$	0.1	0.1	0.0	0.0	0.8	1.7	1.3	0.2	0.8	0.8	0.2	0.0	93.9	0.0	0.1
$\tilde{b}_t$	1.3	18.3	0.0	31.6	18.2	0.0	3.0	22.5	1.3	2.2	0.0	0.0	1.6	0.0	0.0
$\widetilde{Q}_t$	0.6	0.7	0.0	0.8	9.1	33.3	3.3	1.1	2.0	4.4	15.8	0.0	25.7	2.1	1.1
$\tilde{r}_t^k$	4.2	1.0	0.0	16.3	51.8	2.6	1.1	14.9	2.1	2.7	0.6	0.0	2.7	0.0	0.1
P_t^x	0.0	0.2	0.0	16.0	7.6	0.0	1.0	74.4	0.2	0.3	0.0	0.0	0.2	0.0	0.0

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

Table 6

Forecast error variance decomposition $t = 10$ (ten quarters)*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}$	0.9	0.3	0.0	1.3	11.4	68.8	3.3	2.4	3.7	1.3	0.6	0.0	5.9	0.0	0.1
$\hat{\pi}^x_t$	0.1	0.2	0.0	10.1	6.8	0.0	0.6	81.7	0.2	0.2	0.0	0.0	0.2	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.1	0.0	0.5	0.8	0.3	95.8	0.9	0.7	0.5	0.1	0.0	0.2	0.0	0.0
$\bar{\pi}^{**}_t$	0.8	0.3	0.0	2.6	10.5	38.1	10.1	3.8	10.3	3.1	1.8	0.0	18.1	0.0	0.5
π^{***}_t	0.8	0.3	0.0	1.8	10.8	56.2	6.2	2.8	6.4	2.0	1.1	0.0	11.1	0.0	0.3
$\tilde{q}_t$	0.1	2.3	0.0	12.3	0.8	11.0	0.9	0.2	2.5	63.6	3.5	0.0	2.2	0.0	0.7
$\tilde{e}_t$	0.2	3.2	0.0	6.6	2.9	3.0	1.5	2.1	0.9	72.3	4.7	0.0	2.3	0.0	0.3
$d\tilde{e}_t$	0.0	0.7	0.0	2.8	0.3	2.5	0.3	0.2	0.5	85.3	6.9	0.0	0.5	0.0	0.1
$\hat{\imath}_t$	0.6	0.4	0.0	0.9	7.9	25.9	7.6	3.2	7.2	7.9	24.8	0.0	13.4	0.0	0.2
$\tilde{c}^{no}_t$	1.2	2.0	0.0	4.5	15.1	7.3	56.1	6.9	0.9	4.5	0.5	0.0	0.8	0.0	0.1
$\tilde{c}^o_t$	0.3	4.2	0.0	4.9	4.3	1.9	10.2	4.2	65.9	0.6	0.7	0.0	2.0	0.0	0.9
$\widetilde{w}_t$	0.0	0.1	0.0	0.1	0.2	2.8	95.8	0.4	0.2	0.3	0.0	0.0	0.0	0.0	0.1
$\tilde{x}_t$	0.0	0.2	0.0	41.2	5.7	0.0	1.9	49.9	0.4	0.4	0.0	0.0	0.2	0.0	0.1
$\widetilde{m}^d_t$	6.2	6.3	0.0	7.6	39.4	1.4	13.3	2.0	7.6	5.5	0.1	0.0	10.5	0.0	0.1
$\widetilde{m}^x_t$	0.2	2.8	0.0	33.4	23.9	0.0	3.9	33.6	0.3	1.5	0.1	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	6.1	2.7	0.0	1.2	37.2	2.4	34.3	1.1	2.8	4.6	1.0	0.0	6.6	0.0	0.0
$\tilde{l}^x_t$	0.1	2.9	0.0	16.9	16.9	0.2	33.1	22.8	0.5	6.0	0.4	0.0	0.2	0.0	0.0
$\tilde{n}_t$	0.2	1.0	0.0	1.2	2.3	0.8	42.8	4.1	0.2	2.0	0.2	44.3	1.0	0.0	0.0
$\tilde{k}_t$	0.1	0.1	0.0	0.0	1.0	1.8	2.1	0.3	1.3	1.1	0.1	0.0	58.7	0.0	33.4
$\tilde{I}_t$	0.1	0.3	0.0	0.0	1.7	3.0	4.7	0.7	3.0	2.2	0.2	0.0	83.6	0.0	0.4
$\tilde{b}_t$	0.7	17.7	0.0	40.0	10.5	0.2	7.1	16.3	2.1	2.6	0.0	0.0	2.8	0.0	0.0
$\widetilde{Q}_t$	0.4	0.6	0.0	0.6	7.1	20.8	6.1	2.0	4.2	4.5	8.6	0.0	43.1	1.1	0.8
$\tilde{r}^k_t$	4.0	0.9	0.0	15.4	49.0	3.6	1.1	16.8	2.3	2.7	0.6	0.0	3.1	0.0	0.6
P^x_t	0.0	0.2	0.0	22.4	7.3	0.0	2.2	66.5	0.4	0.4	0.0	0.0	0.2	0.0	0.2

* Calculated at mean parameter values.

** $d\bar{q}_t = -\bar{\pi}_t$ in IT.

*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

Table 7

Unconditional variance decomposition*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}$	0.8	0.4	0.0	1.3	11.0	65.2	4.1	2.6	4.4	1.4	0.7	0.0	8.0	0.0	0.1
$\hat{\pi}^x_t$	0.1	0.2	0.0	10.2	6.8	0.0	0.6	81.6	0.2	0.2	0.0	0.0	0.2	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.1	0.0	0.5	0.7	0.3	95.8	0.9	0.8	0.5	0.1	0.0	0.2	0.0	0.0
$\bar{\pi}^{**}_t$	0.7	0.8	0.0	2.4	10.0	32.2	10.5	4.1	9.9	2.6	1.8	0.0	23.6	0.0	1.3
π^{***}_t	0.8	0.6	0.0	1.7	10.6	51.6	6.8	3.2	6.7	1.9	1.2	0.0	14.4	0.0	0.6
$\tilde{q}_t$	0.3	12.9	0.0	30.8	4.4	4.2	9.1	6.5	2.4	23.8	1.2	0.0	3.8	0.0	0.8
$\tilde{e}_t$	0.4	5.6	0.0	7.3	6.4	4.5	13.7	5.8	6.5	33.3	2.7	0.0	11.9	0.0	1.9
$d\tilde{e}_t$	0.0	0.7	0.0	2.8	0.3	2.5	0.3	0.2	0.5	84.9	6.8	0.0	0.8	0.0	0.2
$\hat{\imath}_t$	0.5	0.6	0.0	1.4	7.5	24.3	7.9	3.1	7.1	7.7	22.5	0.0	16.6	0.0	0.8
$\tilde{c}^{no}_t$	1.1	3.2	0.0	7.6	13.6	6.8	52.3	6.7	1.0	4.5	0.5	0.0	2.2	0.0	0.6
$\tilde{c}^o_t$	0.4	13.1	0.0	24.6	6.3	0.7	17.7	8.4	24.3	1.6	0.2	0.0	1.0	0.0	1.6
$\widetilde{w}_t$	0.0	0.3	0.0	0.6	0.2	2.8	93.3	0.5	0.2	0.4	0.0	0.0	1.0	0.0	0.5
$\tilde{x}_t$	0.1	1.2	0.0	40.7	5.6	0.1	2.5	46.7	0.5	0.7	0.0	0.0	1.5	0.0	0.3
$\widetilde{m}^d_t$	4.3	10.4	0.0	17.9	28.4	1.2	13.6	4.9	6.1	4.9	0.1	0.0	8.1	0.0	0.3
$\widetilde{m}^x_t$	0.2	3.1	0.0	33.4	23.6	0.0	4.1	33.3	0.3	1.6	0.1	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	5.7	2.5	0.0	1.2	34.9	2.3	37.5	1.2	2.8	4.5	0.9	0.0	6.4	0.0	0.1
$\tilde{l}^x_t$	0.2	5.8	0.0	21.0	14.8	0.3	30.7	20.0	1.0	5.4	0.3	0.0	0.5	0.0	0.1
$\tilde{n}_t$	0.2	2.8	0.0	5.3	2.6	0.7	43.8	4.3	0.6	1.9	0.2	36.7	0.8	0.0	0.1
$\tilde{k}_t$	0.1	0.6	0.0	0.2	2.1	2.9	11.2	1.2	6.1	3.8	0.1	0.0	54.5	0.0	17.3
$\tilde{I}_t$	0.1	0.3	0.0	0.1	1.9	3.2	7.9	1.0	4.7	3.1	0.2	0.0	76.4	0.0	1.1
$\tilde{b}_t$	0.5	18.3	0.0	38.6	7.1	0.4	15.9	10.7	3.2	3.5	0.0	0.0	1.3	0.0	0.5
$\widetilde{Q}_t$	0.4	0.6	0.0	0.5	6.3	18.4	6.0	1.9	4.1	4.2	7.6	0.0	48.3	1.0	0.7
$\tilde{r}^k_t$	3.6	1.3	0.0	14.8	45.1	3.4	2.3	15.7	2.5	2.6	0.6	0.0	6.7	0.0	1.3
P^x_t	0.1	1.3	0.0	23.3	7.1	0.1	2.7	62.1	0.5	0.8	0.0	0.0	1.6	0.0	0.3

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

7 An alternative model without real time adaptive learning

Disinflation was endogenously detemined by an adaptive learning mechanism in the model outlined above (henceforth called *Baseline Model*) In the *baseline model* the underlying component of inflation was made endogenous by introducing an adaptive learning scheme.

It should be emphasised that the solution in the *Baseline model* does not assume non-rationality: agents take into account that inflation has a permanent component (on top of the exogenously set rate of currency depreciation in the crawling peg regime) and all agents fully index their prices and wages to it first. Optimizing agents set their prices and wages to the optimal level, so this is simply a convenient way of writing Phillips-curves.

As mentioned before, this solution enabled us to explain long term variance of inflation without adding an extra shock. A natural question arises: what are the consequences of choosing this type of 'filtering'. One can suspect that inserting the adaptive learning of '*perceived underlying inflation*' would have created an '*intrinsic*' inertia in both price and wage setting and indexation parameters are estimated to be low. For this purpose, we estimated an alternative model which filters inflation in a different way. The '*intrinsic*' inertia in price and wage setting was switched off and an '*extrinsic*' shock was introduced .

As an alternative model, we experimented with estimating the model by estimating the shock of *'perceived underlying inflation'* with Bayesian methods. For this, raw inflation data were simply demeaned after substracting the exogenous rate of crawl. Then, equation (72) was switched off and a simple equation determining the change in '*underlying*' inflation (see equation (53)) was used. This way, the model was estimated on the same data set as the *Baseline Model.*

$$d\bar{q}_t = \tilde{\chi}_t, \tag{53}$$

As mentioned before, in *the alternative model* the shocks to the *'perceived underlying inflation'* was estimated and the learning rule was switched off. Apart from this, the model has the same properties as the baseline model. *The alternative model* was then estimated by Bayesian method with exactly the same prior distributions and number of draws that of the *Baseline Model*. The only exception is that the gain parameter (g) was set to zero and that the standard error of the 'perceived underlying inflation' shock was given a prior of Inverse Gamma distribution with mean 0.5 and degrees of freedom of 2. *Table 8* and *9* show the estimation results of the alternative model.

Almost all estimated structural parameters in *the alternative model* were found to be very close to that in the *Baseline Model*. The only slight difference is a lower degree of indexation in consumer prices for the inflation targeting regime. Hence, one can conclude that the role of *intrinsic*' inertia in the *Baseline Model* was not generated by the way of how 'perceived underlying' inflation is formed.

Table 8

Estimated parameters of shocks in the alternative model

		Prior distribution			Baseline model Estimated posterior			Alternative model Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.	Mode	Mean	90% prob. int.
Standard errors										
productivity	σ_A	I.Gam.	0.5	2*	2.080	2.152	[1.82,2.55]	2.037	2.110	[1.78,2.51]
export demand	σ_x	I.Gam.	0.5	2*	2.352	2.464	[2.06,2.95]	2.345	2.456	[2.07,2.92]
cons. pref.	σ_c	I.Gam.	0.5	2*	0.146	0.203	[0.11,0.33]	0.151	0.213	[0.12,0.33]
cons. price markup	σ_p	I.Gam.	0.5	2*	0.397	0.420	[0.34,0.52]	0.351	0.380	[0.28,0.50]
export price markup	σ_{px}	I.Gam.	0.5	2*	1.873	2.182	[1.64,2.87]	1.862	2.198	[1.66,2.92]
labor market	σ_w^{cr}	I.Gam.	0.5	2*	0.824	0.922	[0.65,1.26]	0.755	0.862	[0.59,1.21]
labor market	σ_w^{it}	I.Gam.	0.5	2*	0.609	0.932	[0.54,1.42]	0.662	1.016	[0.58,1.57]
investments	σ_I	I.Gam.	0.5	2*	0.969	1.003	[0.76,1.27]	0.945	0.979	[0.74,1.24]
Equity premium	σ_Q	I.Gam.	0.5	2*	0.167	0.393	[0.11,0.99]	0.167	0.393	[0.10,1.30]
policy rule	σ_r^{cr}	I.Gam.	0.5	2*	0.574	0.602	[0.48,0.76]	0.573	0.601	[0.48,0.76]
policy rule	σ_r^{it}	I.Gam.	0.5	2*	0.229	0.247	[0.19,0.32]	0.190	0.210	[0.13,0.30]
fin. premium	σ_{pr}^{cr}	I.Gam.	0.5	2*	0.221	0.372	[0.17,0.67]	0.237	0.366	[0.17,0.65]
fin. premium	σ_{pr}^{it}	I.Gam.	0.5	2*	0.486	0.666	[0.36,1.06]	0.340	0.506	[0.27,0.85]
employment	σ_n	I.Gam.	0.5	2*	0.369	0.383	[0.30,0.49]	0.341	0.357	[0.27,0.46]
perceived *underlying* inflation	σ_χ	I.Gam.	0.5	2*		Fixed at 0.12		0.415	0.456	[0.29,0.67]
Autoregressive coefficients										
productivity	ρ_A	Beta	0.8	0.1	0.577	0.552	[0.43,0.68]	0.530	0.507	[0.38,0.63]
export demand	ρ_x	Beta	0.8	0.1	0.616	0.625	[0.51,0.73]	0.617	0.621	[0.50,0.73]
cons. pref.	ρ_c	Beta	0.8	0.1	0.833	0.767	[0.60,0.88]	0.830	0.758	[0.60,0.88]
labor market	ρ_w^{cr}	Beta	0.8	0.1	0.486	0.480	[0.32,0.64]	0.501	0.498	[0.33,0.66]
labor market	ρ_w^{it}	Beta	0.8	0.1	0.797	0.661	[0.43,0.87]	0.779	0.656	[0.44,0.86]
export markup	ρ_x	Beta	0.5	0.15	0.317	0.318	[0.15,0.50]	0.308	0.308	[0.15,0.49]
investments	ρ_I	Beta	0.8	0.1	0.497	0.488	[0.33,0.65]	0.490	0.484	[0.33,0.65]
fin. premium	ρ_{pr}^{cr}	Beta	0.8	0.1	0.899	0.833	[0.69,0.93]	0.895	0.837	[0.71,0.93]
fin. premium	ρ_{pr}^{it}	Beta	0.8	0.1	0.872	0.820	[0.70,0.92]	0.907	0.853	[0.75,0.94]
employment	ρ_n	Beta	0.8	0.1	0.790	0.770	[0.64,0.89]	0.824	0.796	[0.67,0.91]
perceived *underlying* inflation	σ_χ		Fixed			0.53			0	

