TARGET ZONE REARRANGEMENTS AND EXCHANGE RATE BEHAVIOUR IN AN OPTIONS-BASED MODEL

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# The views expressed are those of the authors and do not necessarily reflect the official view of the Magyar Nemzeti Bank. This disclaimer is particularly important in the case of the applied numerical values of the future conversion rate of the Hungarian forint. These values were determined on the basis of the expectations of market analysts as reported in the Reuters polls. Numerical values of the final conversion rate were necessary to solve the model, and were utilized exclusively for illustrative purposes. None of the presented numerical values of the final conversion rate necessarily coincide with the preferred final conversion rate of the Magyar Nemzeti Bank.
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Summary

In this paper we develop an options-based model of target zone arrangements. A shift in an exchange rate target zone or the widening of the band presumably has an effect on the exchange rate. In the model the exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options. The novelty of our model is that the options are interrelated and we develop the appropriate option-pricing method. The model is able to deal with exchange rates with future fixing, such as the EMU-entry of the currencies we study. The strike prices of the options are the limits of the band, therefore, the direct effect of the band realignment on the exchange rate can be measured by the change of the option prices caused by the change of the strike prices. Exchange rate changes can be decomposed into (a) the direct effect of the band shift or band widening; (b) changing expectations; and (c) changing uncertainty. We apply the model to exchange rate band realignments (i.e. band shift or band widening) of Denmark, France, Hungary and Portugal.

*JEL classification:* F31; F33; G12; C63

*Keywords:* target zone system, options, option pricing, target zone realignment, EMU entry, EMS
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1. Introduction

The relationship between the exchange rate in a target zone system and options can be considered in two different ways. First, the currency of a target zone system can be regarded as a composite financial asset consisting of an underlying asset and two options; second, the currency of a target zone system itself can be the underlying product of an option. The target zone literature typically deals with the second type of relationship.\(^1\) We concentrate on the first type of connection.

In his seminal paper Krugman [1991] mentioned the possible link between the theory of the target zone system and the theory of option pricing, but he did not develop an option pricing model. The idea he set is that the exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options.\(^2\) According to our best knowledge, this topic has not been explored yet. There are only two papers, besides Krugman’s, which have touched on the issue. The first is Mikolasek [1998], in which the type of options is specified similarly to our model, but the underlying asset differs from the one we apply here. The other is Copeland [2000] (Chapter 15.), which demonstrated on binomial trees how the processes of the exchange rate and the fundamental are related to each other. Copeland studied the effect of only one side of the band, hence did not deal with the issues of joint determination of the two options.

Our options-based model is closely related to the seminal target zone model of Krugman [1991]. In the Krugman model the curvature of the exchange rate as a function of the fundamental is an S-shaped curve. This shape also characterizes our options-based model, in which we derive the target zone exchange rate as a function of the underlying floating exchange rate. We regard the underlying floating exchange rate as the shadow exchange rate, which can be thought of as the level of the exchange rate if there was no target zone system.\(^3\) In our options-based model the process of the target zone exchange rate is limited by the two options. At the expiration date of the options the target zone exchange rate is a broken

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\(^2\) “Now the actual exchange rate may be viewed as the price of a compound asset. This asset consists of the imaginary asset […], plus the right to sell the asset at a price \(s\), plus the obligation to sell at the price \(\bar{s}\) on demand.” Krugman [1991], p. 677.

\(^3\) Rangvid-Sørensen [2001] applies the similar concept of the shadow exchange rate for a different problem. They filter out the shadow exchange rate of some ERM currencies and investigate which fundamental macroeconomic factors are able to explain the behaviour of the filtered shadow exchange rate.
linear function of the underlying floating exchange rate (a flipped Z-curve), which is also the starting point of Krugman’s model. In the Krugman model the expected change of the exchange rate leads to an S-curve instead of the flipped Z-curve, whereas in our options-based model the non-linear feature of the options explains the shape of the curve. The close relationship between the Krugman model and the options-based model is also indicated by the fact that the graph of an option’s price as a function of the underlying asset has a curve shape caused by the expectations. Therefore, the options-based model can be thought of as a short cut to the process of the exchange rate, which utilizes the results of the option pricing literature. The difference between the Krugman model and our options-based model is that in the former the monetary authority should adjust the money supply to defend the target zone, while in the options-based model the floating exchange rate cannot be influenced by the central bank. Therefore, instead of the change of the fundamental it is the change of the option prices which limits the exchange rate. However, the options are created by the promise of the intervention.

The novelty of our paper is the accurate description of the options: the underlying assets of the options are not only the freely floating currency, but the freely floating currency combined with the other option. If a currency holder exercises the put option, then he or she is free not only from the floating currency, but also from the obligation incorporated in the call option. In addition, if the central bank exercises its call option, then the central bank buys the floating currency and withdraws the currency holder’s right incorporated in the put option. Therefore, the underlying product of the call option is the floating currency along with the put option, and the underlying product of the put option is the floating currency along with the call option. So each option is a part of the underlying product of the other option.

A special iterative option-pricing method is developed for these interrelated options in the binomial tree framework. To give a closed form solution to the target zone exchange rate as a function of the floating exchange rate is difficult, though it is possible in the case where some special processes are assumed for the floating exchange rate. Since we do not insist on deriving a closed form solution and we are aiming at determining the option prices and the target zone exchange rate numerically, we can freely choose among the theoretically appealing processes for the floating exchange rate. One of the chosen processes is applicable to floating exchange rates with future fixing.

The model is successfully applied to decompose and estimate the exchange rate changes after the band realignments: depreciations calculated by the model deviate only slightly from its historical level in the case of the Danish krone and Hungarian forint, which might be attributed to the fact that in these two cases the realignments were surely unanticipated by the market. In the case of the studied French and Portugal realignments, the model somewhat overestimated the actual depreciation.

Obviously, the options-based approach is only one of the possible approaches to the exchange rate of a target zone system, but it is rather appealing. The accurate description of the currency in the target zone system and its process is an exciting
and challenging objective. The solution presented in this paper is simple compared to the difficulties of the alternative models based on stochastic differential equations. Here we develop a numerical option-pricing method that does not require the solution of stochastic differential equations.

The rest of the paper is organized as follows. Section 2 describes the options-based model. Section 3 deals with the option pricing method. In section 4 we apply the model to the band realignments of Denmark (1979), France (1986), France (1993), Hungary (2003) and Portugal (1995). Section 5 deals with the possible objections to our model. Section 6 contains the conclusions.

2. The options-based model

According to the options-based model, the currency in a target zone system is nothing else than a currency in a freely floating system with two options. One of the options is a long put option with the strike price equal to the weak edge of the band. The other option is a short call option with the strike price equal to the strong edge of the band.

The strike prices, which coincide with the edges of the band, should not be stochastic variables, otherwise we would not be able to apply the options-based description of the currency in a target zone system. Therefore, we have to assume that the commitment to the exchange rate system is fully credible. This assumption also implies that in the case of countries aiming to join a currency union the band enjoys and is expected to enjoy full credibility up to the time of entry to the monetary union.