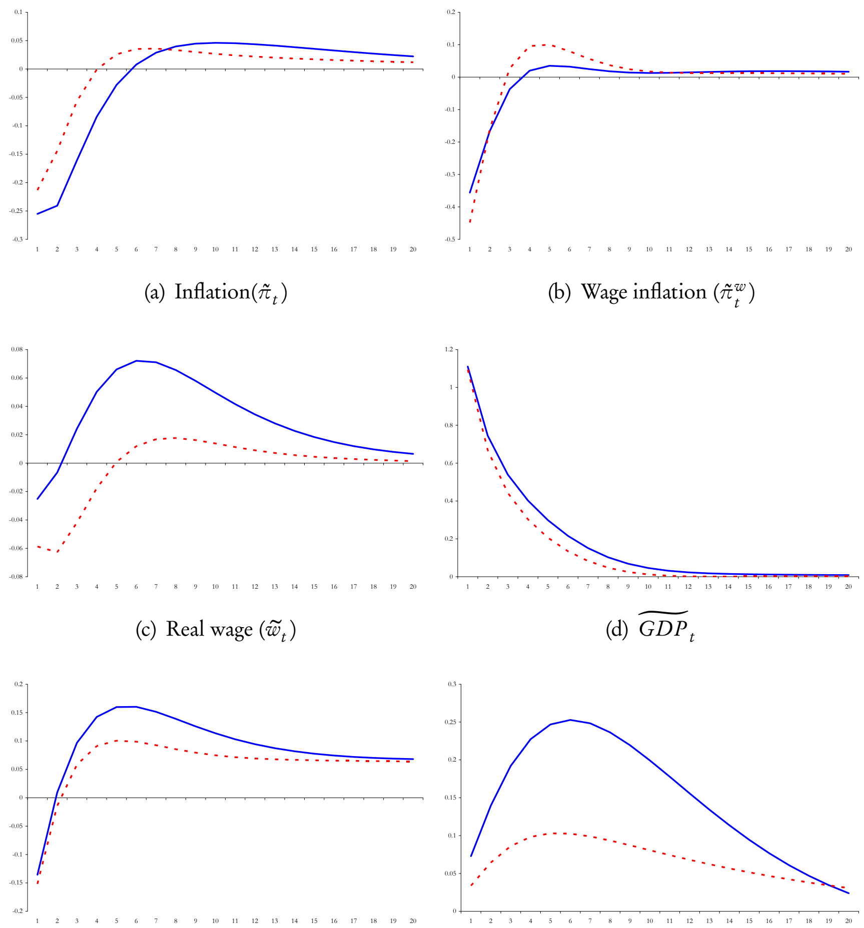
Table 9

Estimated structural parameters in the alternative model

		Prior distribution			Baseline model Estimated posterior			Alternative model Estimated posterior		
		Type	Mean	Stand. err.	Mode	Mean	90% prob. int.	Mode	Mean	90% prob. int.
Utility function parameters										
consumption	σ	Norm.	2.00	0.40	1.680	1.814	[1.18,2.46]	1.812	1.807	[1.16,2.47]
habit	h	Beta	0.75	0.15	0.597	0.646	[0.45,0.83]	0.546	0.619	[0.42,0.81]
Price and wage setting parameters										
ind. cons. prices	ϑ_p^{cr}	Beta	0.60	0.20	0.873	0.821	[0.63,0.96]	0.783	0.756	[0.52,0.94]
ind. cons. prices	ϑ_p^{it}	Beta	0.60	0.20	0.416	0.431	[0.22,0.66]	0.280	0.328	[0.13,0.56]
ind. exp. prices	ϑ_x^{cr}	Beta	0.60	0.20	0.215	0.290	[0.10,0.55]	0.201	0.281	[0.10,0.53]
ind. exp. prices	ϑ_x^{it}	Beta	0.60	0.20	0.383	0.494	[0.18,0.83]	0.380	0.481	[0.18,0.82]
ind. wages	ϑ_w^{cr}	Beta	0.60	0.20	0.112	0.169	[0.05,0.34]	0.105	0.166	[0.05,0.34]
ind. wages	ϑ_w^{it}	Beta	0.60	0.20	0.107	0.185	[0.05,0.40]	0.121	0.211	[0.06,0.44]
Calvo cons. prices	γ_p^{cr}	Beta	0.50	0.20	0.939	0.938	[0.92,0.96]	0.924	0.918	[0.89,0.94]
Calvo cons. prices	γ_p^{it}	Beta	0.50	0.20	0.929	0.921	[0.88,0.95]	0.932	0.922	[0.88,0.96]
Calvo exp. prices	γ_x^{cr}	Beta	0.50	0.20	0.850	0.824	[0.75,0.89]	0.851	0.823	[0.74,0.90]
Calvo exp. prices	γ_x^{it}	Beta	0.50	0.20	0.827	0.810	[0.73,0.89]	0.829	0.808	[0.73,0.89]
Calvo wages	γ_w^{cr}	Beta	0.50	0.20	0.714	0.698	[0.58,0.81]	0.690	0.662	[0.55,0.77]
Calvo wages	γ_w^{it}	Beta	0.50	0.20	0.711	0.657	[0.46,0.88]	0.679	0.617	[0.44,0.81]
Other parameters										
exp. elasticity	θ_x	Beta	0.50	0.10	0.510	0.534	[0.40,0.67]	0.504	0.530	[0.40,0.67]
exp. smooth.	h_x	Beta	0.75	0.15	0.503	0.507	[0.35,0.66]	0.486	0.492	[0.34,0.64]
ir. smooth.	ζ_i	U(0,1)	0.50	0.29	0.766	0.761	[0.67,0.84]	0.675	0.679	[0.53,0.80]
policy rule	ζ_π	Norm.	1.50	0.16	1.375	1.379	[1.12,1.65]	1.417	1.394	[1.14,1.65]
gain	g	Beta	0.167	0.03	0.229	0.234	[0.17,0.30]		fixed at 0	

Figure 18

Productivity shock in the baseline model and in the alternative model*



(a) Inflation($\tilde{\pi}_t$) (b) Wage inflation ($\tilde{\pi}_t^w$)

(c) Real wage ($\widetilde{w}_t$) (d) $\widetilde{GDP}_t$

(e) Consumption ($\tilde{c}_t$) (f) Investments ($\tilde{I}_t$)

* solid: baseline model, dashed: alternative model

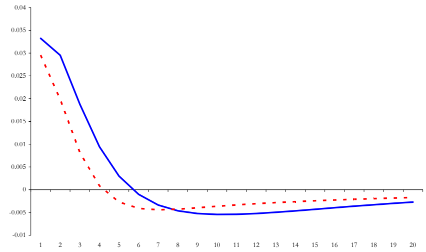
7.1 A COMPARATIVE IMPULSE RESPONSE ANALYSIS

A comparative impulse response analysis between the *alternative* and the *baseline* model is also performed with the same setting of shocks as described before (see *Figure 17 - 21*). Generally, most of the impulse response functions are close to each other. One can observe slight differences in nominal wage reactions: in the *alternative* model wages are somewhat more flexible. Moreover, consumer prices in the *alternative* model generally respond to a lesser extent. This can be explained by two factors. First, the degree of price indexation is somewhat higher in the *alternative* model. Therefore, prices move less in the short run as inflation changes only gradually. The second factor lies exactly on the learning properties of *'perceived underlying inflation'*. While in the *baseline* model, *'perceived underlying inflation'* is also modified for a prolonged period of time, it does not change in the *alternative* model. Therefore, the *alternative* model generally features a more modest reaction in prices. In turn, this also modifies the evolution of real wages.

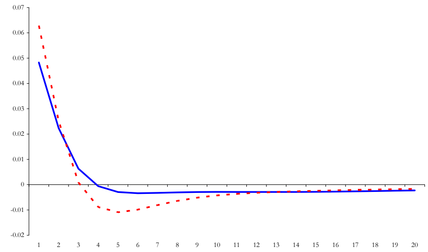
A monetary policy rule shock leads to a weaker drop in inflation in the *alternative* model. The evolution of real wages are markedly different under productivity, government spending shocks. It can also be observed that investments respond to a smaller extent in the *alternative* than in the *baseline model*.

Figure 19

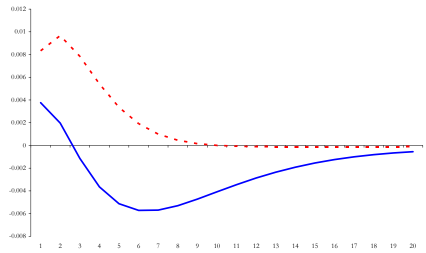
Gov't spending shock in the baseline model and in the alternative model*



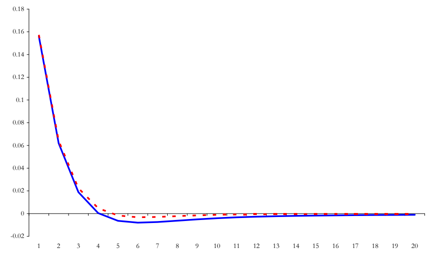
(a) Inflation ($\tilde{\pi}_t$)



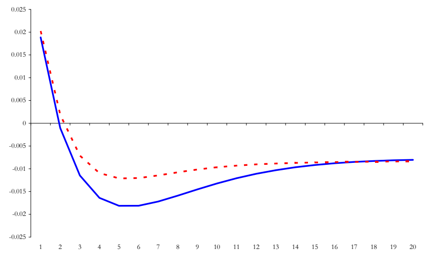
(b) Wage inflation ($\tilde{\pi}_t^w$)



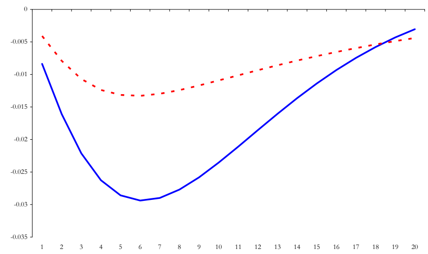
(c) Real wage ($\widetilde{w}_t$)



(d) $\widetilde{GDP}_t$



(e) Consumption ($\tilde{c}_t$)

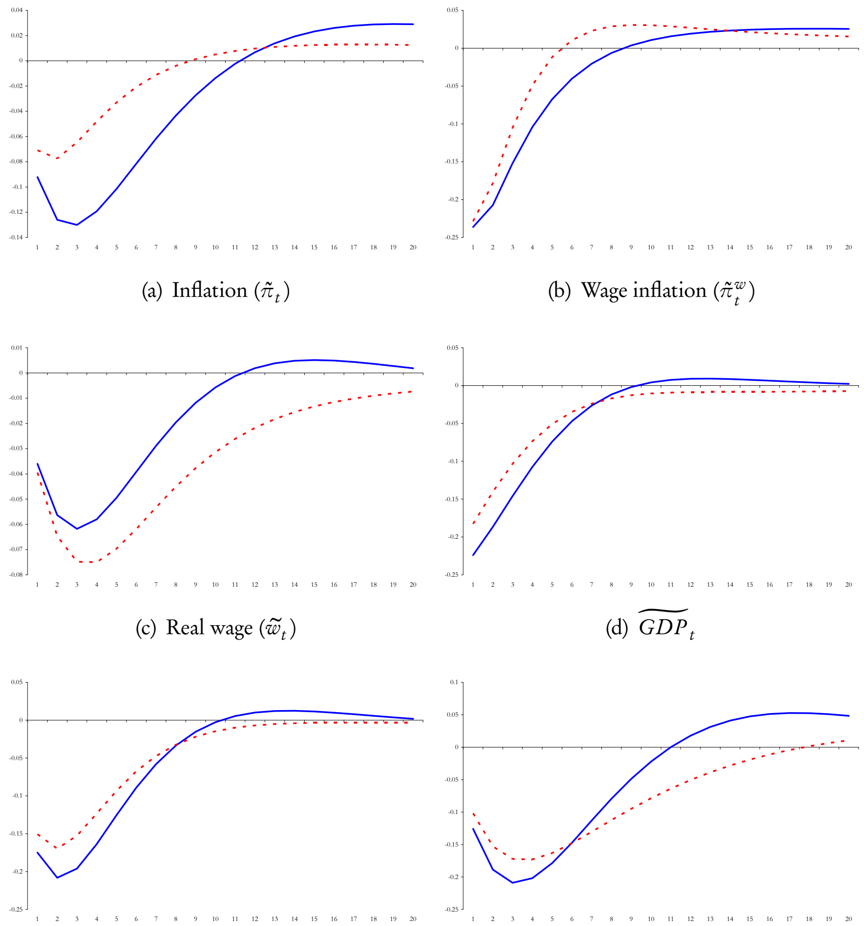


(f) Investments ($\tilde{I}_t$)

* solid: baseline model, dashed: alternative model

Figure 20

Monetary policy rule shock in the baseline model and in the alternative model*



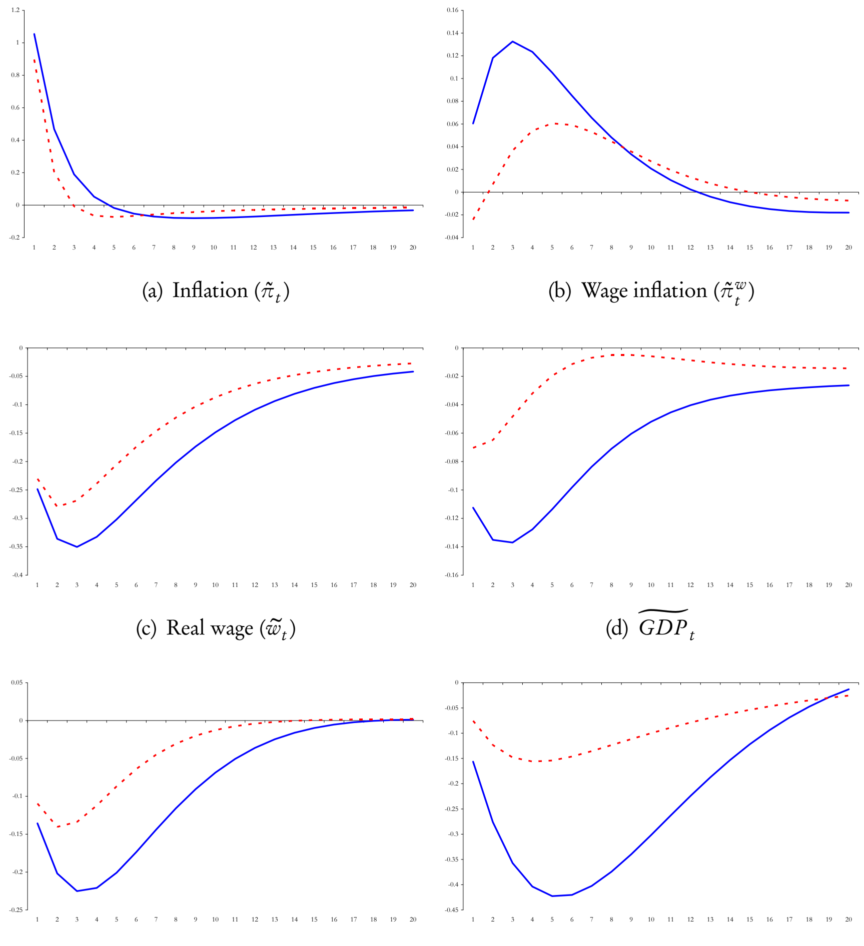
(a) Inflation ($\tilde{\pi}_t$)　(b) Wage inflation ($\tilde{\pi}_t^w$)

(c) Real wage ($\widetilde{w}_t$)　(d) $\widetilde{GDP}_t$

(e) Consumption ($\tilde{c}_t$)　(f) Investments ($\tilde{I}_t$)

*solid: baseline model, dashed: alternative model

Figure 21
Consumer price markup shock in the baseline model and in the alternative model*



(a) Inflation ($\tilde{\pi}_t$) (b) Wage inflation ($\tilde{\pi}^w_t$)

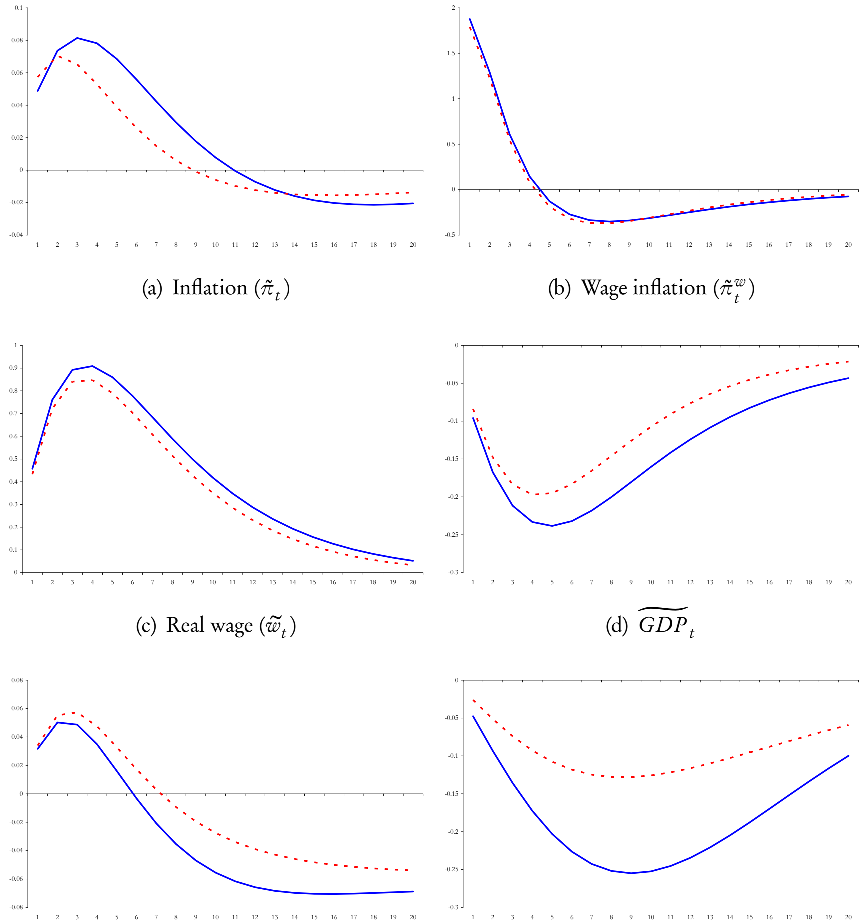
(c) Real wage ($\widetilde{w}_t$) (d) $\widetilde{GDP}_t$

(e) Consumption ($\tilde{c}_t$) (f) Investments ($\tilde{I}_t$))

*solid: baseline model, dashed: alternative model

Figure 22

Labor market shock in the baseline model and in the alternative model*



(a) Inflation ($\tilde{\pi}_t$)

(b) Wage inflation ($\tilde{\pi}_t^w$)

(c) Real wage ($\widetilde{w}_t$)

(d) $\widetilde{GDP}_t$

(e) Consumption ($\tilde{c}_t$)

(f) Investments ($\tilde{I}_t$)

*solid: baseline model, dashed: alternative model

Table 10

Forecast error variance decomposition, model without adaptive learning $t = 1$ (one quarter)*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^p_t r$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}_t$	0.7	0.2	7.7	1.1	7.8	71.8	2.1	1.0	2.2	1.1	0.1	0.0	4.1	0.0	0.1
$\hat{\pi}^x_t$	0.1	0.1	0.0	5.5	5.4	0.0	0.2	88.5	0.1	0.1	0.0	0.0	0.1	0.0	0.0
$\hat{\pi}^w_t$	0.2	0.2	0.6	0.8	1.6	0.0	93.5	0.8	1.1	0.7	0.1	0.0	0.5	0.0	0.0
$\bar{\pi}^{**}_t$	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π^{***}_t	0.5	0.1	34.5	0.8	5.5	50.9	1.5	0.7	1.5	0.8	0.1	0.0	2.9	0.0	0.0
$\tilde{q}_t$	0.0	1.0	0.6	4.1	0.0	4.1	0.0	0.2	0.3	85.5	3.9	0.0	0.1	0.0	0.2
$\tilde{e}_t$	0.0	1.1	0.4	3.6	0.1	0.7	0.0	0.4	0.1	89.3	4.2	0.0	0.0	0.0	0.2
$d\tilde{e}_t$	0.0	1.1	0.4	3.6	0.1	0.7	0.0	0.4	0.1	89.3	4.2	0.0	0.0	0.0	0.2
$\hat{\imath}_t$	0.3	0.1	0.8	0.3	3.7	32.0	1.0	0.5	0.9	2.3	56.0	0.0	1.9	0.0	0.0
$\tilde{c}^{no}_t$	5.0	4.6	0.1	7.7	55.8	1.9	13.9	2.4	1.0	6.0	0.5	0.0	1.2	0.0	0.0
$\tilde{c}^o_t$	0.1	1.2	0.0	1.5	0.9	0.5	2.0	1.0	91.2	0.4	0.7	0.0	0.3	0.0	0.2
$\widetilde{w}_t$	0.0	0.2	0.0	0.5	0.5	3.7	93.5	0.5	0.5	0.4	0.0	0.0	0.1	0.0	0.0
$\tilde{x}_t$	0.0	0.0	0.0	83.6	0.9	0.0	0.0	15.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\widetilde{m}^d_t$	12.0	3.7	0.0	4.5	68.2	0.0	2.2	0.2	2.3	3.7	0.0	0.0	3.1	0.0	0.0
$\widetilde{m}^x_t$	0.6	2.3	0.0	35.2	53.2	0.0	0.9	5.4	0.2	2.0	0.1	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	12.5	4.0	0.1	1.2	70.6	0.2	2.1	0.3	1.3	4.7	0.5	0.0	2.7	0.0	0.0
$\tilde{l}^x_t$	0.5	5.5	0.0	24.6	52.5	0.1	3.8	5.7	0.0	6.9	0.4	0.0	0.0	0.0	0.0
$\tilde{n}_t$	1.4	2.2	0.1	3.7	15.7	0.1	10.4	3.9	0.4	3.4	0.2	56.9	1.5	0.0	0.0
$\tilde{k}_t$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.9	0.0	78.9
$\tilde{I}_t$	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.1	0.2	0.1	0.0	99.2	0.0	0.0
$\tilde{b}_t$	3.0	22.1	0.0	14.9	41.3	0.0	0.9	14.5	0.6	2.0	0.0	0.0	0.7	0.0	0.0
$\widetilde{Q}_t$	0.0	0.2	1.8	3.9	0.8	26.4	0.4	5.4	0.0	4.1	27.6	0.0	14.6	13.3	1.5
$\tilde{r}^k_t$	6.4	1.2	0.0	11.3	73.8	0.4	0.4	2.2	0.9	1.7	0.2	0.0	1.4	0.0	0.0
P^x_t	0.1	0.1	0.0	5.5	5.4	0.0	0.2	88.5	0.1	0.1	0.0	0.0	0.1	0.0	0.0

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

7.2 VARIANCE DECOMPOSITION IN THE ALTERNATIVE MODEL

As far as variance decompositions are concerned, one can observe that inflation is highly determined by the shock to the *'perceived underlying inflation'*, both in the short and in the long run (see *Table 10-13*). In the long run, more than 90 per cent of variance of inflation is explained by the consumer price markup and the 'perceived underlying inflation' shock. Disturbingly, the shock to the 'perceived underlying inflation' explains the variance of cyclical inflation by more than 25 per cent in 1 year and by around 80 per cent in the long run. Hence, the alternative model gives very little explanatory role for all other shocks. This is in sharp contrast to the case with the *baseline* model, where only around 52 per cent is explained by the consumer price markup shock. This clearly shows, that the *baseline* model explains inflation to a larger extent by structural shocks while in the *alternative* model large part of inflation variance can only be captured with shifts in the Phillips curve (with exogenous - not modelled - shocks). That is, the endogenous learning process in this *baseline* model was able to capture longer term inflation movements without introducing an additional exogenous shock related to disinflation. At the same time, it was also shown that this solution did not biased the indexation parameters downwards.

Similarly to the *baseline* model: real variables are highly influenced by external demand and productivity shocks in the long run and the real effects of financial premium and monetary-policy shocks are found negligible in the *alternative* model.

Table 11

Forecast error variance decomposition, model without adaptive learning $t = 4$ (one year)*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}_t$	0.6	0.2	27.7	1.8	6.2	39.6	5.1	2.0	4.8	2.0	0.2	0.0	9.6	0.0	0.2
$\hat{\pi}^x_t$	0.1	0.1	0.0	9.0	5.6	0.0	0.6	84.2	0.1	0.2	0.0	0.0	0.1	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.2	2.0	0.8	1.2	0.0	90.8	1.4	1.4	1.0	0.1	0.0	1.1	0.0	0.0
$\bar{\pi}^{**}_t$	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π^{***}_t	0.5	0.1	40.4	1.5	5.1	32.6	4.2	1.6	4.0	1.7	0.2	0.0	7.9	0.0	0.2
$\tilde{q}_t$	0.0	1.6	0.9	7.2	0.0	3.9	0.1	0.2	0.7	81.9	2.5	0.0	0.4	0.0	0.5
$\tilde{e}_t$	0.1	2.0	0.4	4.9	0.6	0.4	0.2	0.9	0.1	87.2	2.8	0.0	0.2	0.0	0.3
$d\tilde{e}_t$	0.0	1.0	0.3	3.3	0.2	1.2	0.1	0.4	0.2	88.2	4.6	0.0	0.2	0.0	0.2
$\hat{\imath}_t$	0.6	0.4	0.8	1.2	6.5	25.7	5.9	2.7	5.3	8.7	31.0	0.0	11.1	0.0	0.2
$\tilde{c}^{no}_t$	1.9	2.9	0.2	6.6	21.0	2.4	46.0	9.0	2.0	5.5	0.3	0.0	2.1	0.0	0.1
$\tilde{c}^o_t$	0.1	2.2	0.0	2.6	1.6	0.5	4.1	2.1	84.3	0.7	0.5	0.0	0.8	0.0	0.5
$\widetilde{w}_t$	0.0	0.1	0.1	0.4	0.1	1.6	95.4	0.9	0.6	0.5	0.0	0.0	0.1	0.0	0.0
$\tilde{x}_t$	0.0	0.1	0.0	49.3	4.0	0.0	0.6	45.6	0.1	0.2	0.0	0.0	0.1	0.0	0.0
$\widetilde{m}^d_t$	7.6	4.9	0.0	7.9	43.9	0.2	11.5	2.0	7.3	4.5	0.0	0.0	10.0	0.0	0.0
$\widetilde{m}^x_t$	0.3	2.6	0.0	37.7	25.5	0.0	3.2	28.8	0.3	1.4	0.0	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	8.8	3.8	0.1	1.6	50.4	0.4	15.7	1.2	3.8	5.1	0.5	0.0	8.4	0.0	0.0
$\tilde{l}^x_t$	0.2	3.8	0.0	21.6	23.1	0.0	19.7	24.7	0.1	6.4	0.2	0.0	0.0	0.0	0.0
$\tilde{n}_t$	0.5	1.6	0.1	2.1	5.3	0.0	22.3	6.1	0.5	3.0	0.2	56.3	2.0	0.0	0.0
$\tilde{k}_t$	0.0	0.0	0.1	0.0	0.1	0.1	0.2	0.0	0.1	0.2	0.0	0.0	45.0	0.0	54.3
$\tilde{I}_t$	0.0	0.0	0.1	0.0	0.2	0.2	0.4	0.0	0.2	0.6	0.1	0.0	97.9	0.0	0.1
$\tilde{b}_t$	1.4	19.0	0.0	31.0	17.0	0.0	3.4	23.4	1.3	1.8	0.0	0.0	1.7	0.0	0.0
$\widetilde{Q}_t$	0.4	0.6	1.8	1.5	5.2	19.0	3.4	2.3	1.6	7.7	13.9	0.0	36.7	4.3	1.6
$\tilde{r}^k_t$	4.5	1.1	0.1	16.7	50.9	0.7	1.7	15.3	2.7	2.5	0.3	0.0	3.5	0.0	0.1
P^x_t	0.1	0.2	0.0	16.3	6.6	0.0	1.1	75.0	0.3	0.3	0.0	0.0	0.2	0.0	0.0

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

Table 12

Forecast error variance decomposition, model without adaptive learning $t = 10$ (ten quarters)*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}_t$	0.4	0.2	49.4	1.2	4.4	26.2	3.8	1.4	3.5	1.5	0.2	0.0	7.4	0.0	0.4
$\hat{\pi}^x_t$	0.1	0.2	0.0	10.1	6.1	0.0	0.6	82.4	0.2	0.2	0.0	0.0	0.2	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.2	4.2	0.8	1.1	0.0	88.8	1.4	1.3	0.9	0.1	0.0	1.1	0.0	0.0
$\bar{\pi}^{**}_t$	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π^{***}_t	0.4	0.2	55.5	1.1	3.8	23.0	3.4	1.3	3.1	1.3	0.2	0.0	6.5	0.0	0.3
$\tilde{q}_t$	0.0	3.7	1.3	12.1	0.6	3.1	0.2	1.1	0.7	73.7	1.9	0.0	0.5	0.0	0.9
$\tilde{e}_t$	0.2	3.5	0.3	5.8	2.4	1.2	2.7	2.9	1.2	72.9	2.0	0.0	4.7	0.0	0.3
$d\tilde{e}_t$	0.0	1.0	0.3	3.3	0.2	1.2	0.3	0.4	0.3	87.6	4.5	0.0	0.5	0.0	0.2
$\hat{\imath}_t$	0.5	0.3	0.6	1.1	5.1	19.0	9.1	2.9	7.4	12.8	22.9	0.0	17.6	0.0	0.7
$\tilde{c}^{no}_t$	1.3	2.3	0.4	4.5	14.0	2.1	57.1	8.3	2.0	5.1	0.3	0.0	2.5	0.0	0.1
$\tilde{c}^o_t$	0.2	4.6	0.0	6.4	2.5	0.4	8.4	4.2	69.6	0.8	0.4	0.0	1.3	0.0	1.2
$\widetilde{w}_t$	0.0	0.2	0.1	0.2	0.1	1.2	95.2	1.0	0.7	0.8	0.0	0.0	0.3	0.0	0.1
$\tilde{x}_t$	0.0	0.2	0.0	43.0	4.7	0.0	1.9	49.1	0.4	0.4	0.0	0.0	0.2	0.0	0.1
$\widetilde{m}^d_t$	6.3	5.9	0.1	8.2	36.6	0.4	15.1	2.0	8.2	4.7	0.1	0.0	12.5	0.0	0.1
$\widetilde{m}^x_t$	0.3	3.0	0.0	34.5	23.7	0.0	4.0	32.7	0.3	1.3	0.0	0.0	0.2	0.0	0.0
$\tilde{l}^d_t$	7.0	3.1	0.2	1.3	40.1	0.3	28.5	1.1	3.7	4.2	0.5	0.0	9.9	0.0	0.1
$\tilde{l}^x_t$	0.2	3.2	0.0	17.7	17.6	0.1	31.9	22.9	0.4	5.7	0.2	0.0	0.1	0.0	0.0
$\tilde{n}_t$	0.3	1.2	0.2	1.2	2.6	0.0	34.5	4.6	0.3	2.2	0.1	50.8	2.1	0.0	0.0
$\tilde{k}_t$	0.0	0.1	0.2	0.0	0.2	0.2	0.7	0.0	0.3	0.8	0.1	0.0	64.9	0.0	32.5
$\tilde{I}_t$	0.0	0.1	0.5	0.0	0.3	0.3	1.5	0.1	0.7	1.6	0.1	0.0	94.2	0.0	0.5
$\tilde{b}_t$	0.8	18.4	0.0	38.1	9.5	0.1	7.6	17.8	2.3	2.2	0.0	0.0	3.2	0.0	0.0
$\widetilde{Q}_t$	0.2	0.5	1.3	0.9	3.4	10.8	4.7	1.9	2.4	7.1	7.8	0.0	55.2	2.4	1.3
$\tilde{r}^k_t$	4.2	1.0	0.2	15.7	47.7	0.8	2.0	17.6	3.2	2.9	0.3	0.0	3.7	0.0	0.6
P^x_t	0.1	0.2	0.0	22.6	6.2	0.0	2.3	67.2	0.5	0.4	0.0	0.0	0.3	0.0	0.2

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$

Table 13

Unconditional variance decomposition, model without adaptive learning*

	$\tilde{g}_t$	$\widetilde{P}^{m*}_t$	$\tilde{\chi}_t$	$\tilde{x}^*_t$	$\widetilde{A}_t$	$\tilde{\upsilon}_t$	$\tilde{\upsilon}^w_t$	$\tilde{\upsilon}^x_t$	$\tilde{\varepsilon}^c_t$	$\tilde{\varepsilon}^{pr}_t$	$\tilde{\varepsilon}^r_t$	$\tilde{\varepsilon}^n_t$	$\tilde{\varepsilon}^I_t$	$\tilde{\varepsilon}^Q_t$	$\tilde{\varepsilon}_{kt}$
$\hat{\pi}_t$	0.1	0.1	83.6	0.4	1.3	7.6	1.3	0.5	1.1	0.5	0.1	0.0	3.2	0.0	0.2
$\hat{\pi}^x_t$	0.1	0.2	0.0	10.1	6.1	0.0	0.7	82.2	0.2	0.2	0.0	0.0	0.2	0.0	0.0
$\hat{\pi}^w_t$	0.1	0.2	17.9	0.6	0.9	0.0	75.8	1.2	1.2	0.8	0.1	0.0	1.1	0.0	0.0
$\bar{\pi}^*_t$	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
π^{***}_t	0.1	0.1	84.2	0.4	1.3	7.3	1.3	0.5	1.1	0.4	0.1	0.0	3.1	0.0	0.2
$\tilde{q}_t$	0.3	13.7	4.8	28.4	4.1	1.1	8.6	8.0	1.9	23.2	0.5	0.0	4.6	0.0	0.7
$\tilde{e}_t$	0.4	6.2	0.6	8.2	4.0	1.4	11.9	6.0	5.1	39.7	1.1	0.0	13.1	0.0	2.4
$d\tilde{e}_t$	0.0	1.0	0.3	3.3	0.2	1.2	0.3	0.4	0.3	87.3	4.5	0.0	0.8	0.0	0.2
$\hat{\imath}_t$	0.5	0.7	0.6	1.8	4.8	17.5	8.6	3.0	7.0	12.4	20.6	0.0	21.3	0.0	1.4
$\tilde{c}^{no}_t$	1.1	3.3	4.8	6.6	12.2	1.9	51.2	7.8	1.8	4.9	0.2	0.0	4.0	0.0	0.3
$\tilde{c}^o_t$	0.5	14.7	0.1	26.2	5.3	0.2	15.1	9.8	23.7	1.8	0.1	0.0	0.8	0.0	1.7
$\widetilde{w}_t$	0.0	0.3	2.5	0.7	0.1	1.2	90.4	1.1	0.7	1.0	0.0	0.0	1.6	0.0	0.4
$\tilde{x}_t$	0.1	1.3	0.4	41.8	4.6	0.0	2.5	45.5	0.5	0.7	0.0	0.0	2.1	0.0	0.4
$\widetilde{m}^d_t$	4.3	10.4	0.2	17.7	26.0	0.3	14.5	5.5	6.4	4.3	0.0	0.0	10.0	0.0	0.3
$\widetilde{m}^x_t$	0.3	3.3	0.0	34.5	23.4	0.0	4.2	32.4	0.3	1.3	0.0	0.0	0.3	0.0	0.0
$\tilde{l}^d_t$	6.7	3.0	0.9	1.3	38.4	0.3	30.0	1.1	3.5	4.1	0.4	0.0	10.1	0.0	0.2
$\tilde{l}^x_t$	0.2	6.5	0.2	21.6	15.2	0.1	28.8	20.4	1.0	5.2	0.1	0.0	0.6	0.0	0.1
$\tilde{n}_t$	0.3	3.2	0.9	5.3	2.8	0.0	33.9	5.0	0.6	2.2	0.1	43.7	1.9	0.0	0.2
$\tilde{k}_t$	0.0	0.5	13.9	0.2	0.5	0.4	3.9	0.5	1.7	3.1	0.1	0.0	61.4	0.0	13.7
$\tilde{I}_t$	0.0	0.2	3.2	0.1	0.3	0.3	2.4	0.3	1.1	2.5	0.1	0.0	87.8	0.0	1.6
$\tilde{b}_t$	0.5	19.0	0.5	36.5	6.4	0.2	14.7	12.4	3.4	3.5	0.0	0.0	2.2	0.0	0.6
$\widetilde{Q}_t$	0.2	0.4	1.1	0.7	2.8	9.0	4.1	1.6	2.1	6.1	6.5	0.0	62.1	2.0	1.1
$\tilde{r}^k_t$	3.8	1.4	0.4	15.2	44.1	0.8	2.5	16.5	3.2	2.9	0.3	0.0	7.5	0.0	1.5
P^x_t	0.1	1.4	0.4	23.3	6.1	0.0	2.8	61.9	0.6	0.8	0.0	0.0	2.2	0.0	0.4

* Calculated at mean parameter values.
** $d\bar{q}_t = -\bar{\pi}_t$ in IT.
*** $\pi_t = \hat{\pi}_t + \bar{\pi}_t$.