The existence of the two options can be explained in the following way. If the central bank promises to keep the exchange rate in the determined band until the final fixing, then, on one hand, the bank assumes the obligation of repurchasing its currency at the rate equal to the weak edge of the band. This provides a long put option from the viewpoint of the currency holders. On the other hand, the central bank does not let the exchange rate strengthen beyond the strong edge of the band. From the viewpoint of the currency holders it looks as if the central bank had a purchasing right at the rate equal to the strong edge of the band. Since foreign exchange market participants could exercise their put option by trading with the central bank, which promised to buy the weak domestic currency at the weak edge of the band, the existence of the put option is straightforward. On the other hand, the central bank cannot force anybody to sell its strong domestic currency at the strong edge of the band. Instead, the central bank has the obligation to buy an unlimited amount of the weak foreign currency at the strong edge of the band, which has the same effect on the exchange rate as a short call option.

It is also important to determine the type of these options. Obviously these are American-type options, because they can be exercised at any time within the existing target zone system. The two options are incorporated in the currency, because they do not exist separately. Moreover, the underlying assets of the
options are also far from being trivial. If a currency holder exercises the put option,\(^4\) then he or she is free not only from the floating currency, but also from the obligation incorporated in the call option. In addition, if the central bank exercises its call option, then the central bank buys the floating currency and withdraws the currency holder’s right incorporated in the put option. Therefore, the underlying product of the call option is the floating currency along with the put option, and the underlying product of the put option is the floating currency along with the call option. So each option is a part of the underlying product of the other option:

\[
s_t = f_t + P_{t,Kp,a}(f - C_{t,Kc,a}) - C_{t,Kc,a}(f + P_{t,Kp,a}) \tag{1a}
\]

where \(s_t\) is the exchange rate of the currency in the target zone system at time \(t\) measured as the foreign currency price of a unit of domestic currency, \(f_t\) is the exchange rate of the floating currency at time \(t\), \(P_{t,Kp,a}(f - C_{t,Kc,a})\) is the value of the American-type put option at time \(t\) with the strike price \(K_p\). Its underlying product is the floating currency along with the short call option. \(K_p\) equals the weak edge of the band. \(C_{t,Kc,a}(f + P_{t,Kp,a})\) is the value of the American-type call option at time \(t\) with the strike price \(K_c\). Its underlying asset is the floating currency along with the long put option. \(K_c\) equals the strong edge of the band.

This options-based model is, in fact, very unusual. What is special about this model is that both options are partly the underlying products of the others. Thus, the options are also partly underlying products of themselves. These self-references necessitate applying a special option pricing method, which is developed in the next section.

Krugman’s idea relating the options-based description can be formalized similarly to (1a):

\[
s_t = f_t + P_{t,Kp,a}(f) - C_{t,Kc,a}(f) \tag{1b}
\]

This expression differs from (1a) as the underlying product of the options is simply the floating currency. The difference between the two models is insignificant if the band is wide enough, because in that case only one of the options is worth much, while the other’s value is marginal. Consequently, the accurate and complex determination of the underlying product does not change the option prices significantly in a wide band, but might change it in a narrow band. Even if the simplified model (1b) works well in practice, it might lead to theoretical problems, such as having an exchange rate outside the band. To show that possibility, suppose that the simple put option \((P_{t,Kp,a}(f))\) is exercised. Then this put option along with its underlying product is worth as much as the strike

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\(^4\) Exercising the put option is equivalent to the intervention of the central bank at the weak edge of the band. The ability to intervene is limited by the foreign exchange reserves and imperfect cooperation among central banks. Here we ignore this fact.
price: $f_i + P_{i,Kp,a}(f) = Kp$. By plugging this expression into (1b), we get:

$s_i = f_i + P_{i,Kp,a}(f) - C_{i,Kc,a}(f) = Kp - C_{i,Kc,a}(f)$. Having a positively valued call option, the exchange rate gets outside its band, being less than the weak edge of the band ($Kp$).

3. Option pricing method

Although the option pricing literature discusses the problem of determining the price of a compound option, these results are not applicable here. The option pricing method in this case is not obvious for two reasons. First, a part of the underlying assets of each of these options is the other option itself, and not only one of the underlying assets contains an option. Second, these are American-type options, causing a little complication compared to the European-type options. Our option pricing method is a new iterative method developed in the basic option pricing framework, the binomial framework. Given the binomial tree of the floating exchange rate, this method produces the binomial tree of the call and put components of the target zone exchange rate and their combination – the binomial tree of the target zone exchange rate. In the first step of the iterative method we should determine the binomial tree of the options as if their underlying products were simply the floating part ($f_i$). We get a $\text{put}^{(1)}$ and a $\text{call}^{(1)}$ binomial tree.

No value of the $\text{put}^{(1)}$ binomial tree will be greater than the corresponding value of the binomial tree of the put component of the target zone exchange rate, because the value of the underlying asset of the put component is not greater than the value of the floating part ($f - C_{Kc,a} \leq f$).

The same consideration can be applied to the call option. No value of the $\text{call}^{(1)}$ binomial tree will be greater than the corresponding value of the binomial tree of the call component of the target zone exchange rate, because the value of the underlying asset of the call component is not less than the value of the floating part ($f + P_{Kp,a} \geq f$).

The second step of the iterative method will supply the $\text{put}^{(2)}$ and the $\text{call}^{(2)}$ binomial trees. The underlying product of the $\text{put}^{(2)}$ binomial tree will be $f - C_{Kc,a}^1$, where $C_{Kc,a}^1$ is the call option, the process of which is described by the $\text{call}^{(1)}$ binomial tree. Since the value of the underlying product is not less than the value of the underlying product of the put component of the target zone exchange rate ($f - C_{Kc,a}^1 \geq f - C_{Kc,a}$), no node value of the $\text{put}^{(2)}$ binomial tree will be greater than the corresponding node value of the binomial tree of the put component. Similarly, no value of the $\text{put}^{(2)}$ binomial tree will be less than the corresponding value of the $\text{put}^{(1)}$ binomial tree, which can also be derived from the comparison of the underlying products.

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5 See, for example, Geske [1979].
The underlying product of the call\(^{(2)}\) binomial tree will be \( f + P_{kp,a}^{(1)} \), where \( P_{kp,a}^{(1)} \) is the put option with the process \( put^{(1)} \). As the value of the underlying product is not greater than the value of the underlying product of the call component of the target zone exchange rate \( f + P_{kp,a}^{(1)} \leq f + P_{kp,a}^{(2)} \), no node value of the call\(^{(2)}\) binomial tree will be greater than the corresponding node value of the binomial tree of the call component of the target zone exchange rate. Similarly, no value of the call\(^{(2)}\) binomial tree will be less than the corresponding value of the call\(^{(1)}\) binomial tree, which can also be derived from the comparison of the underlying products.

Continuing the iterative method: in the \( i \)-th step the underlying product of the put\(^{(i)}\) will be \( f - C_{kc,a}^{i-1} \) and the underlying product of the call\(^{(i)}\) will be \( f + P_{kp,a}^{i-1} \). The sequence of the created put and call binomial trees will be monotonously increasing on each node, but they will never grow over the binomial trees of the put and call components of the target zone exchange rate. According to a convergence theorem,\(^{6}\) these sequences of binomial trees are convergent on each node, as they are both monotonous and bounded. We will refer to the limits of the sequence of the put and call binomial trees as put-limit, and call-limit binomial trees.

The put-limit binomial tree is identical to the binomial tree of the put component of the target zone exchange rate and the call-limit binomial tree is identical to the binomial tree of the call component of the target zone exchange rate.

4. Application of the model

First we show how the model is applied to analyse band realignments and then we discuss the analysed range of currencies and periods.

According to the options-based model the target zone exchange rate is a function of the floating exchange rate. The relationship between the spot value of the floating exchange rate and the spot value of the target zone exchange rate can be obtained by applying the option pricing method. First we have to specify the process of the floating exchange rate. Then, by applying a comparative static analysis, the derived functional relationships can be used to determine the effects of the realignment on the exchange rate by taking into account the changing band, the changing expectations and changing uncertainty. The changing band changes the strike prices of the options resulting in a different functional relationship between the floating and target zone exchange rates. Comparing the original and this modified functional relationship, and assuming that the floating exchange rate is not affected by the realignment, we can determine the direct effect of the realignment on the exchange rate.\(^{7}\)

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\(^{6}\) See for example: Pownall [1994], p.162 Theorem 2.6.1 part (iii).

\(^{7}\) Supposing that after the band shift the fundamental/floating exchange rate remains unchanged and its process as well, the function \( g_0: \log(F) \rightarrow \log(S) \) describing the relationship
The indirect effects of the realignment are related to growing uncertainty and in the case of countries aiming to join a currency union, to changing expectations about the final conversion rate. An unexpected realignment almost surely motivates the participants of the market to revise their expectations about the final conversion rate, and the realignment might cause disturbance on the market as well. These modify the functional relationship between the exchange rate and the floating exchange rate by changing the process of the floating exchange rate. Comparing again the modified and the original functional relationship, one can draw inferences about the indirect effects of the realignment.

The EMS was a multilateral target zone exchange rate system. Typically the bilateral bandwidths were ±2.25% at the start of the system, with a wider margin for the Italian Lira. After the currency crises in 1993, the bands were widened to ±15%.

There were fifty-six realignments during the 1979-1997 period, implemented in seventeen discrete adjustments. Consequently, the EMS has witnessed plenty of devaluation episodes, but many of them do not meet the assumptions of the model. In many cases the exchange rate realignments took place when the exchange rates were very near to their limits. In this set-up one should be suspicious as to whether the market forced the band realignment. In these cases we cannot adopt the options-based model for two reasons. First, the options-based model assumes the credibility of the band. Second, even if the exchange rate system was credible, when the exchange rate stays close to the edge of the band for a longer period, then it is impossible to determine the floating exchange rate from

\[ g_1(\log(f)) = g_0(\log(f) - x\%) + x\% \]

The derivation is based on the homogeneity property of the general

\[ b: (\log(K_p), \log(K_e), \log(f)) \rightarrow \log(S) \]

8 The effective fluctuation bands of a multilateral target zone system are narrower than the official bands. We can illustrate this idea if we think of three currencies: A, B, C, with each pairwise exchange rate fluctuating in bands of the same size. It is impossible that A is maximally appreciated against B, while maximally depreciated against C, otherwise the exchange rate between B and C would be out of its band. Consequently, the intervention points do not coincide with the edges of the official bands. See Flandreau [1998] about modelling the effective bands in a multilateral target zone system. Here we do not distinguish between the effective and the official bands.

9 The band width of the Italian Lira as well as the band width of the currencies of the newcomers, like the peseta, the sterling and the escudo were ±6%.

10 Except for the exchange rate of the Dutch guilder versus German mark, which remained in the narrow band of ±2.25%.

the observed exchange rate. Consequently, the model cannot analyse all the fifty-six realignments.

Therefore, we selected certain episodes from the EMS experience: the French band widening in 1993, and the Portuguese (1995), French (1986) and Danish (1989) band shifts. We also study the 2003 Hungarian band shift. Figures 3. and 4. show the time series of the exchange rates.

Since the histories of these currencies diverge, we assume two different processes for the floating exchange rate in different cases. One of the processes is mainly driven by the convergence to the expected final conversion rate, while the other is assumed to be a random walk without drift. Whichever process is applied, the graph of the exchange rate as a function of the floating exchange rate is an $S$-shaped curve. First we discuss the case with convergence toward the final conversion rate, then we turn to the random walk case.

The French band widening took place approximately 6 years before its irrevocable fixing and the analysed Portuguese band shift was approximately 4 years prior to its fixing. Hungary will presumably be a member of the EMU in 5-6 years, so the exchange rate of the forint will also be fixed in the relatively near future. The expected final conversion rate surely has a strong influence on the spot exchange rate in the run-up period to currency union. Motivated by this, we assume that in these above-mentioned three cases the process of the floating exchange rate is mainly driven by the convergence to the expected final conversion rate. After the irrevocable “disappearance” of these currencies it does not make sense to talk about an alternative exchange rate system, and about the floating exchange rate, which would be the actual exchange rate if the exchange rate system were fully flexible. Consequently, we assume that the underlying floating exchange rate will be fixed at the same time as the target zone system will be replaced by the fixed system and the numerical value of the underlying fixed rate is equal to the final conversion rate. The assumed continuous process of the floating exchange rate is:

\[ df_t = \mu_t \cdot dt + \sigma_t \cdot dz \]  
\[ dz \text{ is a standard Wiener-process} \]  
\[ \mu_t = \frac{s_T - f_t}{T - t} \]  
\[ \sigma_t = \sigma_0 \cdot \frac{T - t}{T} \]  

where the floating exchange rate at time $t$ is denoted by $f_t$. The exchange rate will be fixed at time $T$ at the value $s_T$. The $\mu_t$ and the $\sigma_t$ are the instantaneous expected value and standard deviation respectively.

The expression for $\mu_t$ ensures the convergence of the floating exchange rate toward the final conversion rate ($s_T$). The monotonously decreasing $\sigma_t$ over time
ensures that the deviation from the expected value is diminishing over time, and at
time $T$ the exchange rate should be equal to its final conversion rate.

Since the option pricing method developed here is based on the discrete
binomial model, the continuous process of the floating exchange rate given above
needs to be discretized. The discretized version of this process is illustrated by
figure 1. The initial value of the floating exchange rate is denoted by $f_0$; the final
conversion rate applied at time $T$ is denoted by $s_T$. The spread of the process is
determined by parameter $b$, playing a similar role as parameter $\sigma_0$ in the continuous
version (2a). We divide the time interval $[0,T]$ into $N$ number of small time
intervals of length $dt$; consequently parameter $N$ determines the “fineness” of the
binomial model showing the number of subperiods until the final fixing. By
refining the discrete model ($N \to \infty$) we get the asymptotic process of the discrete
model, which is equivalent to the process determined by the continuous version.
At each subperiod the exchange rate can shift either upward or downward, as is
usual in the binomial framework. The nodes are equidistantly distributed at the
initial time around the initial value of the floating exchange rate ($f_0$); nodes are on
the “radials” starting at the initial nodes and ending at the node representing the
final fixing. As a consequence of the equidistant distribution and of the
assumption that shifts towards both directions are equally likely, the expected shift
of the exchange rate at any node is along the “radial” starting from the given node.