8 Conclusions

In this paper we presented an estimated two-sector dynamic stochastic general equilibrium (DSGE) small-open-economy model for the Hungarian economy. The specialty of the model is that agents' perception on *underlying* inflation is made endogenous by a real-time adaptive-learning algorithm. In addition, the monetary regime shift occured in 2001 is explicitely takin into account. The model is estimated by Bayesian methods. Throughout the estimations we explicitely took into account the fact that there were two different monetary regimes in Hungary.

The model's special feature is that inflation to which rule-of-thumb price setters partly indexate is generated by an adaptive learning mechanism. In this model, agents' perception on *'underlying'* inflation heavily influences long term inflation developments.

According to the estimates the Calvo parameters of consumer prices are similar to those estimated for the euro-area. On the other hand, nominal wage rigidities are less important in Hungary than in the euro-area. An interesting result is that the change in monetary regime mostly influenced the price indexation mechanisms in the economy. Less role for indexation in consumer prices are estimated for the inflation targeting regime than in the previous crawling peg regime. Wage indexation parameters are estimated to be relatively low compared to euro zone estimates.

Interest-rate smoothing parameter is found significantly lower than the euro-area and US estimates. The real-time adaptive learning process of underlying inflation works as an additional source of inflation inertia and it is also important in the responses of real variables, as well. Adjustment cost of investment is found to be higher usually found in the literature. This results in reactions of investment to shocks being close in magnitude of output or consumption. Comparing impulse responses with other DSGE models, monetary policy and productivity shocks have qualitatively similar effect. The basic difference is that in this model investments are less responsive than usual in the literature. A crowding-out effect of a government-consumption shock in the medium run is also found. Though, the presence of non-optimizer consumers create a weekly Keynesian effect of fiscal shock in the short run.

Estimated impulse responses to an interest-rate shock was also compared to that of other estimated non-DSGE models of Hungarian economy. The model produces similar real exchange rate response in the short run, but we estimated a smaller consumer price adjustment in the longer run. Real responses were found to be slightly different in this model.

According to variance decomposition, both the *cyclical* and the permanent (*'underlying'*) component of inflation can be explained by productivity, investment, consumer preference and markup shocks. Unlike in other estimated DSGE models estimated for disinflation periods, by introducing a simple learning scheme, the model was capable to explain the disinflation process occured in Hungary. As suspected in a small, open economy, real variables are highly influenced by external demand and productivity shocks in the long run. Real effects of financial premium and monetary-policy shock are negligible, which is in contrast with euro-zone estimates of Smets & Wouters (2003). However, it conforms to the results of Vonnák (2007) and Jakab *et al.* (2006) that monetary transmission mechanism in Hungary works less through the change in output.

As a robustness check the estimates of an *alternative* model without endogenous real time adaptive learning of *'underlying inflation'* is also demonstrated. The estimated coefficients in the *baseline* and in the *alternative* model are found to be relatively close to each other. The degree of indexation of consumer prices is estimated to be slightly lower in the alternative model indicating that the presence of adaptive learning is not responsible for an 'intrinsic' inertia in inflation. Impulse responses are more or less similar in different model specifications. Slight differences can be found with respect to nominal wage reactions and consumer prices. Wages behave in a more flexible manner in the *alternative* model, while consumer prices generally respond to a lesser extent in the alternative model than in the baseline model.

However, variance decomposition shows that negliging information content of long-term movements of inflation in a country with disinflation has serious consequences. It would lead to a model which can only explain long term inflationary movements in a limited way. The exogenous shock (inflation target shock) is responsible for a large part of inflation movements either in the short or in the long run.

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Appendices

A The model

A.1 PRODUCTION

First, we show how to derive the log-linearized equations of the demand for production inputs. The marginal-cost equation (5) implies that

$$mc_t^s = A_t^{-1}\left[\bar{\alpha}_s\left(r_t^k\right)^{1-\varrho} + (1-\bar{\alpha}_s)\left(w_t^{zs}\right)^{1-\varrho}\right]^{\frac{1}{1-\varrho}}.$$

Log-linearizing it yields

$$\widetilde{mc}_t^s + \widetilde{A}_t = \bar{\alpha}_s\left(\frac{r^k}{mc^s A}\right)^{1-\varrho}\tilde{r}_t^k + (1-\bar{\alpha}_s)\left(\frac{w^{zs}}{mc^s A}\right)^{1-\varrho}\widetilde{w}_t^{zs}.$$

It can be expressed in the following way.

$$\begin{aligned}\widetilde{mc}_t^s + \widetilde{A}_t = {} & \frac{r^k}{mc^s\left(y^s+\bar{f}_s\right)}\left\{\bar{\alpha}_s\left(\frac{mc^s}{r^k}\right)^{\varrho}\frac{y^s+\bar{f}_s}{A^{1-\varrho}}\right\}\tilde{r}_t^k \\ & + \frac{w^{zs}}{mc^s\left(y^s+\bar{f}_s\right)}\left\{(1-\bar{\alpha}_s)\left(\frac{mc^s}{w^{zs}}\right)^{\varrho}\frac{y^s+\bar{f}_s}{A^{1-\varrho}}\right\}\widetilde{w}_t^{zs}.\end{aligned}$$

Substitute formula (6) and (7) for the expressions in curly brackets,

$$\widetilde{mc}_t^s + \widetilde{A}_t = \frac{r^k k^s}{mc^s\left(y^s+\bar{f}_s\right)}\tilde{r}_t^k + \frac{w^{zs}z^s}{mc^s\left(y^s+\bar{f}_s\right)}\widetilde{w}_t^{zs}.$$

Let us define

$$\alpha_s = \frac{r^k k^s}{mc^s\left(y^s+\bar{f}_s\right)}.$$

Homogeneity of the marginal cost function implies that

$$1-\alpha_s = \frac{w^{zs}z^s}{mc^s\left(y^s+\bar{f}_s\right)}.$$

$$\widetilde{mc}_t^s = \alpha_s\tilde{r}_t^k + (1-\alpha_s)\widetilde{w}_t^{zs} - \widetilde{A}_t. \tag{54}$$

Let us log-linearize (6),

$$\tilde{k}_t^s + \tilde{u}_t = \varrho\left(\widetilde{mc}_t^s - \tilde{r}_t^k\right) + \frac{y^s}{y^s+\bar{f}_s}\tilde{y}_t^s - (1-\varrho)\widetilde{A}_t.$$

where it was used that $\widetilde{DP}_t^s = 0$. It is a corollary of equation (2), since it implies $\widetilde{P}_t = \int_0^1 \widetilde{P}_t(i)\,di$. Substitute (54)

$$\tilde{k}_t^s + \tilde{u}_t = \varrho(1-\alpha_d)\left(\widetilde{w}_t^{zs} - \tilde{r}_t^k\right) + \frac{\tilde{y}_t^s}{1+f_s} - \widetilde{A}_t,$$

where $f_s = \bar{f}_s / y_s$. According to equation (30) $r_t^k = \Psi'(u_t)$. Log-linearizing this formula yields $r^k \tilde{r}_t^k = \Psi''(1) u \tilde{u}_t$. Since $r^k = \Psi'(1) u$,

$$\tilde{r}_t^k = \frac{\Psi''(1)}{\Psi'(1)} \tilde{u}_t = \frac{1}{\psi} \tilde{u}_t.$$

Hence,

$$\tilde{k}_t^s = \varrho (1-\alpha_s) \left(\widetilde{w}_t^{zs} - \tilde{r}_t^k \right) - \psi \tilde{r}_t^k + \frac{\tilde{y}_t^s}{1+f_s} - \widetilde{A}_t,$$

which is identical to the log-linearized demand equations of (83) in *section A.4.* One can show the same way that equation (7) implies that

$$\tilde{z}_t^s = \varrho \alpha_d \left(\tilde{r}_t^k - \widetilde{w}_t^{zs} \right) - \psi \tilde{r}_t^k + \frac{\tilde{y}_t^s}{1+f_s} - \widetilde{A}_t.$$

The cost minimization problem of producers of z^s is the following.

$$K_s \left(W_t, e_t P_t^{m*}, z_t^s \right) = \min_{l_t^s, m_t^s} W_t l^s + e_t P_t^{m*},$$

subject to

$$\left(z_t^s + k_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} = \bar{a}_s^{\frac{1}{\varrho_z}} \left(m_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} + (1-\bar{a}_s)^{\frac{1}{\varrho_z}} \left(l_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}},$$

where $k_t^s = z_t^s \Phi_{zs} \left(z_t^s \right)$. The Lagrangian is

$$W_t l^s + e_t P_t^{m*} + \varsigma \left[\left(z_t^s + k_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} - \bar{a}_s^{\frac{1}{\varrho_z}} \left(m_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} - (1-\bar{a}_s)^{\frac{1}{\varrho_z}} \left(l_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} \right].$$

The first-order conditions are

$$W_t l_t^s = \varsigma \bar{a}_s^{\frac{1}{\varrho_z}} \frac{\varrho_z - 1}{\varrho_z} \left(l_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}},$$

$$e_t P^{m*} m_t^s = \varsigma (1-\bar{a}_s)^{\frac{1}{\varrho_z}} \frac{\varrho_z - 1}{\varrho_z} \left(m_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}}.$$

This implies

$$K_s \left(W_t, e_t P_t^{m*}, z_t^s \right) = W_t l_t^s + e_t P^{m*} m_t^s = \varsigma \frac{\varrho_z - 1}{\varrho_z} \left(\bar{a}_s^{\frac{1}{\varrho_z}} \left(m_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} + (1-\bar{a}_s)^{\frac{1}{\varrho_z}} \left(l_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}} \right)$$
$$= \varsigma \frac{\varrho_z - 1}{\varrho_z} \left(z_t^s + k_t^s \right)^{\frac{\varrho_z - 1}{\varrho_z}}.$$

Hence the Lagrange-multiplier is

$$\varsigma = K_s(\cdot) \frac{\varrho_z - 1}{\varrho_z} \left(z_t^s + k_t^s \right)^{\frac{1-\varrho_z}{\varrho_z}}.$$

Substituting it back to the first-order conditions results in

$$l_t^s = \bar{a}_s \left(\frac{K(\cdot)}{W_t} \right)^{\varrho_z} \left(z_t^s + k_t^s \right)^{1-\varrho_z}, \tag{55}$$

$$m_t^s = (1-\bar{a}_s) \left(\frac{K(\cdot)}{e_t P_t^{m*}} \right)^{\varrho_z} \left(z_t^s + k_t^s \right)^{1-\varrho_z}. \tag{56}$$

Substituting the above expression into the constraint of the minimization problem and rearranging it yields the closed form solution of the cost function,

$$K_s\left(W_t, e_t P_t^{m*}, z_t^s\right) = \left[\bar{a}_s W_t^{1-\varrho_z} + (1-\bar{a}_s)\left(e_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}} \left(z_t^s + k_t^s\right).$$

The accompanying marginal cost function is

$$W_t^{zs} = \frac{\partial K_s(\cdot)}{\partial z_t^s} = \left[\bar{a}_s W_t^{1-\varrho_z} + (1-\bar{a}_s)\left(e_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}} \left(1 + \frac{\partial k_t^s}{\partial z_t^s}\right),$$

which is equivalent with equation equation (9) in *section A.1.* Substituting K_s into equation (55) and (56) yields the input demand functions,

$$l_t^s = \bar{a}_s \left(\frac{\left[\bar{a}_s w_t^{1-\varrho_z} + (1-\bar{a}_s)\left(q_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}}}{W_t} \right)^{\varrho_z} \left(z_t^s + k_t^s\right),$$

$$m_t^s = (1-\bar{a}_s) \left(\frac{\left[\bar{a}_s w_t^{1-\varrho_z} + (1-\bar{a}_s)\left(q_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}}}{e_t P_t^{m*}} \right)^{\varrho_z} \left(z_t^s + k_t^s\right),$$

recall that $w_t = W_t/P_t$ and $q_t = P_t/e_t$. The above two expressions are identical to equations (10) and (11) in *section A.1.*

Define $w^{zs} = W_t^{zs}/P_t$. Then equation (9) implies that

$$w_t^{zs} = \left[\bar{a}_s w_t^{1-\varrho_z} + (1-\bar{a}_s)\left(q_t P_t^{m*}\right)^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}} \left[1 + \Phi_{zs}\left(z_t^s\right) + z_t^s \Phi'_{zs}\left(z_t^s\right)\right].$$

As above, one can show that

$$\widetilde{w}_t^{zs} = a^s \widetilde{w}_t + (1-a_s)\left(\tilde{q}_t + \widetilde{P}_t^{m*}\right) + \tilde{z}_t^s,$$

where

$$a_s = \frac{w l^s}{w^{zs} z^s},$$

and

$$z_t^s = 1 + \Phi_{zs}(z^s) + z^s \Phi'_{zs}(z^s).$$

Log-linearizing z_t^s yields

$$z^s \tilde{z}_t^s = \left[2\Phi'_z(z^s) + z^s \Phi''_{zs}(z^s)\right] z_t^s \tilde{z}_t = (z^s)^2 \Phi''_{zs}(z^s)\tilde{z}_t,$$

where the second equation is a consequence of the assumptions $\Phi_{zs}(z^s) = \Phi'_{zs}(z^s) = 0$. They also imply that $z^s = 1$. Hence,

$$\tilde{z}_t^s = (z^s)^2 \Phi''_z(z^s)\tilde{z}_t^s = \phi_z \tilde{z}_t^s.$$

As a consequence,

$$\widetilde{w}_t^{zs} = a^s \widetilde{w}_t + (1-a_s)\left(\tilde{q}_t + \widetilde{P}_t^{m*}\right) + \phi_z \tilde{z}_t^s, \tag{57}$$

$$\widetilde{\bar{w}}_t^{zs} = a^s \widetilde{w}_t + (1-a_s)\left(\tilde{q}_t + \widetilde{P}_t^{m*}\right). \tag{58}$$

Equations (10) and (11) imply that

$$\tilde{l}_t^s = \varrho_z \left(\tilde{\bar{w}}_t^{zs} - \tilde{w}_t \right) + \tilde{y}_t^s,$$
$$\tilde{m}_t^s = \varrho_z \left(\tilde{\bar{w}}_t^{zs} - \tilde{q}_t - \tilde{P}_t^{m*} \right) + \tilde{y}_t^s,$$

where

$$y_t^s = z_t^s + z^s \Phi_{zs} \left(z_t^s \right).$$

Substituting formula (58) into the above expressions yields

$$\tilde{l}_t^s = \varrho_z (1 - a_s) \left(\tilde{q}_t + \tilde{P}_t^{m*} - \tilde{w}_t \right) + \tilde{y}_t^s,$$
$$\tilde{m}_t^s = \varrho_z a_s \left(\tilde{w}_t - \tilde{q}_t - \tilde{P}_t^{m*} \right) + \tilde{y}_t^s.$$

These equations are equivalent with formulas (84) and (85) in *section A.4*, since

$$y^s \tilde{y}_t^s = z^s \tilde{z}_t^s + \left[\Phi_{zs}(z^s) + z^s \Phi'_{zs}(z^s) \right] z_t^s \tilde{z}_t = z^s \tilde{z}_t^s,$$

and $y^s = z^s$.

Price setting

Equations (13) and (14) imply that

$$\mathscr{X}_t^1 = \Lambda_t^o y_t^d mc_t^d + \sum_{T=t+1}^{\infty} (\beta \gamma_d)^{T-t} \mathrm{E}_t \left[\Lambda_T^o y_T^d mc_T^d \left(\frac{P_T}{P_t \Pi_t^I} \right)^{\theta} \right],$$
$$\mathscr{X}_t^2 = \tau_t^d \Lambda_t^o y_t^d + \sum_{T=t+1}^{\infty} (\beta \gamma_d)^{T-t} \mathrm{E}_t \left[\tau_T^d \Lambda_T^o y_T^d \left(\frac{P_T}{P_t \Pi_t^I} \right)^{\theta-1} \right].$$

With some manipulations it is easy to show that $\mathscr{X}_t^1$ can be expressed in the recursive way of formula (15),

$$\mathscr{X}_t^1 = \Lambda_t^o y_t^d mc_t^d + \beta \gamma_d \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^I} \right)^{\theta} \sum_{T=t+1}^{\infty} (\beta \gamma_d)^{T-t-1} \Lambda_T^o y_T^d mc_T^d \left(\frac{P_T}{P_{t+1} \Pi_{t+1}^I} \right)^{\theta} \right]$$
$$= \Lambda_t^o y_t^d mc_t^d + \beta \gamma_d \mathrm{E}_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^I} \right)^{\theta} \mathscr{X}_{t+1}^1 \right],$$

Similarly, one can show that $\mathscr{X}_t^2$ can be expressed as in equation (16).