The process of the floating exchange rate defined above is in accordance with
our intuitive requirements: as time passes the ex ante range of the process grows
until the effect of the final fixing becomes dominant, turning the process to
“constrict”. Finally, the process should end up at its conversion rate. The process
of the floating exchange rate is illustrated by figure 2, which shows the expected
symmetrical confidence interval of the exchange rate and of the floating exchange
rate in the case of the Hungarian forint.

After the geometric description of the assumed discrete process of the floating
exchange rate, we present its algebraic definition by determining its value for an
arbitrary node. At time $i \cdot dt$ the floating exchange rate for the grid representing $k$
times “upward” shifts and $i-k$ times “downward” shifts is:

$$f_{i \cdot dt, up, k} = \frac{i}{N} \cdot s_T + \frac{N-i}{N} \cdot \left\{ f_0 + h \cdot \left[ k - (i-k) \right] \right\}$$

(2b)

Before discussing the other process (the case without monetary union), we will
describe how some of the relevant parameters are set. In the case of the French
franc and the Portuguese escudo we assume that the final conversion rate ($s_T$), and
the expected final conversion rate are always equal to the actual central parity,
before and after the realignment as well. This assumption is related to the fact that
the central parity of the French franc at the time of band widening and the actual
central parity after the band shift of the Portuguese escudo became the final
conversion rate some years later. We also assume that the length of time period
until fixing is equal to the observed one, in the case of the French franc it is
approximately 6 years, and in case of the Portuguese escudo it is 3.8 years. With
this simplification we do not take into account the uncertainty related to the final conversion rate and the uncertainty related to the time of fixing.

In the Hungarian case not only the final conversion rate but also the date of euro introduction was highly uncertain. In the case of Hungary the monthly Reuters Poll,\textsuperscript{12} which surveys market analysts, provided some information that we can use to calibrate the model. First, it contained a question regarding the expected date of euro introduction and second, it contained a question regarding the exchange rate two years ahead. The latter could be closely related to the expected final conversion rate.

Denmark has not joined the Euro-area even by now, and the studied band widening of the French franc in 1986 was much earlier than Euro-area entry in 1999. Consequently it would not be plausible to assume that the process of the floating exchange rate of the Danish krone in 1979 and of the French franc in 1986 were driven by the convergence to any fixed value. In these cases we assume that the process of the floating exchange rate is a geometric Brownian motion:

\[ \frac{df}{f} = \sigma \cdot dz \]  
\[ dz \] is a standard Wiener-process \hspace{1cm} (3a)

Since the option pricing method is based on the discrete binomial model, we have to discretize the process (3a) as well. We work with the process of the Cox-Ross-Rubinstein (CRR) binomial model\textsuperscript{13} with no drift, which provides a possible discretization of the geometric Brownian motion (3a). At time \(i \cdot dt\) the floating exchange rate for the grid representing \(k\) times “upward” shifts and \(i-k\) times “downward” shifts is:

\[ f_{i\cdot dt, u=k} = u^k \cdot d^{i-k} \cdot f_0 \] \hspace{1cm} (3b)

\[ u = \exp(\sigma \sqrt{T/N}) \]

\[ d = 1/u \]

\[ p = \frac{1-d}{u-d} \]

where \(f_0\) is the initial value of the floating exchange rate, \(p\) denotes the probability of an “upward” shift and parameter \(\sigma\) denotes the constant instantaneous volatility of the floating exchange rate. Applying the process (3b),

\textsuperscript{12} Preceding the Hungarian band shift of 4 June 2003. The Reuters Poll interviewed market analysts on 22 May. The next Reuters Poll was on 19 June. Consequently, the polls reports were not exactly about the expectations just before and just after the realignment. On the other hand, this is our best source of information about the expectations.

\textsuperscript{13} The most often used binomial model is the Cox-Ross-Rubinstein model. See: Cox-Ross-Rubinstein [1979].
almost the same set of parameters \((T, \sigma, f_0)\) has to be given as with process (2b), but there is no roll for the final conversion rate \((s_f)\).

In order to determine the process of the exchange rate we have to set not only the parameters belonging to the process of the floating exchange rate \((T, N, f_0, s_T, b/\sigma)\), but also the foreign and domestic yield curves to be able to price the options. The following simplifications are made: we assume that the yield curves are horizontal in each case except with that of the Hungarian forint. This simplification does not modify our results since the interest rate differential does not affect the exchange rate directly in this model, as the interest rates influence the exchange rate only through the option prices. This indirect effect is marginal.

4.1 The band widening of the French franc in 1993

On 3 August 1993 the exchange rate band was widened from its original width \(\pm 2.25\%\) to \(\pm 15\%\), while the central parity remained unchanged. The band widening had a similar effect on the French franc, Belgian franc and Danish krone. Here we study only the case of French franc.

The exchange rate on the day before the band widening already reached the edge of the band. (See figure 4/a). An exchange rate equal to the edge of the band does not determine the floating exchange rate uniquely. On the previous days the exchange rate did not stick to the edge of the band, therefore we determine the exchange rate before \((s_b)\) and after the band widening \((s_a)\) as the arbitrarily chosen 15 days average exchange rates. Thus, calculating with the average exchange rate before the band widening has technical reasons. After the band widening the volatility grew, so an average exchange rate better describes the exchange rate after the band widening, than the single exchange rate data of the consecutive day.

The annualised volatility characterizing the period before the band widening was \(2.8\%\), and it grew substantially to \(6.6\%\) after the widening. The \(b\) parameter determines the spread of the discretized process of the floating exchange rate. But

\[\text{Source of inter-bank short term interest rates: www.ecu-activities.htm}\]

\[\text{In case of the Hungarian forint the calculations were done with both the simplified horizontal yield curves and with the actual observed forint and euro yield curves. For the latter, more accurate calculations are presented by figure 5/c. However, applying the two assumptions about the interest rates did not lead to a significant difference in the option's prices.}\]

\[\text{We calculated the volatility characterising the period before the band widening, and the period after the realignment from 15 daily exchange rate data. We annualised the volatility by the standard method: the standard error estimated from daily data was multiplied by } \sqrt{250}\text{. This method of annualisation should not be applied on bounded processes, because in that case the volatility is bounded as well. The maximum volatility in a target zone with band width } \pm 15\%\text{ is } 30\%. \text{ If the estimated volatility is } 3\%, \text{ then by multiplying it by } \sqrt{250} \text{ we get unreasonably high, more than } 30\% \text{ annualised volatility. In the case of the Hungarian forint we get over the problem of annualising by setting the three months volatility according to the historical implied volatility of options with maturity of three months.}\]
this variable is unobservable, just like the floating exchange rate, so this parameter has to be set according to the observed volatility and observed exchange rate. Here we note that the instantaneous volatility of the floating exchange rate depends not only on the volatility of the actual exchange rate, but also on its level. For a given value of the volatility of the floating exchange rate, the closer the exchange rate is to the edges of the band the lower is the volatility of the actual exchange rate due to the limiting effect of the band. The exchange rate was near to the edge of its old band, but it moved into the middle range of the new band. For a fixed \( b \) the volatility of the exchange rate tends to be higher if the exchange rate gets toward the middle of the band from both of the edges. Consequently, a growing observed volatility of the exchange rate could be reproduced even with an unchanged \( b \). Moreover, the volatility of the exchange rate can grow even if \( b \) decreases, in the case where it is accompanied by a shift toward the middle of the band. In this particular case of the French franc the exchange rate’s relative position in the band changed substantially so that the \( b \) parameter has to be decreased after the band widening despite the growing volatility. So, in the case of the French band widening both the volatility and the relative position of the exchange rate in the band changed, but the latter’s effect on \( b \) is more dominant.\(^{17}\)

As it was indicated earlier, the final conversion rate \( (s_T) \), as well as the time of fixing \( (T) \) is set, in our calculations, to be equal to the values historically developed later.

Figure 5/a. shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its graph 0 demonstrates the relationship before the band shift. Since the exchange rate before the band widening \( (s_0) \) was approximately 3.41 FRF/DEM the floating exchange rate should have been 3.58 FRF/DEM according to graph 0. Graph 1 deviates from graph 0 mainly because the strike prices of the options were changed. The short-term inter-bank interest rates of the mark and the franc also changed after the realignment, but this had only marginal effect on the shape of graph 1.

Graph 2 shows not only the effect of the changing strike prices, but also of the changing volatility. According to both graph 1 and graph 2, the exchange rate should have weakened approximately to 3.56 FRF/DEM as a result of the band widening if the floating exchange rate remained unchanged. Consequently the model attributes 4.3% weakening to the direct effect of the realignment. No effect is attributed to the changing volatility, since graph 2 does not differ significantly from graph 1.

\(^{17}\) The \( b \) parameter had to be set such that the volatility of model generated exchange rate at its observed level became equal to the measured volatility. In this particular case the exchange rate characterising the period before the widening was \( s_0=3.414 \) FRF/DEM and the volatility was 2.8%, indicating the value of the parameter \( b \) to be set equal to 4.5; whereas the parameter \( b \) after the realignment is set to a different value, 3.4, because the exchange rate and the volatility characterising the period after the realignment is \( s_1=3.503 \) FRF/DEM and 6.6% respectively.
The result of the model-based analyses does not coincide with the observed weakening: the 15 days average exchange rate of the French franc was about 3.51 FRF/DEM after the band widening (s0), meaning 2.7% weakening. On the other hand, the difference between the model-based (4.3%) and the observed (2.7%) depreciation is not extremely large if we take into account the fact that the exchange rate could fluctuate in a relatively wide, ±15% range after the widening.\(^{18}\)

A possible reason for the deviation might be based on the changing interest rate differential, which had risen from 0.92% to 1.37%. The changing interest rate differential might have had an effect on the exchange rate, which effect is not represented realistically by the model. Since the process of the floating exchange rate is mainly driven by the expected conversion rate, the interest rate differential plays only a minor role in this model, just like as its change does.

### 4.2 The band shift of the Portuguese escudo in 1995

In 1995 the exchange rate of the Portuguese escudo fell about 1-2% from 104 PTE/DEM to 105-106 PTE/DEM as an effect of the 3.6% devaluation of the band. Preceding the band shift the annualised volatility\(^{19}\) was 2%, which had risen to 5% after the realignment. The relatively low volatility evolved despite the fact that the exchange rate was in the more volatile region of the band, i.e. in the middle range, and the band was already widened allowing the exchange rate to fluctuate in a ±15% band.

We want to study here not only the direct effect of the realignment to the exchange rate, but also the effect of the changing expectations relating to the final conversion rate, and the effect of changing volatility.

Figure 5/b. shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its graph 0 demonstrates the relationship before the band shift. Graph 1 deviates from graph 0 mainly because the strike prices of the options were changed. The inter-bank interest rates of the German mark and the escudo also changed, but this had only marginal effect on the shape of graph 1. The exchange rate was approximately 104 PTE/DEM before the band shift (s0), and the adherent floating exchange rate should have been the same according to graph 0. If we assume that the floating exchange rate remained unchanged after the band shift, then the exchange rate should not have to change according to the graph 1. This is due to the coincidence of graphs 0 and 1 over the important part of their domain. Consequently, the band shift has no direct effect on the exchange rate. It is not surprising that according to the model the direct effect of the band shift is insignificant, since the exchange rate was near to the central parity, the band was relatively wide and the volatility was low. In that case the limits of the

\(^{18}\) The maximum weakening, independent of any theory and calculated from the difference of the band widths, could be approximately 15%-2.25%=12.75%.

\(^{19}\) The volatility was estimated from 15 daily data.
band, as well as the change of the limits, should only marginally influence the exchange rate.

An important parameter i.e. the final conversion rate, assumed to be the same with graph 0 and graph 1, equals the central parity before the band shift. The next graph, graph 2, differs from graph 1 because the change of the final conversion rate was also taken into account. We assumed that the final conversion rate changed to the new central parity. Graph 2 seems to coincide with graph 1 over the entire domain, so, in the case where the floating exchange rate remains unchanged, the changing expectations relating to the final conversion rate has no effect on the exchange rate. We made the assumption that the floating exchange rate will be fixed at the same rate and same time as the exchange rate. Based on this assumption one can argue that, as a result of the band shift, not only the expected final conversion rate should change, but also the floating exchange rate, because its expected final conversion rate changes as well. We can disregard the assumption of an unchanged floating exchange rate and assume, for instance, that the floating exchange rate weakens by the same rate as its expected final conversion rate. As a result of the almost linear relationship between the floating exchange rate and the actual exchange rate inside the band, the latter should weaken by the same rate. As a consequence, the escudo should have weakened by the rate of the band shift, namely 3.6% due to the changing expectations. (See the arrow of figure 5/b.)

Graph 3 embodies not only the changing strike prices and the changing final conversion rate, but also the changing volatility. The changing volatility affects the shape of the graph only in the irrelevant range, near to the edges of the band by bending it. Consequently, the changing volatility should not have any effect on the exchange rate either.

The observed weakening of the exchange rate was 1-2%, which is less than 3.6%. One reason for the deviation might be that the floating exchange rate weakens by less than its expected final conversion rate. Another possible explanation is based on the changing interest rate differential, which had risen from 4.86% to 5.99%. The same explanation was given in the case of the French band widening. The changing interest rate differential might have had an effect on the exchange rate, which became insignificant according to the model. Since the process of the floating exchange rate is mainly driven by the expected conversion rate and not, for instance, by the uncovered interest rate parity, the interest rate differential plays only a minor role in this model.