The log-linear version of equation (17) is

$$\tilde{\mathscr{P}}_t = \frac{\gamma_d}{1 - \gamma_d} \breve{\pi}_t, \tag{59}$$

where $\breve{\pi}_t = \pi_t - \bar{\pi}_t - \vartheta_d \left(\pi_{t-1} - \bar{\pi}_{t-1} \right)$, $\pi_t = \tilde{P}_t - \tilde{P}_{t-1}$ and $\bar{\pi}_t = \tilde{\bar{\Pi}}_t$. Equation (14) implies that

$$\tilde{\mathscr{P}}_t = \tilde{\mathscr{X}}_t^1 - \tilde{\mathscr{X}}_t^2. \tag{60}$$

The log-linearization of formulas (15) and (16) results in

$$\tilde{\mathscr{X}}_t^1 = \frac{\Lambda^o y^d mc^d}{\mathscr{X}^1} \left(\tilde{\Lambda}_t^o + \tilde{y}_t^d + \widetilde{mc}_t^d \right) + \beta \gamma \mathrm{E}_t \left[\theta \breve{\pi}_{t+1} + \tilde{\mathscr{X}}_{t+1}^1 \right], \tag{61}$$

$$\tilde{\mathscr{X}}_t^2 = \frac{\Lambda^o y^d \tau^d}{\mathscr{X}^2} \left(\tilde{\Lambda}_t^o + \tilde{y}^d + \tilde{\tau}_t^d \right) + \beta \gamma \mathrm{E}_t \left[(\theta - 1) \breve{\pi}_{t+1} + \tilde{\mathscr{X}}_{t+1}^2 \right], \tag{62}$$

where it is used that $\Pi/\Pi^I = 1$. Observe that

$$\frac{\Lambda^o y^d mc^d}{\mathscr{X}^1} = \frac{\Lambda^o y^d \tau^d}{\mathscr{X}^2} = (1-\beta\gamma_d),$$

hence combining expressions (60), (61) and (62) yields

$$\widetilde{\mathscr{P}}_t = (1-\beta\gamma_d)\left(\widetilde{mc}_t^d - \tilde{\tau}_t^d\right) + \beta\gamma \mathrm{E}_t\left[\widetilde{\mathscr{P}}_{t+1}\right] + \beta\gamma\mathrm{E}_t\left[\breve{\pi}_{t+1}\right].$$

Formula (59) implies that

$$\breve{\pi}_t = \beta \mathrm{E}_t[\breve{\pi}_{t+1}] + \xi_d \widetilde{mc}_t^d + \tilde{v}_t, \tag{63}$$

where

$$\xi_d = \frac{(1-\beta\gamma_d)(1-\gamma_d)}{\gamma_d} \text{ and } \tilde{v}_t = -\xi_d\tilde{\tau}_t^d.$$

Formulas (54) and (57) imply that

$$\begin{aligned}\widetilde{mc}_t^d &= \alpha_d \tilde{r}_t^k + (1-\alpha_d)a_d\widetilde{w}_t + (1-\alpha_d)(1-a_d)\left(\tilde{q}_t + \widetilde{P}_t^{m*}\right)\\ &+ (1-\alpha_d)\phi_z\tilde{z}_t^d - \widetilde{A}_t,\end{aligned}$$

hence expression (63) is equivalent with the Phillips-curve equation (86) in *section A.4*.

As above, one can prove that the first-order condition of price setting in the export sector can be represented by expressions (19), (20) and (21). Using the log-linear versions of the previous formulas and that of equation (22), it is easy to show that price setting behavior of the export sector can be described by

$$\breve{\pi}_t^{x*} = \beta\mathrm{E}_t[\breve{\pi}_{t+1}^{x*}] + \xi_x\widetilde{mc}_t^{x*} + \tilde{v}_t^x, \tag{64}$$

where where $\breve{\pi}_t^{x*} = \pi_t^{x*} - \bar{\pi}_t^{x*} - \vartheta_x\left(\pi_{t-1}^{x*} - \bar{\pi}_{t-1}^{x*}\right)$, $\pi_t^{x*} = \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}$, $\bar{\pi}_t^{x*} = \tilde{\bar{\Pi}}_t^{x*}$,

$$\xi_x = \frac{(1-\beta\gamma_x)(1-\gamma_x)}{\gamma_x} \text{ and } \tilde{v}_t^x = -\xi_x\tilde{\tau}_t^x.$$

The log-linear version of equation (18) is

$$\widetilde{mc}_t^{x*} = \widetilde{mc}_t^x + \widetilde{P}_t - \tilde{e}_t - \widetilde{P}_t^{x*} = \widetilde{mc}_t^x - \tilde{q}_t - \widetilde{P}_t^{x*}.$$

Applying formulas (54) and (57) yields

$$\begin{aligned}\widetilde{mc}_t^x &= \alpha_x\tilde{r}_t^k + (1-\alpha_x)a_x\widetilde{w}_t + (1-\alpha_x)(1-a_x)\widetilde{P}_t^{m*}\\ &- \left[\alpha_x + (1-\alpha_x)a_x\right]\tilde{q}_t + (1-\alpha_x)\phi_x\tilde{z}_t^x - \widetilde{P}_t^{x*} - \widetilde{A}_t.\end{aligned}$$

Hence equation (64) is equivalent with formula (87) in *section A.4*.

A.2 OPTIMIZING HOUSEHOLDS

The representative household maximizes the objective function (23) subject to the budget constraint (24) the investments equation (25), with respect to c_t, B_t, k_{t+1}, I_t and u_t. The corresponding Lagrangian is given by

$$\begin{aligned}&\sum_{t=0}^{\infty}\int \mathrm{prob}_t(s_t|s_0)\beta^t\eta_t^c\left\{U(c_t^o(j) - hc_{t-1}^o) - \eta_t^l V(l_t(j))\right\}ds_t\\ &+\sum_{t=0}^{\infty}\beta\lambda_t\left\{P_tc_t^o(j) + P_tI_t(j) + B_t()j(1+i_t)^{-1} + P_t^wX_t^w()j\right.\\ &\left.-B_{t-1}(j) - \chi_tX_{t-1}^w(j) - W_t(j)l_t(j) - P_tr_t^ku_t(j)k_t(j) - \Psi(u_t(j)) - Div_t\right\},\\ &+\sum_{t=0}^{\infty}\beta\lambda_tQ_t\left\{k_{t+1}(j) - (1-\delta)k_t(j) + \left[1-\Phi_I\left(\frac{\eta_t^I I_t(j)}{I_{t-1}(j)}\right)\right]I_t(j)\right\},\end{aligned}$$

where λ_t and Q_t are state dependent Lagrange multipliers and s_t denotes the state of the world at date t and $\text{prob}_t(\cdot)$ is the appropriate conditional density function.

Due to the existence of asset X_t^w the wage incomes of all households are the same. As a consequence, all households choose the same solutions. Hence, index j is drooped from subsequent notations.

The first order condition with respect to c_t is

$$P_t\lambda_t = \text{prob}(s_t|s_0)\Lambda_t^o = \text{prob}(s_t|s_0)\eta_t^c(c_t^o - hc_{t-1}^o)^{-\sigma}. \tag{65}$$

The first order condition with respect to B_t takes the form of

$$\beta\lambda_t = (1+i_t)\beta^{t+1}\lambda_{t+1}, \tag{66}$$

where $(1+i_t) = 1/P_t^B$. Combining equations (65) and (66) yields

$$\frac{\Lambda_t^o}{P_t} = \beta(1+i_t)\int \text{prob}(s_{t+1}|s_t)\frac{\Lambda_{t+1}^o}{P_{t+1}}\,ds_{t+1} = \beta\text{E}_t\left[\frac{\Lambda_{t+1}^o}{P_{t+1}}\right].$$

where it is used that

$$\text{prob}(s_{t+1}|s_t) = \frac{\text{prob}_{t+1}(s_{t+1}|s_0)}{\text{prob}_t(s_t|s_0)}.$$

The above equation can be expressed as

$$1 = (1+i_t)\text{E}_t\left[D_{t+1,t}\right], \tag{67}$$

where

$$D_{t+j,1} = \beta^j\frac{\Lambda_{t+j}^o P_t}{\Lambda_t^o P_{t+j}}$$

is the stochastic discount factor.

The first-order condition with respect to k_{t+1} is given by the following equation.

$$\beta^t Q_t\lambda_t = \beta^{t+1}\lambda_{t+1}\left[Q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - \Psi\left(u_{t+1}\right)\right].$$

Substitute equation (65) into the above formula.

$$Q_t = \beta\int \text{prob}(s_{t+1}|s_t)\frac{\Lambda_{t+1}^o}{\Lambda_{t+1}^o}\left[Q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - \Psi\left(u_{t+1}\right)\right]ds_{t+1}.$$

$$Q_t = \beta\text{E}_t\left[D_{t+1,t}\left(Q_{t+1}(1-\delta) + r_{t+1}^k z_{t+1} - \Psi\left(u_{t+1}\right)\right)\right]. \tag{68}$$

Equation (68) is equivalent with expression (29) in *section A.2*.

The first order condition with respect to I_t yields the following formula.

$$0 = \beta^t\lambda^t Q_t\left[1 - \Phi_I\left(\frac{\eta_t^I I_t}{I_{t-1}}\right) - \Phi_I'\left(\frac{\eta_t^I I_t}{I_{t-1}}\right)\frac{\eta_t^I I_t}{I_{t-1}}\right] - \beta^t\lambda_t$$
$$+\beta^{t+1}\lambda_{t+1}Q_{t+1}\Phi_I'\left(\frac{\eta_{t+1}^I I_{t+1}}{I_t}\right)\frac{\eta_{t+1}^I I_{t+1}^2}{I_t^2}.$$

One can show, as previously, that it implies the following expression.

$$Q_t\left[1-\Phi_I\left(\frac{\eta^I_t I_t}{I_{t-1}}\right)-\Phi_I'\left(\frac{\eta^I_t I_t}{I_{t-1}}\right)\frac{\eta^I_t I_t}{I_{t-1}}\right]= \tag{69}$$
$$1-\beta \mathrm{E}_t\left[D_{t+1,1}Q_{t+1}\Phi_I'\left(\frac{\eta^I_{t+1}I_{t+1}}{I_t}\right)\frac{\eta^I_{t+1}I^2_{t+1}}{I^2_t}\right].$$

The above equation is equivalent with expression (28) in *section A.2*.

Finally, the first order condition with respect to u_t is given by

$$r^k_t=\Psi'(u_t). \tag{70}$$

This expression is the same as formula (30) in *section A.2*.

Log-linearization of formulas (67) and (70) is straightforward. Let us log-linearize expression (69). First, observe the steady-state form of the formula.

$$Q\left[1-\Phi_I(1)-\Phi_I'(1)\right]=1-\beta Q\Phi_I'(1).$$

Since, by assumption, $\Phi_I(1)=\Phi_I'(1)=0$, the previous equation implies that $Q=1$. Define

$$\mu(x)=\left.\frac{d\Phi_I'\left(\eta^I_t I_t I^{-1}_{t-1}\right)\eta^I_t I_t I^{-1}_{t-1}}{dx}\right|_{(\eta^I_t=1,I_t=I,I_{t-1}=I)},$$

where $x=\eta^I_t, I_t, I_{t-1}$, and

$$\mu^+(y)=\left.\frac{d\Phi_I'\left(\eta^I_{t+1}I_{t+1}I^{-1}_t\right)\eta^I_{t+1}I^2_{t+1}I^{-2}_t}{dy}\right|_{(\eta^I_{t+1}=1,I_{t+1}=I,I_t=I)},$$

where $y=\eta^I_{t+1}, I_{t+1}, I_t$. Since $Q=1$ and $\Phi_I(1)=\Phi_I'(1)=0$ the log-linear version of equation (69) takes the form of

$$\tilde{Q}_t-\mu(I_t)I\tilde{I}_t-\mu(I_{t-1})I\tilde{I}_{t-1}-\mu\left(\eta^I_t\right)\tilde{\eta}^I_t=$$
$$-\beta\mu^+(I_{t+1})I\mathrm{E}_t\left[\tilde{I}_{t+1}\right]-\beta\mu^+(I_t)I\tilde{I}_t-\beta\mu^+\left(\eta^I_{t+1}\right)\mathrm{E}_t\left[\tilde{\eta}^I_{t+1}\right].$$

Observe that

$$\mu(I_t)=\frac{\Phi_I''(1)}{I},\quad \mu(I_{t-1})=-\frac{\Phi_I''(1)}{I},\quad \mu\left(\eta^I_t\right)=\Phi_I''(1),$$

furthermore,

$$\mu^+(I_{t+1})=\frac{\Phi_I''(1)}{I},\quad \mu^+(I_t)=-\frac{\Phi_I''(1)}{I},\quad \mu^+\left(\eta^I_{t+1}\right)=\Phi_I''(1).$$

As a consequence,

$$\Phi_I''(1)(1+\beta)\tilde{I}_t=\Phi_I''(1)\left\{\beta\mathrm{E}_t\left[\tilde{I}_{t+1}\right]+\tilde{I}_{t-1}-\tilde{\eta}^I_t+\beta\mathrm{E}_t\left[\tilde{\eta}^I_{t+1}\right]\right\}+\tilde{Q}_t.$$

The above expression is equivalent with equation (78) in *section A.4*.

The log-linear version of equation (68) is given by

$$\tilde{Q}_t-\mathrm{E}_t\left[\tilde{D}_{t+1,1}\right]=\frac{Q(1-\delta)\mathrm{E}_t\left[\tilde{Q}_{t+1}\right]+r^k u\mathrm{E}_t\left[\tilde{r}^k_{t+1}+\tilde{u}_{t+1}\right]-\Psi'(1)\mathrm{E}_t\left[u_{t+1}\right]}{Q(1-\delta)+r^k u-\Psi(1)}.$$

Since $Q=1$, $u=1$, $\Psi(1)=0$ and equation (70) implies that $r^k=\Psi'(1)$, it takes the form of

$$\widetilde{Q}_t-\mathrm{E}_t\left[\tilde{D}_{t+1,1}\right]=\frac{1-\delta}{1-\delta+r^k}\mathrm{E}_t\left[\widetilde{Q}_{t+1}\right]+\frac{r^k}{1-\delta+r^k}\mathrm{E}_t\left[\tilde{r}^k_{t+1}\right].$$

The log-linearized version of equation (67) implies that

$$\tilde{i}_t-\pi_{t+1}=\mathrm{E}_t\left[\tilde{D}_{t+1,t}\right],$$

hence

$$\tilde{i}_t-\mathrm{E}_t\left[\pi_{t+1}\right]=\frac{1-\delta}{1-\delta+r^k}\mathrm{E}_t\left[\widetilde{Q}_{t+1}\right]-\widetilde{Q}_t+\frac{r^k}{1-\delta+r^k}\mathrm{E}_t\left[\tilde{r}^k_{t+1}\right].$$

The above equation is equivalent with formula (77) in *section A.4.*

A.3 WAGE SETTING

First order conditions (34) and (35) imply that

$$\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t(j)\Pi^{Iw}_{T,t}}\right)^{\theta_w}\tau^w_T\frac{\Lambda^l_T}{P_T}\Pi^{Iw}_{T,t}\right]=$$
$$\frac{\theta_w}{\theta_w-1}\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t(j)\Pi^{Iw}_{T,t}}\right)^{\theta_w}\eta^c_T\eta^l_T\frac{l_T(j)^{\varphi}}{W_t(j)}\right],$$

where $\Lambda^l_T=\left(\bar{\omega}^o\lambda^o_T+\bar{\omega}^{no}\lambda^{no}_T\right)/(\bar{\omega}^o+\bar{\omega}^{no})$ Substituting the demand equation (33) for $l_T(j)$ results in

$$\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t(j)\Pi^{Iw}_{T,t}}\right)^{\theta_w}\frac{\Pi^{Iw}_{T,t}}{W_T}\tau^w_T\Lambda^l_Tw_T\right]=$$
$$\frac{\theta_w}{\theta_w-1}\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t(j)\Pi^{Iw}_{T,t}}\right)^{\theta_w}\frac{\Pi^{Iw}_{T,t}}{W_T}\eta^c_T\eta^l_T\frac{l^{\varphi}_TW^{\iota}_T}{\left(W_t(j)\Pi^{Iw}_{T,t}\right)^{\iota}}\right],$$

where $\iota=\theta_w\varphi+1$. After some manipulations one can get

$$\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t\Pi^{Iw}_{T,t}}\right)^{\theta_w+\iota-1}\tau^w_T\Lambda^l_Tw_T\left(\frac{W_t(j)}{W_t}\right)^{\iota}\left(\frac{W_t\Pi^{Iw}_{T,t}}{W_T}\right)^{-\iota}\right]=$$
$$\frac{\theta_w}{\theta_w-1}\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[\left(\frac{W_T}{W_t\Pi^{Iw}_{T,t}}\right)^{\theta_w+\iota-1}\eta^c_T\eta^l_Tl^{\varphi+1}_T\right],$$

Rearranging the above expression yields

$$\left(\frac{W_t(j)}{W_t}\right)^{\iota}=\frac{\theta_w}{\theta_w-1}\frac{\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[\left(\frac{W_T}{W_t\Pi^{Iw}_{T,t}}\right)^{\theta_w+\iota-1}\eta^c_T\eta^l_Tl^{\varphi+1}_T\right]}{\sum_{T=t}^{\infty}(\gamma_w\beta)^{T-t}\mathrm{E}_t\left[l_T\left(\frac{W_T}{W_t\Pi^{Iw}_{T,t}}\right)^{\theta_w-1}\tau^w_T\Lambda^l_Tw_T\right]}.\qquad(71)$$

This implies that all unions choose the same $W_t(j) = W^\star$. Denote $\mathscr{W}_t = W_t^\star / W_t$, then equation (71) implies formulas (36), (34) and (35) in *section A.2*.