4.3 The band shift of the Hungarian forint in 2003

At the request of the Hungarian Government, on 4 June 2003 the Magyar Nemzeti Bank agreed to shift the central parity of the forint by 2.26%, with the currency’s ±15% intervention band remaining unchanged. The exchange rate of the Hungarian forint weakened by 2.26%, to 26.77 forint per euro. The rate was in line with expectations, as indicated by a survey conducted by the Central Bank in May 2003. The survey showed that most respondents believed the rate would be in the range of 26.5-27.0 forint per euro.

Hungarian forint fell about 6% calculated on the basis of the weakest exchange rate on the next day. The market was consolidated on this day partly due to a verbal intervention and an ECB rate cut, resulting in depreciation to the extent of at least 3.5%. This depreciation (3.5%-6%) might be interpreted as being relatively large, after a small, but unanticipated shift of the band. The band shift surprised the market, because a few months before the decision a shift in the opposite direction was expected by some participants of the market. At that time the forint’s expected strengthening led to a speculative attack in January 2003.\footnote{See: Barabás [2003].}

The exchange rate of the Hungarian forint had begun to weaken slowly before the realignment, but it was far away from the weak edge of the band, so the realignment was not at all enforced by the market.

We have more information about the recent realignment of the Hungarian forint than about the other studied realignments. For example, the Reuters Poll provides information about the expected time of Hungary’s entry into the EMU, while in case of the French franc and Portuguese escudo we simply assumed that the expected time until the fixing is equal to the historically observed one.

One has to have data about the interest rates in order to price the options. We use the forint and the euro yield curves instead of using only a short-term interest rate and assuming that the yield curve is horizontal. But this does not influence our results, since the interest rates have only marginal effect on the exchange rate. After the realignment, the Hungarian central bank base rate was hiked from 6.5% to 9.5% in two steps, and meanwhile the ECB cut its interest rate by 50 basis points.\footnote{On 10 June the Hungarian central bank base rate was increased by 100 basis points and on 19 June a further 200 basis points increase took place. The ECB interest rate cut took place on 5 June. The pre band shift yield curves were set equal to the EUR and HUF yield curves effective on 3 June 2003, and the yield curves effective on date 20 June 2003 were used to characterise the post band shift period.} Although these measures affected the interest rate differential and its term structure, these had only insignificant effect on the exchange rate according to the model.

Data on currency option prices provides us with the implied volatility. Thus, instead of using historical volatility, we could use the implied volatility, which is a forward-looking measure of the variability of the exchange rate. The unexpected realignment caused the jump of both the historical volatility and the implied volatility. The annualised implied volatility had grown from 6% to 11%.

The monthly Reuters Poll informs us that the average expected exchange rate for the end of 2004 was 238.7 HUF/EUR before the realignment, and after the realignment it became weaker by 4%, equal to 248.4 HUF/EUR. At the time of realignment the expected final conversion rate was not questioned by the Reuters Poll, so the expected final conversion rate has to be set equal to its closest
substitute available: the average expected exchange rate for end 2004, the furthest time reported. Hence, we set the parameter of the final conversion rate \( (s_f) \) equal to 238.7 HUF/EUR before the realignment and to 248.4 HUF/EUR after the realignment. The reported average expected date of Hungary’s entry to the EMU was the middle of 2008 and did not change for the Reuters Poll after the realignment. Consequently, we set \( T \) equal to 5 years according to the expectations.

Figure 5/c. shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its graph 0 demonstrates the relationship before the band shift. The exchange rate was approximately 256 HUF/EUR before the band shift, and the adherent floating exchange rate should have been 252.6 HUF/EUR according to graph 0. Graph 1 deviates from graph 0 mainly because the strike prices of the options were changed, but the changing yield curves were also taken into account. For the unchanged floating exchange rate (252.6 HUF/EUR) the exchange rate is 258.1 HUF/EUR according to graph 1. Consequently the direct effect of the band shift is near to 1%.

Taking into account the further effects of the band shift, we plotted graph 2 and graph 3 as well, which demonstrate the effect of the changing final conversion rate and the growing volatility respectively. We assumed that the floating exchange rate weakens by the same rate as its final conversion rate or the expected final conversion rate. Since the final conversion rate of the floating exchange rate is equal to the final conversion rate of the exchange rate by assumption, the floating exchange rate should depreciate by 4% as well, resulting in a floating exchange rate equal to 262.9 HUF/EUR. (See the arrow of figure 5/c.) At that weaker level of the floating exchange rate the exchange rate is 264.8 HUF/EUR according to graph 2, indicating a 3% depreciation as an effect of the changing expectations. Graph 3 differs from graph 2 by taking into account the increased volatility: parameter \( b \) was increased from 2.7 to 6.4. Assuming that the changing volatility has no effect on the floating exchange rate, remaining 262.9 HUF/EUR, the exchange rate depreciated to 273.1 HUF/EUR.

Summarizing the decomposition: 1% depreciation can be explained by the direct effect, a further 3% exchange rate fall is due to the changing expectations, and another 3% depreciation is due the growing volatility. The latter two can only be measured by using information available after the band shift, but the direct effect can be estimated contemporaneously. Consequently the competent authorities, which decided to shift the band, could have a much clearer picture about the direct effect of the band shift than about the further effects.

The total effect of the band shift on the exchange rate is almost 7% according to the model, which is close to the maximum observed depreciation. If we keep in mind that the volatility of the exchange rate measured by the implied volatility of the currency options, has risen only provisionally, then only the first two effects

\[ 23 \] It is not surprising that the direct effect of a 2.26% band shift is barely 1%: see footnote 7. about the size of the direct effect.
determine the total long-run effect, which is about 4%. This is almost equal to the observed average depreciation.

4.4 The band shift of the French franc in 1986

The French franc weakened approximately 3.4% after its band shift. The franc should have weakened at least to this magnitude, because after the weakening the franc was at the strong edge of its new band. (See figure 5/d.) The exchange rate was 3.08 FRF/DEM before the realignment and it was 3.18 FRF/DEM afterwards.

The euro was born some 13 years after the analysed band shift, so it seems plausible to assume that neither the final conversion rate, nor the expectations concerning it had an effect on the process of the floating exchange rate. However, we set the expiration date of the options equal to 13 years. The annualised volatility characterizing the period before the band shift was 2.4%. In order to have 2.4% volatility for the exchange rate at 3.08 FRF/DEM the volatility of the floating exchange rate needs to be set at 4.5%. Thus, we arrive at a process for the exchange rate where volatility equals the observed volatility at the observed exchange rate before the realignment. After the realignment the franc stuck to the edge of its band (see figure 4/d) and its volatility almost vanished, making it impossible to draw inferences about the volatility of the floating exchange rate characterizing the post-realignment period. In order to analyse the direct effect of the band shift we do not need to know anything about the volatility after the realignment. But the indirect effect, the effect of the possibly changing volatility, cannot be calculated without a reasonable estimate of the post-realignment volatility.