Log-linearization of equations (36) and (39) results in

$$\widetilde{\mathscr{W}}_t = \frac{\gamma_w}{1-\gamma_w}\breve{\pi}_t^w,$$
$$\iota\widetilde{\mathscr{W}}_t = \widetilde{\mathscr{Z}}_t^{w1} - \widetilde{\mathscr{Z}}_t^{w2},$$

where $\breve{\pi}_t^w = \widetilde{w}_t - \widetilde{w}_{t-1} + \pi_t - \bar{\pi}_t - \vartheta_w(\widetilde{w}_{t-1} - \widetilde{w}_{t-2} + \pi_{t-1} - \bar{\pi}_{t-1})$. Expressions (34) and (35) imply that

$$\widetilde{\mathscr{Z}}_t^{w1} - \widetilde{\mathscr{Z}}_t^{w2} = (1-\beta\gamma_w)\left[\varphi\tilde{l}_t + \frac{\sigma}{1-h}\left(\tilde{c}_t^l - h\tilde{c}_{t-1}^l\right) + \tilde{\eta}_t^l - \tilde{\tau}_t^w\right]$$
$$+\beta\gamma_w \mathrm{E}_t\left[\widetilde{\mathscr{Z}}_{t+1}^{w1} - \widetilde{\mathscr{Z}}_{t+1}^{w2}\right] + \iota\beta\gamma_w \mathrm{E}_t\left[\breve{\pi}_{t+1}^w\right],$$

where

$$\tilde{c}_t^l = \frac{\bar{\omega}^o (c^o)^{-\sigma}\,\tilde{c}_t^o + \bar{\omega}^{no}(c^{no})^{-\sigma}\,\tilde{c}_t^{no}}{\bar{\omega}^o (c^o)^{-\sigma} + \bar{\omega}^{no}(c^{no})^{-\sigma}},$$

and it was used that $\Pi^w/\Pi^{Iw} = 1$ and

$$\frac{l^{\varphi+1}}{\mathscr{Z}^{w1}} = \frac{\Lambda^a w l}{\mathscr{Z}^{w2}} = (1-\beta\gamma_w).$$

Combining the above expressions yields equation (88) in *section A.4*.

A.4 LOG-LINEARIZED MODEL

To solve the model we log-linearize it around its steady state. This section reviews the log-linearized model equations. The tilde denotes the log-deviation of a variable from its steady-state value. Variables without time indices represent their steady-state values.

A.5 PERCEIVED *UNDERLYING* INFLATION RATE

As mentioned, if one defines the cyclical component of inflation as $\hat{\pi}_t = \pi_t - \bar{\pi}_t$, then the rela time adaptive mechanism on *'underlying inflation'* ($\bar{\pi}_t$) can be expressed as,

$$\bar{\pi}_t = \frac{\rho_\pi - g}{1-g}\bar{\pi}_{t-1} + \frac{g}{1-g}\hat{\pi}_t. \tag{72}$$

A.6 AGGREGATE DEMAND

Combining and log-linearizing equations (26) and (27) yields the Euler equation of optimizing households'

$$\tilde{c}_t^o = \frac{h}{1+h}\tilde{c}_{t-1}^o + \frac{1}{1+h}\mathrm{E}_t\left[\tilde{c}_{t+1}^o\right] - \frac{1-h}{(1+h)\sigma}\mathrm{E}_t\left[\hat{\imath}_t - \hat{\pi}_{t+1} + d\bar{q}_{t+1}\right] + \tilde{\varepsilon}_t^c, \tag{73}$$

where, $\hat{\imath} = \tilde{\imath}_t - d\bar{e}_t$, $d\bar{e}_t$ is the preannounced rate of depreciation of the central parity of the nominal exchange rate (it is equal to zero in the crawling peg regime), and $d\bar{q}_t = d\bar{e}_t - \bar{\pi}_t$, we call it as the *perceived underlying rate of real depreciation*, furthermore,

$$\tilde{\varepsilon}_t^c = \frac{(1-h)}{(1+h)\sigma}\left(\tilde{\eta}_t^c - \mathrm{E}_t\left[\tilde{\eta}_{t+1}^c\right]\right).$$

Log-linearizing equations (31) and (42) yields the following formula,

$$\tilde{c}_t^{no} = \widetilde{w}_t + \tilde{l}_t. \tag{74}$$

Equation (32) implies that expression

$$\tilde{c}^p = \widetilde{w}_t/2 \tag{75}$$

describes the evolution of log-linear consumption of pensioners. Path of aggregate consumption is determined by

$$\tilde{c}_t = \omega^o \tilde{c}_t^o + \omega^{no}\tilde{c}_t^{no} + \omega^p \tilde{c}_t^p, \tag{76}$$

where $\omega^j = c^j\bar{\omega}^j/c$, $j = o, no, p$.

In *Appendix A.2* it is shown that the log-linearized version of equation (29) is

$$\begin{aligned}\mathrm{E}_t\left[\hat{\imath}_t - \hat{\pi}_{t+1} + d\bar{q}_{t+1}\right] &= \frac{1-\delta}{1-\delta+r^k}\mathrm{E}_t\left[\widetilde{\mathrm{Q}}_{t+1}\right] - \widetilde{\mathrm{Q}}_t \\ &+ \frac{r^k}{1-\delta+r^k}\mathrm{E}_t\left[\tilde{r}_{t+1}^k\right] + \tilde{\varepsilon}_t^{\mathrm{Q}}.\end{aligned} \tag{77}$$

Appendix A.2 explains how to derive from equation (28) the following log-linear formula determining the trajectory of investments.

$$\tilde{I}_t = \frac{1}{1+\beta}\tilde{I}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\left[\tilde{I}_{t+1}\right] + \frac{1}{(1+\beta)\phi_I}\widetilde{\mathrm{Q}}_t + \tilde{\varepsilon}_t^I, \tag{78}$$

where

$$\tilde{\varepsilon}_t^I = \frac{\beta\mathrm{E}_t\left[\tilde{\eta}_{t+1}^I\right] - \tilde{\eta}_t^I}{1+\beta}.$$

Capital accumulation equation is standard.

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{I}_t + \tilde{\varepsilon}_t^k. \tag{79}$$

The log-linear version of the export-demand equation (40) is

$$\tilde{x}_t = h_x\tilde{x}_{t-1} - \theta_x\tilde{P}_t^{x*} + \tilde{x}_t^*. \tag{80}$$

Log-linearizing the equilibrium conditions (49) and (50) yields [10]

$$y^d\tilde{y}_t^d = c\tilde{c}_t + I\tilde{I}_t + g\,\tilde{g}_t + r^k k^d\psi\tilde{r}_t^k, \tag{81}$$

$$y^x\tilde{y}_t^x = x\tilde{x}_t + r^k k^x\psi\tilde{r}_t^k, \tag{82}$$

recall that $\psi = \Psi'(1)/\Psi''(1)$.

A.7 AGGREGATE SUPPLY

In *Appendix A.1* it is shown that *demand for production inputs* is represented by the following log-linear equations,

$$\begin{aligned}\tilde{k}_t^s &= \varrho(1-\alpha_s)\left(\widetilde{w}_t^{zs} - \tilde{r}_t^k\right) - \psi\tilde{r}_t^k + \frac{\tilde{y^s}_t}{1+f_s} - \widetilde{A}_t, \qquad s = d, x, \\ k\tilde{k}_t &= k^d\tilde{k}_t^d + k^x\tilde{k}_t^x.\end{aligned} \tag{83}$$

[10] Equation (30) implies that $\Psi'(1) = r_k$ and $\tilde{u}_t = \tilde{r}_t^k\Psi'(1)/\Psi''(1)$.

where $f^s = \bar{f}^s / y$, and the second line is a consequence of the third equilibrium condition of formula (51).

$$\begin{aligned}\widetilde{w}_t^{zs} &= a_s \widetilde{w}_t + (1-a_s)\left(\tilde{q}_t + \widetilde{P}_t^{m*}\right) + \phi_z \tilde{z}_t^s, \qquad s = d, x. \\ \tilde{q}_t &= \tilde{e}_t - \widetilde{P}_t,\end{aligned}$$

α_s is the steady-state share of capital in production cost, that is,

$$\alpha_s = \frac{r^k k^s}{mc^s\left(y^s + \bar{f}_s\right)}, \qquad s = d, x,$$

and a_s is the steady-state share of labor in w^{zs}, that is,

$$a_s = \frac{w l^s}{w^{zs} z^s}, \qquad s = d, x.$$

Furthermore,

$$\begin{aligned}\tilde{l}_t^s &= \varrho_z(1-a_s)\left(\tilde{q}_t + \widetilde{P}_t^{m*} - \widetilde{w}_t\right) + \tilde{z}_t^s, \qquad s = d, x, \\ l\tilde{l}_t &= l^d \tilde{l}_t^d + l^x \tilde{l}_t^x,\end{aligned} \tag{84}$$

and

$$\begin{aligned}\widetilde{m}_t^s &= \varrho_z a_s\left(\widetilde{w}_t - \tilde{q}_t - \widetilde{P}_t^{m*}\right) + \tilde{z}_t^s, \qquad s = d, x, \\ m\widetilde{m}_t &= m^d \widetilde{m}_t^d + m^x \widetilde{m}_t^x,\end{aligned} \tag{85}$$

where

$$\tilde{z}_t^s = \varrho\alpha_s\left(\tilde{r}_t^k - \widetilde{w}_t^{zs}\right) + \frac{\tilde{y}^s{}_t}{1+f_s} - \widetilde{A}_t, \qquad s = d, x,$$

and the equilibrium conditions of formula (51) are used again.

It is shown in *Appendix A.1* that the Calvo price-setting rule with indexation to lagged inflation implies the following log-linear *hybrid Phillips curve*.

$$\begin{aligned}\hat{\pi}_t = {}& \frac{\beta}{1+\beta\vartheta_d}\mathrm{E}_t\left[\hat{\pi}_{t+1}\right] + \frac{\vartheta_d}{1+\beta\vartheta_d}\hat{\pi}_{t-1} \\ &+ \frac{\xi_d}{1+\beta\vartheta_d}\left[\alpha_d \tilde{r}_t^k + (1-\alpha_d)\widetilde{w}_t^{zd} - \widetilde{A}_t\right] + \tilde{\upsilon}_t^d,\end{aligned} \tag{86}$$

where

$$\xi_d = \frac{(1-\gamma_d)(1-\beta\gamma_d)}{\gamma_d}, \qquad \tilde{\upsilon}_t = -\frac{\xi_d}{1+\beta\vartheta_d}\tilde{\tau}_t.$$

The *Phillips curve* of the exports sector is given by

$$\begin{aligned}\hat{\pi}_t^{x*} = {}& \frac{\beta}{1+\beta\vartheta_d}\mathrm{E}_t\left[\hat{\pi}_{t+1}^{x*}\right] + \frac{\vartheta_x}{1+\beta\vartheta_x}\hat{\pi}_{t-1}^{x*} \\ &+ \frac{\xi_x}{1+\beta\vartheta_x}\left[\alpha_x \tilde{r}_t^k + (1-\alpha_x)a_x\widetilde{w}_t - \left[\alpha_x + (1-\alpha_x)a_x\right]\tilde{q}_t + (1-\alpha_x)(1-a_x)\widetilde{P}_t^{m*}\right] \\ &+ \frac{\xi_x}{1+\beta\vartheta_x}\left[(1-\alpha_x)\phi_z\tilde{z}_t^x - \widetilde{P}_t^{x*} - \widetilde{A}_t\right] + \tilde{\upsilon}_t^x,\end{aligned} \tag{87}$$

where $\hat{\pi}_t^{x*} = \pi_t^{x*} - \bar{\pi}_t^{x*}$, $\pi_t^{x*} = \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}$ and

$$\xi_x = \frac{(1-\gamma_x)(1-\beta\gamma_x)}{\gamma_x}, \qquad \tilde{\upsilon}_t^x = -\frac{\xi_x}{1+\beta\vartheta_x}\tilde{\tau}_t^x.$$

Wage setting in the model is based on similar assumptions as price formation. *Appendix A.3* shows that the log-linear *wage Phillips curve* is given by

$$\begin{aligned}\hat{\pi}_t^w &= \frac{\beta}{1+\beta\vartheta_w}\mathrm{E}_t\left[\hat{\pi}_{t+1}^w\right] + \frac{\vartheta_w}{1+\beta\vartheta_w}\hat{\pi}_{t-1}^w \\ &+ \frac{\xi_w}{1+\beta\vartheta_w}\left[\frac{\sigma}{1-h}\left(\tilde{c}_t^l - h\tilde{c}_{t-1}^l\right) + \varphi\tilde{l}_t - \widetilde{w}_t\right] + \tilde{\upsilon}_t^w,\end{aligned} \tag{88}$$

where

$$\xi_w = \frac{(1-\gamma_w)(1-\beta\gamma_w)}{\gamma_w(1+\theta_w\varphi)}, \qquad \tilde{\upsilon}_t = \frac{\xi_w}{1+\beta\vartheta_w}\left(\eta_t^l - \tau_t^w\right)$$

and

$$\tilde{c}_t^l = \frac{\omega^o\left(c^o\right)^{-\sigma-1}\tilde{c}_t^o + \omega^{no}\left(c^{no}\right)^{-\sigma-1}\tilde{c}_t^{no}}{\omega^o\left(c^o\right)^{-\sigma-1} + \omega^{no}\left(c^{no}\right)^{-\sigma-1}},$$

furthermore,

$$\hat{\pi}_t^w = \widetilde{w}_t - \widetilde{w}_{t-1} + \hat{\pi}_t. \tag{89}$$

A.8 CURRENT ACCOUNT

Equation (43) implies that

$$\tilde{b}_t = (1+i^*)\tilde{b}_{t-1} + \frac{P^{x*}x}{GDP^*}\left(\widetilde{P}_t^{x*} + \tilde{x}_t\right) - \frac{P^{m*}m}{GDP^*}\left(\widetilde{P}_t^{m*} + \widetilde{m}_t\right), \tag{90}$$

since it is assumed that $b = 0$, $\tilde{b}_t = b_t/GDP^*$, where $GDP^* = Py^d/e + P^{x*}x - P^{m*}m$.

A.9 THE INTEREST RATE AND THE EXCHANGE RATE

We decompose nominal depreciation of the nominal exchange rate as

$$\tilde{e}_t - \tilde{e}_{t-1} = d\bar{e}_t + d\hat{e}_t = d\bar{e}_t + \hat{e}_t - \hat{e}_{t-1},$$

where $d\bar{e}_t$ is the exogenously given deterministic part of depreciation and $\hat{e}_t$ is the cyclical part of the nominal exchange rate. In the crawling-peg regime we assume that $d\bar{e}_t$ is the announced rate of the crawl, in the inflation-targeting regime $d\bar{e}_t = 0$.

Uncovered interest rate parity with financial premium shock can be expressed as

$$\hat{\imath}_t = \mathrm{E}_t\left[d\hat{e}_{t+1}\right] + \tilde{\imath}_t^* + \tilde{\varepsilon}_t^{pr}, \tag{91}$$

following Schmitt-Grohe & Uribe (2002), it is assumed that $\tilde{\imath}_t^* = -\nu\tilde{b}_t$, this assumption ensures stationary of $\tilde{b}_t$.

We assume that in the crawling-peg regime the main focus of monetary policy is determination of the rate of crawl. Hence the behavior of cyclical part of the nominal interest rate is captured by the following simple equation.

$$\hat{\imath}_t = \zeta_e^{cr}\hat{e}_t + \tilde{\varepsilon}_t^r, \tag{92}$$

where $\tilde{\varepsilon}^{r}_{t}$ is an exogenous stochastic shock, and $\zeta^{cr}_{e} > 0$ ensures that $\hat{e}_t$ is stationary. Since the presence of ζ^{cr}_{e} is due to this technical requirement its magnitude is set to be negligible.

In the inflation-targeting regime the behavior of the monetary authority is captured by the following *interest-rate rule*.

$$\hat{\imath}_t = \zeta_i \hat{\imath}_{t_1} + (1-\zeta_i)\left[\zeta_\pi \left(\hat{\pi}_t - d\bar{q}_t\right) + \zeta^{it}_{e}\hat{e}_t\right] + \tilde{\varepsilon}^{r}_{t}. \tag{93}$$

Recall that $-d\bar{q}_t = \bar{\pi}_t$ in the inflation targeting regime. Again, the only role of $\zeta^{it}_{e} > 0$ is to ensure the stationarity of $\hat{e}_t$.

The domestic component of the real exchange rate is determined by the following identity.

$$\tilde{q}_t - \tilde{q}_{t-1} = d\hat{e}_t - \hat{\pi}_t + d\bar{q}_t. \tag{94}$$

Finally equation (72) implies the following law of motion for $d\bar{q}_t = d\bar{e}_t - \bar{\pi}_t$,

$$d\bar{q}_t = \frac{\rho_\pi - g}{1-g} d\bar{q}_{t-1} - \frac{g}{1-g}\hat{\pi}_t + \tilde{\chi}_t, \tag{95}$$

where

$$\tilde{\chi}_t = d\bar{e}_t - \frac{\rho_\pi - g}{1-g} d\bar{e}_{t-1}$$

is an exogenous shock.

A.10 COMPLEMENTARY EMPLOYMENT EQUATION

Since there is no consistent data available on aggregate hours worked in the euro area, we need to use employment instead. Hence, following Adolfson *et al.* (2005) and Smets & Wouters (2003), the model is complemented by the following Calvo-type *measurement equation* for *employment*.

$$\Delta\tilde{n}_t = \beta \mathrm{E}_t\left[\Delta\tilde{n}_{t+1}\right] + \frac{(1-\gamma_n)(1-\beta\gamma_n)}{\gamma_n}\left(\tilde{l}_t - \tilde{n}_t\right) + \tilde{\varepsilon}^{n}_{t}, \tag{96}$$

where $\Delta\tilde{n}_t = \tilde{n}_t - \tilde{n}_{t-1}$, $\tilde{n}_t$ denotes the number of people employed at date t, and γ_n is a parameter and $\tilde{\varepsilon}^{n}_{t}$ is an error term.[11]

A.11 THE STEADY STATE

Variables without time indices represent their steady-state values.