Figure 5/d. shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its graph 0 demonstrates the relationship before the band shift. Since the exchange rate before the band widening was 3.08 FRF/DEM, the floating exchange rate should have been 3.11 FRF/DEM according to graph 0. Graph 1 deviates from graph 0 mainly because the strike prices of the options were changed and partly because the French and German short-term interest rates were changed as well. According to graph 1 the exchange rate should have to weaken to 3.21 FRF/DEM at the unchanged floating exchange rate. All in all, the model attributes 4.2% weakening to the direct effect of the realignment, consequently it overestimates slightly the true effect. If the unobservable volatility of the floating exchange rate decreased after the

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24 Here we applied the process (3a, 3b).

25 We calculated the volatility characterising the period before the band widening and the period after the realignment from 15 daily data.

26 In contrast to the above studied forint and escudo, it is not possible to analyse the other indirect effect, the effect of the changing final conversion rate, because here final fixing has no role at all.
realignment then the indirect effect of the realignment would strengthen the exchange rate. Thus, it is possible that also taking into account the indirect effect would modify our result toward the lower historical depreciation, which was about 3.4%.

4.5 The band shift of the Danish krone in 1979

In 1979 the exchange rate of the Danish krone fell about 3.6% from 2.98 DKK/DEM to 3.09 DKK/DEM as an effect of the 5% devaluation of the band. It is documented that the devaluation of the band was unanticipated,27 similarly to the Hungarian realignment. This is good for the model, because it is applicable exclusively for credible target zone systems with no possibility of realignments. Obviously this no realignment assumption could not be perfectly fulfilled, since the band was rearranged between 1979 and 1999 six times. But these further realignments might not have a significant effect on the exchange rate at the analysed realignment.

Studying the Danish krone in 1979 well before the Monetary Union, it would not be plausible to assume that the process of the floating exchange rate and the process of the exchange of the Danish krone are driven by convergence to the final conversion rate. We applied the same process (3a, 3b) as in the case of the analysed French band shift in 1986. However, since it was not foreseeable in 1979 that the Danish krone would not join the Euro-area in 1999, the expiration date of the options is set equal to 20 years.

The annualised volatility28 characterizing both the periods before and after the band shift was about 4%. In order to have 4% volatility for the exchange rate at 2.98 DKK/DEM exchange rate the volatility of the floating exchange rate needs to be set 4% as well. This is not surprising because the exchange rate was near to the central parity, where the band has the smallest stabilizing effect on the exchange rate. The unchanged volatility implies that the band shift had no effect on the exchange rate through the changing uncertainty. However, it is still worth calculating the direct effect.

Figure 5/e. shows the relationships between the floating exchange rate and the exchange rate in the target zone. Its graph 0 demonstrates the relationship before the band shift, whereas graph 1 demonstrates the relationship after the band shift. Since the exchange rate before the band shift was 2.98 DKK/DEM, the floating exchange rate should have been 3.02 FRF/DEM according to graph 0. To the assumed unchanged floating exchange rate graph 1 assigns an exchange rate of 3.07 DKK/DEM. Consequently the model attributes 3% weakening to the direct

27 The decision about the band shift was not made at a usual meeting of the Council of Ministers, but the Danish request was approved de facto by telephone. See: Gros–Thygesen [1992] p74.

28 We calculated the volatility characterising the period before the band widening and the period after the realignment from 15 daily data.
effect of the realignment, which differs from the observed weakening (3.6%) only marginally. The accuracy of the model is very attractive, but it should be evaluated in the light of the fact that the band was relatively narrow at a width of ±2.25% at the time and the exchange rate was close to the central parity. (See figure 4/b). If the exchange rate was exactly at the central parity before the realignment, then a band shift of 5% should depreciate the exchange rate at least of 2.25%, but not more than 7.25% due to the limiting effect of the new band.

5. Critique of the model

The model outlined here can meet various objections both from theoretical and practical points of view. Here we will discuss the drawbacks of assuming a totally credible target zone system with intervention exclusively at the edges of the band, the exogenous nature of the interest rates and the definition of the floating exchange rate.

The model introduced in the paper describes the target zone exchange rate as a combination of a floating exchange rate and two options. The floating exchange rate would be the true exchange rate if the exchange rate system would not be a target zone system but a floating system, and all the real variables and interest rates remained unchanged. This ceteris paribus way of thinking is not appropriate, however, if either the target zone system influences the real variables by its very existence, or, if it demands a special interest rate policy.

As the target zone system reduces the volatility of the exchange rate and as it also moderates the uncertainty in the economy, it may have a favourable effect on real variables. Stockman [1999] challenges this assumption. Baldwin-Krugman [1989] points out another connection between the real variables and the exchange rate system. The authors of this work think that excessive exchange rate shocks, which do not occur in a target zone system, have a persistent effect on the trade and the equilibrium exchange rate.

29 Stockman argues that for most of the countries the floating exchange rate system is favourable. Although Stockman admits that the uncertainty could cause real effects, but the macro indicators in the last decades support the opposite. This fact can be explained by the improvement of the financial markets. The risk can be easily eliminated in an improved market by different hedging opportunities. Consequently, the real effects are significant only if the agents cannot hedge at a low price or if the practice of hedging is not widespread.

Albeit for the total elimination of the risk it would be also necessary to have the opportunity to hedge for whatever long term.

30 Their reasoning is based on the fact that the entry and withdrawal of companies to/from the market and the beginning/halting of export activities depend on excessive changes in the exchange rates. Due to an overestimation of the domestic currency following an economic shock, foreign companies appear in the domestic market, and later, when the exchange rate returns to the original level, they will not withdraw from the market. Therefore the trade balance along with the equilibrium exchange rate permanently changes.
In order to maintain the target zone exchange rate system, a central bank does not exclusively have the opportunity to intervene in the foreign exchange market but it can influence the exchange rate through its interest rate policy. Consequently, interest rates have a different role in a target zone system than in a floating system.

The above reasons compel us to take into consideration the endogenous nature of the interest rate policy and of some real variables when determining the process of the exchange rate in the options-based model. But we should be cautious as well when applying models other than the options-based model, and determining the process of the fundamental, because similar problems might show up there. It seems hardly possible to take into account these feedbacks of the exchange rate system concerning the main variables such as interest rates, and complete with them the models.