The steady state of the model is calculated in two stages. First, given the values of $\beta = 0.99$, $\delta = 0.025$, $\theta = \theta^d = \theta^x = 6$, $p^{m*} = P^{m*}/P = 1$, $e = 1$, $S^{m^d}_{gdp^d} = 0.702$ (the share of imports in value-added in sector d), $S^{m^x}_{gdp^x} = 1.237$ (the share of imports in value-added in sector x), $S^{I}_{y} = 0.112$ (the share of investments in total output), $f_d = f_x = 0.2$ we calculate r^k, $\bar{\alpha}_d$, $\bar{\alpha}_x$, $\bar{a}_d$, $\bar{a}_x$,$w = W/P$, $\varkappa = x/y^d$.

The steady-state value of the rental rate of capital is given by. $r^k = \beta^{-1} - 1 + \delta$.

[11] Smets & Wouters (2003) applied first a similar employment equation in their estimated model. The particular form of equation (96) is taken from Adolfson *et al.* (2005). Equation (37) of Smets & Wouters (2003) is slightly different. It contains terms $\tilde{n}_t$ and $\tilde{n}_{t+1}$ instead of $\Delta\tilde{n}_t$ and $\Delta\tilde{n}_{t+1}$.

Formula (9) implies that

$$w^{zd} = \left[\bar{a}_d w^{1-\varrho_z} + (1-\bar{a}_d)(p^{m*})^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}},$$
$$w^{zx} = \left[\bar{a}_x w^{1-\varrho_z} + (1-\bar{a}_x)(p^{m*})^{1-\varrho_z}\right]^{\frac{1}{1-\varrho_z}},$$

where $w^{zd} = W^{zd}/P$, $w^{zx} = W^{zx}/P$.

The steady-state form of formula (5) is

$$mc^d = \left[\bar{\alpha}_d \left(r^k\right)^{1-\varrho} + (1-\bar{\alpha}_d)(w^{zs})^{1-\varrho}\right]^{\frac{1}{1-\varrho}},$$
$$mc^x = \left[\bar{\alpha}_x \left(r^k\right)^{1-\varrho} + (1-\bar{\alpha}_x)(w^{zs})^{1-\varrho}\right]^{\frac{1}{1-\varrho}},$$

where $mc^d = MC^d/P$, $mc^x = MC^x/P$, $r^k = R^k/P$.

Demand equations (6), (7), and (11) imply that

$$k^d = \bar{\alpha}_d \left(\frac{mc^d}{r^k}\right)^{\varrho} y^d \left(1+f_d\right),$$
$$k^x = \bar{\alpha}_x \left(\frac{mc^x}{r^k}\right)^{\varrho} xy^d \left(1+f_x\right),$$
$$z^d = (1-\bar{\alpha}_d)\left(\frac{mc^d}{w^d}\right)^{\varrho} y^d \left(1+f_d\right),$$
$$z^x = (1-\bar{\alpha}_x)\left(\frac{mc^x}{w^x}\right)^{\varrho} xy^d \left(1+f_x\right),$$
$$m^d = (1-\bar{a}_d)\left(\frac{w_t^{zd}}{p^{m*}}\right)^{\varrho_z} z^d, \quad m^x = (1-\bar{a}_x)\left(\frac{w_t^{zx}}{p^{m*}}\right)^{\varrho_z} z^x.$$

Furthermore,

$$S_{gdp^d}^{m^d} = \frac{p^{m*}m^d}{y^d - p^{m*}m^d}, \qquad S_{gdp^x}^{m^x} = \frac{p^{m*}m^x}{xy^d - p^{m*}m^x}, \qquad S_y^I = \frac{\delta k}{y^d(1+x)},$$
$$1 = \frac{\theta}{\theta-1}mc^d, \qquad 1 = \frac{\theta}{\theta-1}mc^x,$$
$$x = p^{m*}m,$$

where the last equality is due to the assumption that the steady-state debt of the country is zero.

One can use the above formulas to calculate the required quantities. It is important to note that the homogeneity of production functions imply that at this stage of calculations one does not need the level y^d, hence we set $y^d = 1$.

The calculated steady-state values are $\bar{\alpha}_d = 0.330$, $\bar{\alpha}_x = 0.259$, $\bar{a}_d = 233$, $\bar{a}_x = 146$,$w = 10.808$, $x = 0.923$.

In the next stage we take as given $\omega^o = 0.75$, $\omega^{no} = 0.1625$, $\bar{c}^p = c^p/c = 0.248$, $S_y^g = g/y = 0.142$, $\theta_w = 3$, and calculate the values of y^d, c^o, c^p, $\bar{\omega}^o$, $\bar{\omega}^{no}$.

In this stage we use the above steady-sate input demand equations, furthermore the following steady-state labor demand

$$l^d = \bar{a}_d\left(\frac{w^{zd}}{w}\right)^{\varrho_z} z^d, \quad l^x = \bar{a}_x\left(\frac{w^{zx}}{w}\right)^{\varrho_z} z^x,$$

derived form formula (10).

Beyond this we have the following equations.

$$
\begin{aligned}
w &= \frac{\theta_w}{\theta_w - 1}\left[(1-h)c\right]^{\sigma}\left(l^d + l^x\right)^{\varphi},\\
y^d &= c + S_y^g y^d(1+x) + \delta\left(k^d + k^x\right),\\
\omega^o &= \frac{\bar{\omega}^o c^o}{c}, \qquad \omega^{no} = \frac{\bar{\omega}^{no} c^{no}}{c},\\
c &= \bar{\omega}^o c^o + \bar{\omega}^{no}\left[w\left(l^d + l^x\right) - S_y^g y^d(1+x)\right] + (1-\bar{\omega}^o - \bar{\omega}^{no})c^p,\\
\bar{c}^p &= \frac{c^p}{c}.
\end{aligned}
$$

Using the above formulas it is possible to calculate the required steady-state values. If the estimated mode is applied the numerical values are the following: $y^d = 14.616$, $c^o = 10.629$, $c^p = 1.856$, $\bar{\omega}^o = 0.528$, $\bar{\omega}^{no} = 0.119$. Given these values the steady-state values of the rest of variables can be calculated straightforwardly.

A.12 CONSTRUCTION OF THE KALMAN FILTER

This section describes the construction of the Kalman filter used for evaluating the likelihood function. The rational-expectation solution log-linearized model can be express by the following time-varying-coefficients difference equations

$$\mathscr{X}_t = \mathscr{P}_t \mathscr{X}_{t-1} + \mathscr{Q}_t \mathscr{Z}_t, \qquad z_t = \mathscr{R}_t z_{t-1} + \mathscr{E}_t$$

$\mathscr{X}_t$ is the vector of endogenous variables[12], $\mathscr{Z}_t$ is the vector of exogenous shocks[13] and $\mathscr{E}_t$ is the vector of innovations. In the crawling-peg period $\mathscr{P}_t = \mathscr{P}^{cr}$, $\mathscr{Q}_t = \mathscr{Q}^{cr}$, where matrices $\mathscr{P}^{cr}$ and $\mathscr{Q}^{cr}$ are the solutions of the system of equations 73)–(91), (94) and (96). While in the inflation-targeting period $\mathscr{P}_t = \mathscr{P}^{it}$, $\mathscr{Q}_t = \mathscr{Q}^{it}$, where matrices $\mathscr{P}^{it}$ and $\mathscr{Q}^{it}$ are the solutions of the system of equations (73)–(91) and (93)–(96).

The state-space form of the above difference equations is

$$\mathscr{V}_t = T_t \mathscr{V}_{t-1} + G_t \mathscr{E}_t,$$

where $\mathscr{V}_t' = [\mathscr{X}_t', \mathscr{Z}_t']$ and

$$T_t = \begin{bmatrix} \mathscr{P}_t & \mathscr{Q}_t \mathscr{P}_t \\ 0 & \mathscr{R} \end{bmatrix}, \qquad \text{and} \qquad G_t = \begin{bmatrix} \mathscr{Q}_t \\ 0 \end{bmatrix}.$$

The observation equation is given by

$$\mathscr{S}_t = H \mathscr{V}_t,$$

where $\mathscr{S}_t$ is the column vector of observed variables and H is a selection matrix. As Chapter 13 in Hamilton (1994) shows, it is possible to use a Kalman filter derived from a time-varying model for likelihood evaluation, only if the time-varying parameters are functions of exogenous and predetermined variables. The estimated AL versions fulfil this condition.

---

[12] Namely, $\hat{\pi}_t$, $\hat{\pi}_t^w$, $\tilde{c}_t$, $\tilde{c}_t^o$, $\tilde{c}_t^{no}$, $\widetilde{w}_t$, $\tilde{x}_t$, $\widetilde{m}_t$, $\tilde{l}_t$, $\tilde{n}_t$, $\tilde{k}_t$, $\tilde{I}_t$, $\tilde{q}_t$, $\tilde{e}_t$, $\hat{\imath}_t$, $\tilde{b}_t$, $\widetilde{Q}_t$ and $\tilde{r}_t^k$.

[13] That is, $\tilde{g}_t$, $\widetilde{P}_t^{m*}$, $d\bar{q}_t$, $\tilde{x}_t^*$, $\widetilde{A}_t$, $\tilde{\upsilon}_t$, $\tilde{\upsilon}_t^w$, $\tilde{\varepsilon}_t^c$, $\tilde{\varepsilon}_t^{de}$, $\tilde{\varepsilon}_t^{pr}$, $\tilde{\varepsilon}_t^n$, $\tilde{\varepsilon}_t^I$ and $\tilde{\varepsilon}_t^Q$ in the crawling peg period, and $\tilde{\varepsilon}_t^{de}$ is replaced by $\tilde{\varepsilon}_t^r$ in the inflation-targeting period.

Following Koopman & Durbin (2003), the Kalman filter is generated by the following recursive formulas.

$$\begin{aligned}
\mathcal{U}_t &= \mathcal{S}_t - H\xi_t, \\
F_t &= HP_tH', \\
\bar{\xi}_t &= \xi_t + P_tH_t'^{-1}\mathcal{U}_t, \\
\xi_{t+1} &= T_t\bar{\xi}_t, \\
K_t &= TP_tH_t'^{-1} \\
P_{t+1} &= T_tP_t(T_t - K_tH)' + T_tVT_t',
\end{aligned}$$

P_0 and ξ_0 are given. The matrix V is the variance-covariance matrix of $\mathcal{E}_t$. Series of the forecast error $\mathcal{U}_t$ and the matrix F_t are used to construct the logarithm of the likelihood function. This is given by

$$\mathcal{L}(\cdot) = -\frac{TN}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\det(F_t) - \frac{1}{2}\sum_{t=1}^{T}\mathcal{U}_t'F_t^{-1}\mathcal{U}_t,$$

where N is the number of observed variables, and T is the number of time periods used for estimation.

To initialize the above algorithm we set $\xi_0 = 0$, and P_0 is the unconditional variance-covariance matrix of the constant-coefficient process,

$$\mathcal{V}_t = T_0V_{t-1} + G_0\mathcal{E}_t,$$

that is, using formulas (10.2.17) and (10.2.18) of Hamilton (1994), it can be expressed as

$$\text{vec}(P_0) = (I - T_0 \otimes T_0)^{-1}\,\text{vec}(G_0VG_0')$$

where symbol ⊗ represents the *Kronecker product*, and operator *vec* transforms a quadratic matrix into a column vector by stacking the columns of the matrix one below the other, with the columns ordered from left to right.

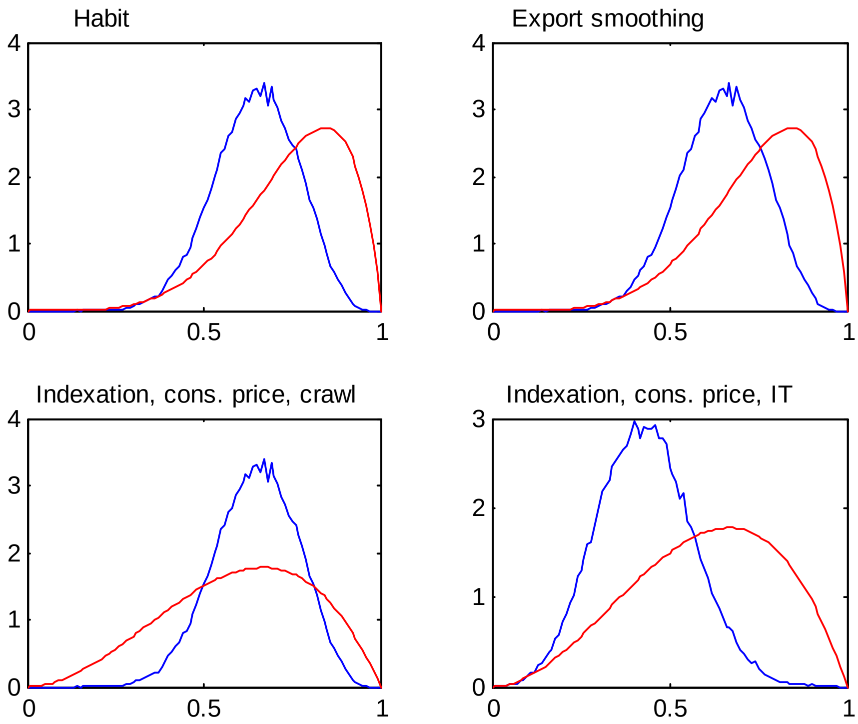
B Data

We estimated the log-linearized model on the sample between 1995Q2 and 2007Q2. We used twelve data series as observed variables. All data are quarterly, seasonally adjusted and imported from the database of the November, 2007 version of the Quarterly Projection Model (NEM) of the Magyar Nemzeti Bank (Benk *et al.* (2006)).

HP-filtered data were used in the case of GDP, capital stock, employment, consumption, investments, export, imports and real wages (private wages deflated by CPI inflation, WP/CPI in NEM) ($\lambda = 1600$). Consumption is defined as private 'consumption expenditures' (CE in the NEM). Investments contain all private investments (household and corporate investments - HI+CI in NEM). Government consumption equals to the sum of public investments, government purchases of goods and services and transfers in kind (GC+GI+TRAN in NEM). Employment is constructed as total (private plus public) employment. Capital stock is defined as private capital stock, excluded housing (KP in NEM).

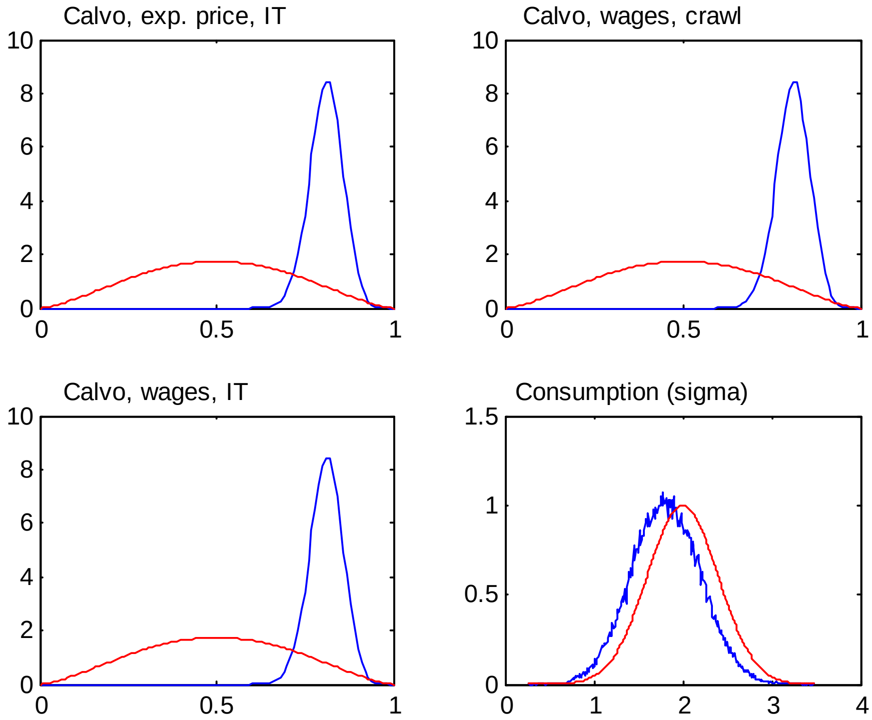
Price inflation data were calculated by a two step method. In Hungary there is a trend difference between non-traded and traded inflation caused by systematic productivity differential (Balassa-Samuelson effect) the real exchange rate has an appreciating trend (see Kovács (2002)). Our model, however, does not have two-sectors and thus unable to account for this, we first filtered out a trend (around 1.6 percent annually) from inflation data (quarterly non-traded inflation was reduced by 1 percent), and then a 'Balassa-Samuelson' filtered inflation series was calculated. As a second step we deduced the average nominal exchange rate change and demeaned inflation data. By this transformation we filtered out foreign inflation and the remaining trend in real exchange rate. Nominal wage inflation is then real wages plus this type of transformed consumer price inflation.

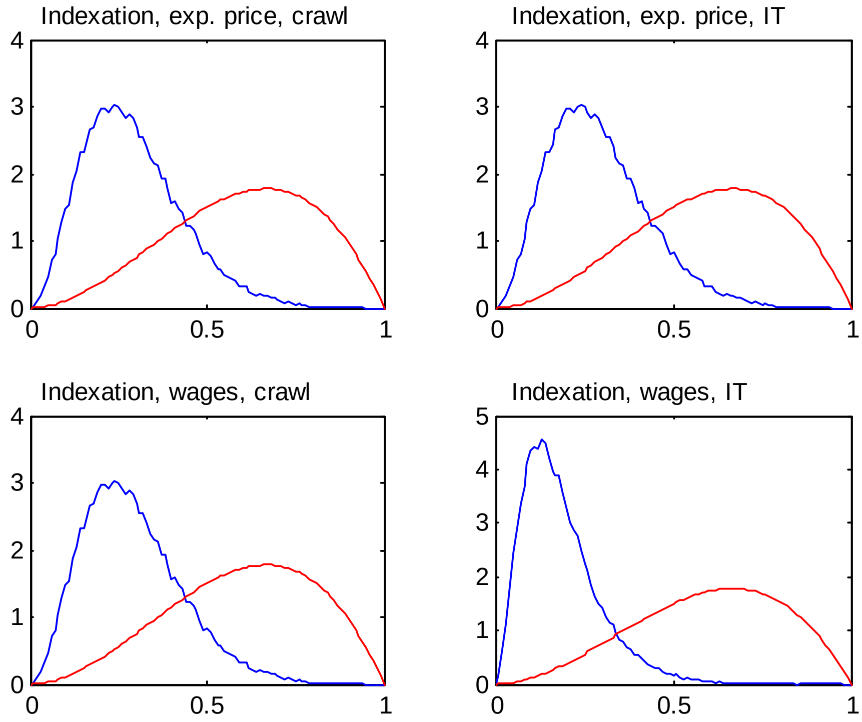
Export and import prices are defined in foreign currency units, that is export and import deflators divided by the nominal effective exchange rate of the Forint (PX/EFEX and PM/EFEX in NEM).

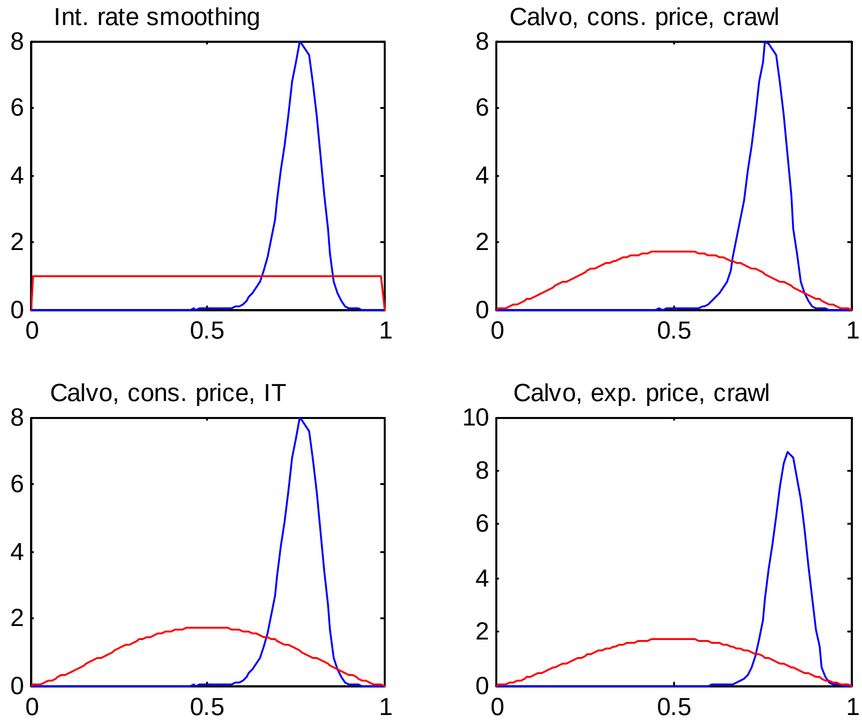


C Metropolis-Hastings Monte Carlo graphs

red line: prior density, blue line: posterior density


Calvo, exp. price, IT
Calvo, wages, crawl
Calvo, wages, IT
Consumption (sigma)


Indexation, exp. price, crawl
Indexation, exp. price, IT
Indexation, wages, crawl
Indexation, wages, IT


Int. rate smoothing
Calvo, cons. price, crawl
Calvo, cons. price, IT
Calvo, exp. price, crawl

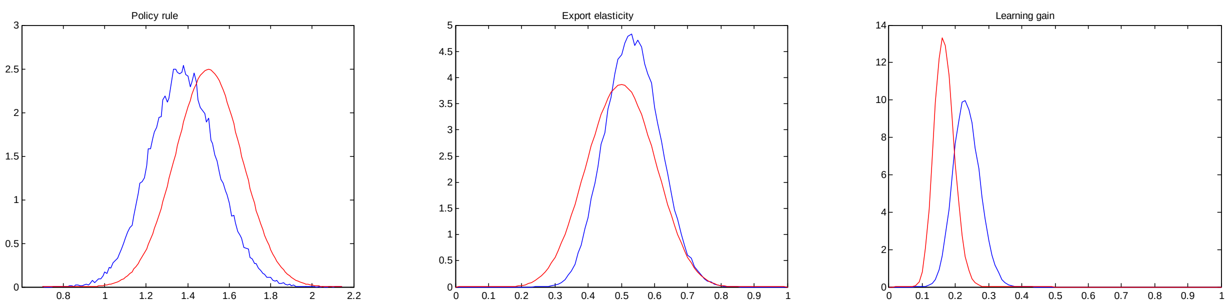
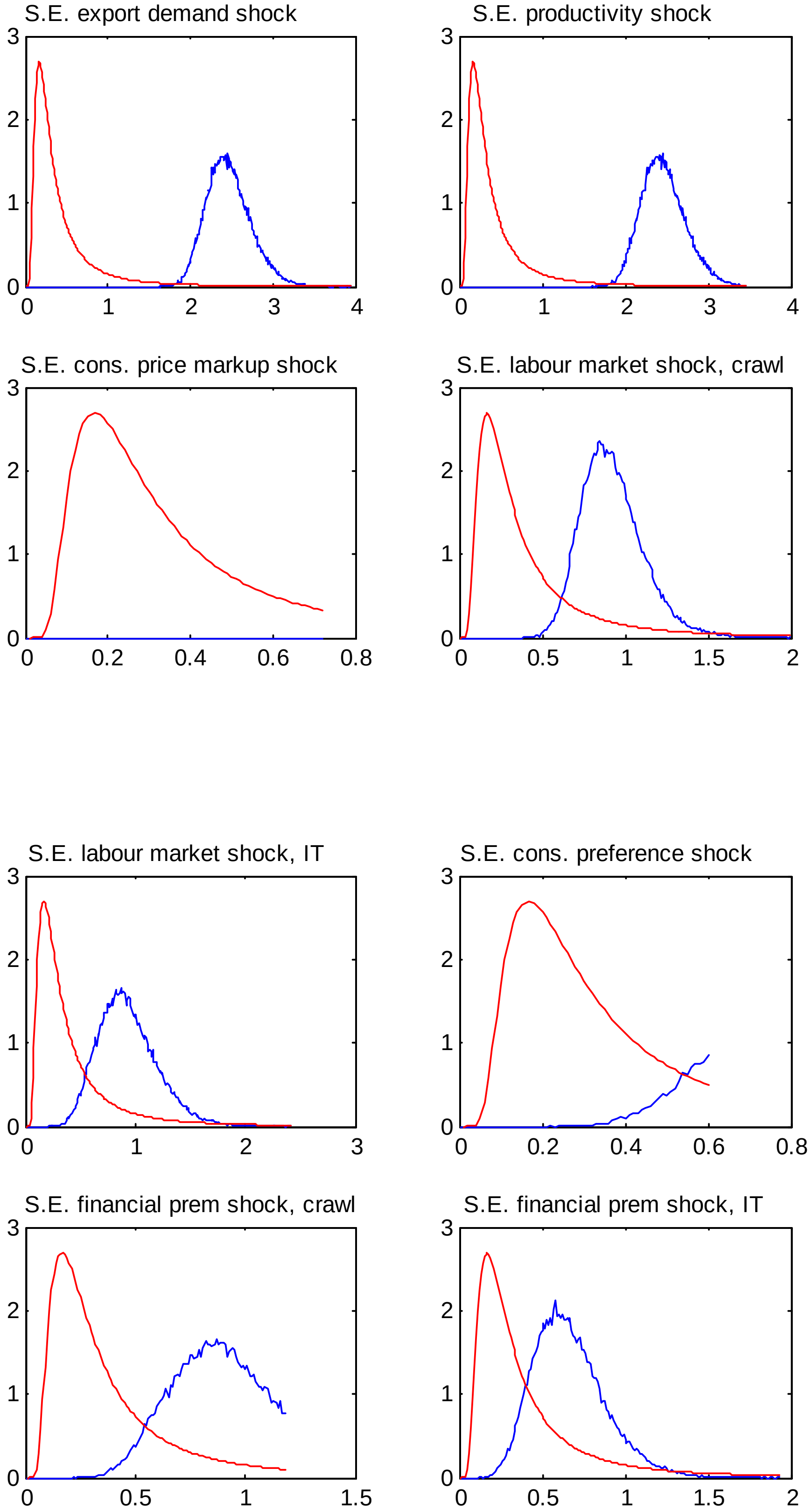
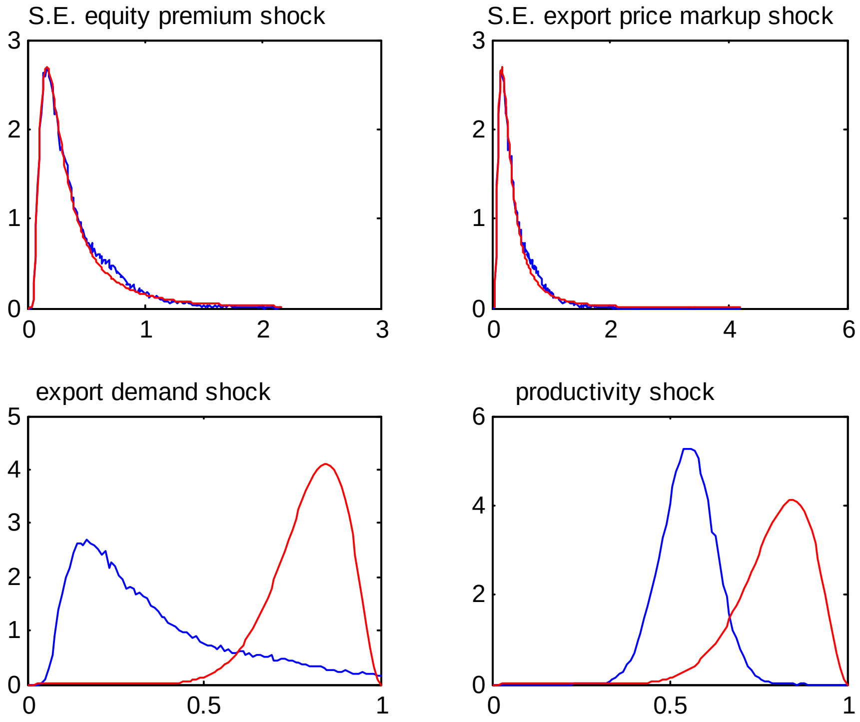

Policy rule
Export elasticity
Learning gain

Figure 23
Standard errors of shocks




S.E. equity premium shock
S.E. export price markup shock
export demand shock
productivity shock

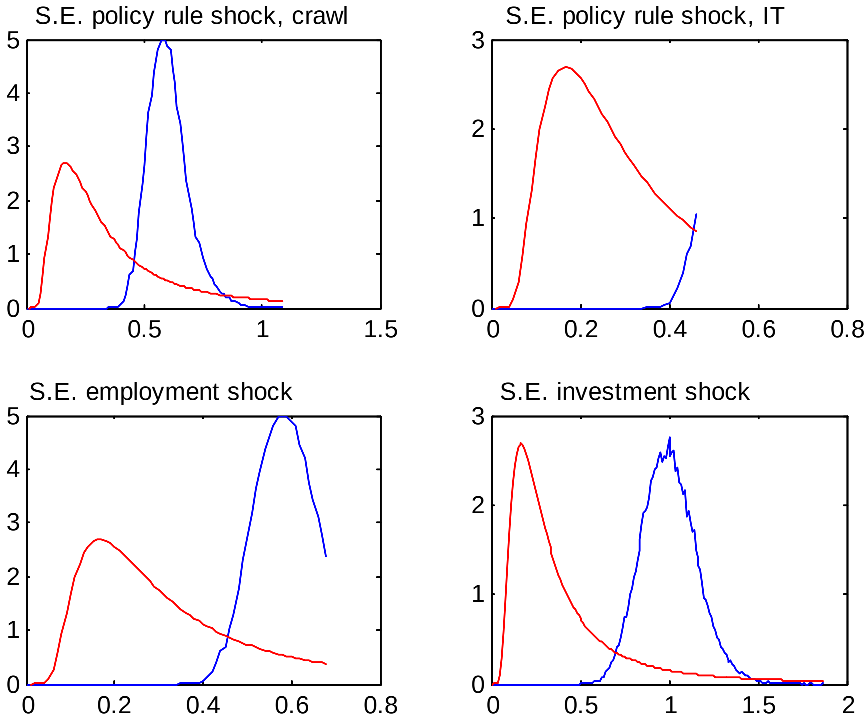
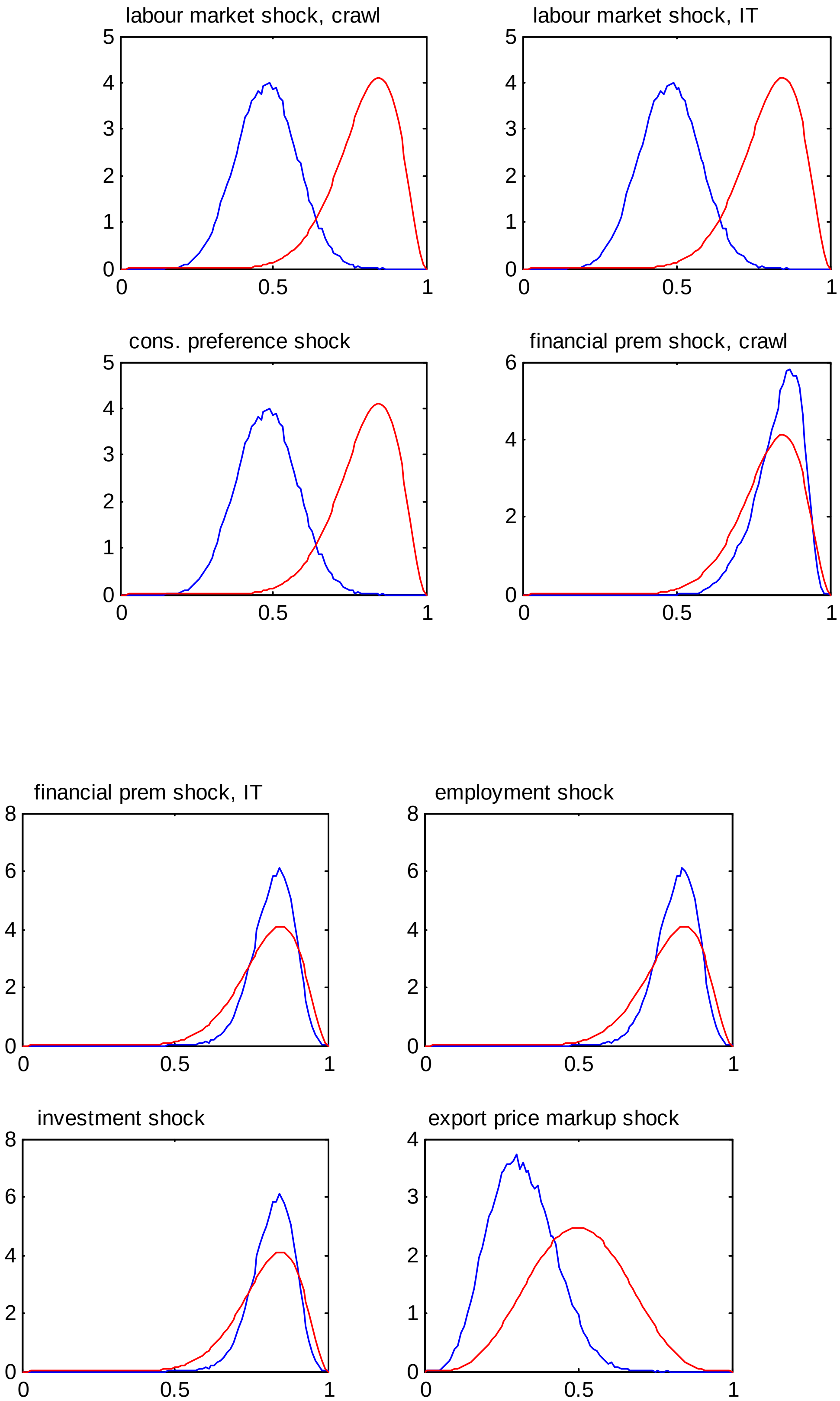

S.E. policy rule shock, crawl
S.E. policy rule shock, IT
S.E. employment shock
S.E. investment shock

Figure 24
Autoregressive coefficients of shocks



MNB Working Papers 2008/9
An estimated DSGE model of the Hungarian economy

Print: D-Plus
H–1037 Budapest, Csillaghegyi út 19–21.