One of the assumed processes of the floating exchange rate is mainly driven by the convergence to the expected final conversion rate, and the other is assumed to be a random walk without drift. With both cases interest rates have only a marginal effect on the exchange rate through the options prices. In order to have significant effect, it might be plausible to assume that the often applied uncovered interest rate parity\(^{31}\) holds either with the floating or the true exchange rate. The uncovered interest rate parity can be taken into account by a drift term, which depends on the interest rate differential. Although the fact is that either the floating or the true exchange rate with a time-invariant drift would result in the exchange rate arriving sooner or later to one of its limits. This could be avoided by making the interest rate differential endogenous or assuming the possibility of band realignments. But by allowing the possibility for band realignments the final conversion rate becomes endogenous. One drawback of an endogenously determined interest rate differential could be that it differs from its observed value. Similarly, an endogenously given final conversion rate might be very different from its expected value reported, for example, by the Reuters Polls or from the historical final conversion rate in the case of the euro currencies. Simplifying the problem, here we have not dealt with drift and a non-credible exchange rate system. As a consequence, even a dramatic change in the interest rate differential has only marginal effect on the exchange rate. Therefore, the effect of the interest rate cannot be analysed by this specification of the model and this drawback might be a

\[^{31}\text{Assuming the uncovered interest rate parity (UIP) on the floating exchange rate, a higher interest rate differential (}r_q\text{)}\]

\[^{\uparrow}\text{ would strengthen the exchange rate at an unchanged floating exchange rate. This is due to the fact that the floating exchange rate will have a weakening trend in accordance with the higher interest rate differential and the UIP. Meanwhile, the put option becomes more valuable and the potential loss represented by the call option decreases. At the same time, the expected future value of the major constituent of their underlying assets decreases. All in all, at an unchanged current value of the floating exchange rate a rise in the interest rate differential strengthens the current exchange rate. This relationship between the interest rate and the exchange rate results in the model becoming more realistic, since in practice an interest rate hike has a strengthening effect on the exchange rate in the short run.}\]
partial cause of the deviation of the model-based depreciations from the observed ones.

As to the assumed perfect credibility of the target zone system, we should note that in a realistic target zone system with the possibility of band realignments the exchange rate moves toward the direction of the band shift prior to the realignment, provided that the band shift is expected. By assuming a totally credible target zone with unexpected band shift the model overestimates the effect of the band realignment on the exchange rate in a real target zone system. This overestimation can only be avoided by complementing the model with the possibility of band realignments and by modelling the expectations for realignments. Among the analysed band realignments the Danish and Hungarian band shifts are likely to be unanticipated. With these cases the historical depreciation and the one based on the model were relatively close to each other, which supports the existence of an expectation bias in our estimates.

Our assumption that the central bank intervenes only in the case of the exchange rate already reaching the limit of the band might contradict the practice, especially when the central bank prefers more exchange rate stability than is declared by the official wider band. Most of the exchange rates in the EMS after the band widening in 1993 could fluctuate in a ±15% range, although they were restricted into a narrower band. Since the interventions inside the band and the implicit bands are confidential, it is difficult to analyse the exchange rate in such systems.

6. Conclusions

In this paper we developed an options-based model of target zone arrangements.

The basic idea of the model was mentioned by Krugman [1991]: the exchange rate in a target zone system is equivalent to the exchange rate of a currency in an underlying freely floating system adjusted by the price of two options. Here we have discussed the topic in detail; we have described the type of options and have determined the underlying assets of the options accurately. These are American-type options with strike price equal to the edges of the band. The underlying asset of each option is the floating exchange rate combined with the other option. An option pricing method applicable for these special options was developed in the binomial framework.

The pricing of the options constituting the currency in a target zone system enabled us to derive the process of the exchange rate from an assumed process of the floating exchange rate. Moreover, option pricing yields a relationship between the spot value of the exchange rate and the spot value of the floating exchange rate. Under certain specifications of the process of the floating exchange rate this relationship depends on the band, on the expected final conversion rate and on the volatility of the exchange rate. A band realignment not only changes the band, it can also affect the expectations relating to the final conversion rate and the volatility of the exchange rate, enabling us to decompose the effect of a band
realignment on the exchange rate into (a) the direct effect; (b) changing expectations; and (c) changing uncertainty.

By assuming a process, characteristic of the floating exchange rate before a foreseeable fixing of an exchange rate, we analysed the following band realignments: French band widening in 1993, Portuguese band shift in 1995, and the recent Hungarian band shift in 2003. The band shift of the Danish krone in 1979 and of the French franc in 1986 was also studied, but a different process of the floating exchange rate was assumed, since the final fixing might have only minor relevance at the time of the band shifts.

We conclude that the depreciation of the French franc after the band widening and after the studied band shift can be attributed mainly to the direct effect of the realignment. The same holds for the Danish krone, whereas the Portuguese escudo was affected mainly by changing expectations. The Hungarian forint depreciated partly due to the changing expectations relating to the final conversion rate and partly due to the growing uncertainty, while the direct effect was minor. (Table 1.)

Table 1. Depreciation after target zone realignments and its decomposition with the options-based model

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<tr>
<td></td>
<td>Band widening from +2.25% to -15%</td>
<td>Band shift of 3.6%</td>
<td>Band shift of 6.2%</td>
<td>Band shift of 5%</td>
<td>Band shift of 2.26%</td>
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<tr>
<td>Historical depreciation</td>
<td>2.7%</td>
<td>1%-2%</td>
<td>3.4%</td>
<td>3.6%</td>
<td>3.5%-6%</td>
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<tr>
<td>Total depreciation according to the model decomposition</td>
<td>4.6%</td>
<td>3.6%</td>
<td>4.2%</td>
<td>3%</td>
<td>6.6%</td>
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<td>Direct effect</td>
<td>4.3%</td>
<td>0%</td>
<td>4.2%</td>
<td>3%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Changing expectations</td>
<td>-</td>
<td>3.6%</td>
<td>-</td>
<td>-</td>
<td>2.6%</td>
</tr>
<tr>
<td>Changing volatility</td>
<td>0.3%</td>
<td>0%</td>
<td>-</td>
<td>0%</td>
<td>3.1%</td>
</tr>
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The depreciations calculated by the model deviate only slightly from the historical level in the case of the Danish krone and Hungarian forint, which might be attributed to the fact that in these two cases the realignments were surely unanticipated by the market. With the other three studied currencies the model overestimated the actual depreciation, which can be attributed either to the possibility that the realignments were not totally unexpected or to deficiencies of the model.
Figures

Figure 1. Discretized process of the floating exchange rate demonstrated on binomial tree

Figure 2. The forint/euro exchange rate and the expected symmetrical confidence interval of the exchange rate and of the floating exchange rate determined at the time of band shift – under the assumption: the hypothetical final conversion rate equals 248.4 HUF/EUR
Figure 3/a. Exchange rate of the French franc and its band in the EMS (13 March 1979–31 Dec 1998)

Figure 3/b. Exchange rate of the Portugal escudo and its band in the EMS (2 Jan 1985–31 Dec 1998)

Figure 3/c. Exchange rate of the Hungarian forint and its band (4 Jan 2000–30 Apr 2004)

Figure 3/d. Exchange rate of the Danish krone and its band in the EMS (13 March 1979–31 Dec 1998)
Figure 4/a. Exchange rate of the French frank and its band in the vicinity of the band widening (3 Aug 1993)

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Figure 4/d. Exchange rate of the French frank and its band in the vicinity of the 6.2% band shift (7 Apr 1986)

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Figure 5/b. Depreciation of the Portugal escudo after the band shift (6 March 1995)

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Figure 5/d. Depreciation of the French frank after the 6.2% band shift (7 Apr 1986)

Figure 5/e. Depreciation of the Danish krone after the 5% band shift (30 Nov 1979)
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